AREA OF POLYGONS AND CIRCLES

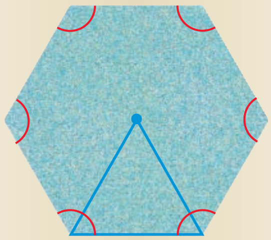
Where do hexagons occur in nature?

CHAPTER 11

APPLICATION: Area of Columns

Basaltic columns are geological formations that result from rapidly cooling lava.

Most basaltic columns are hexagonal, or six sided. The Giant's Causeway in Ireland, pictured here, features hexagonal columns ranging in size from 15 to 20 inches across and up to 82 feet high.



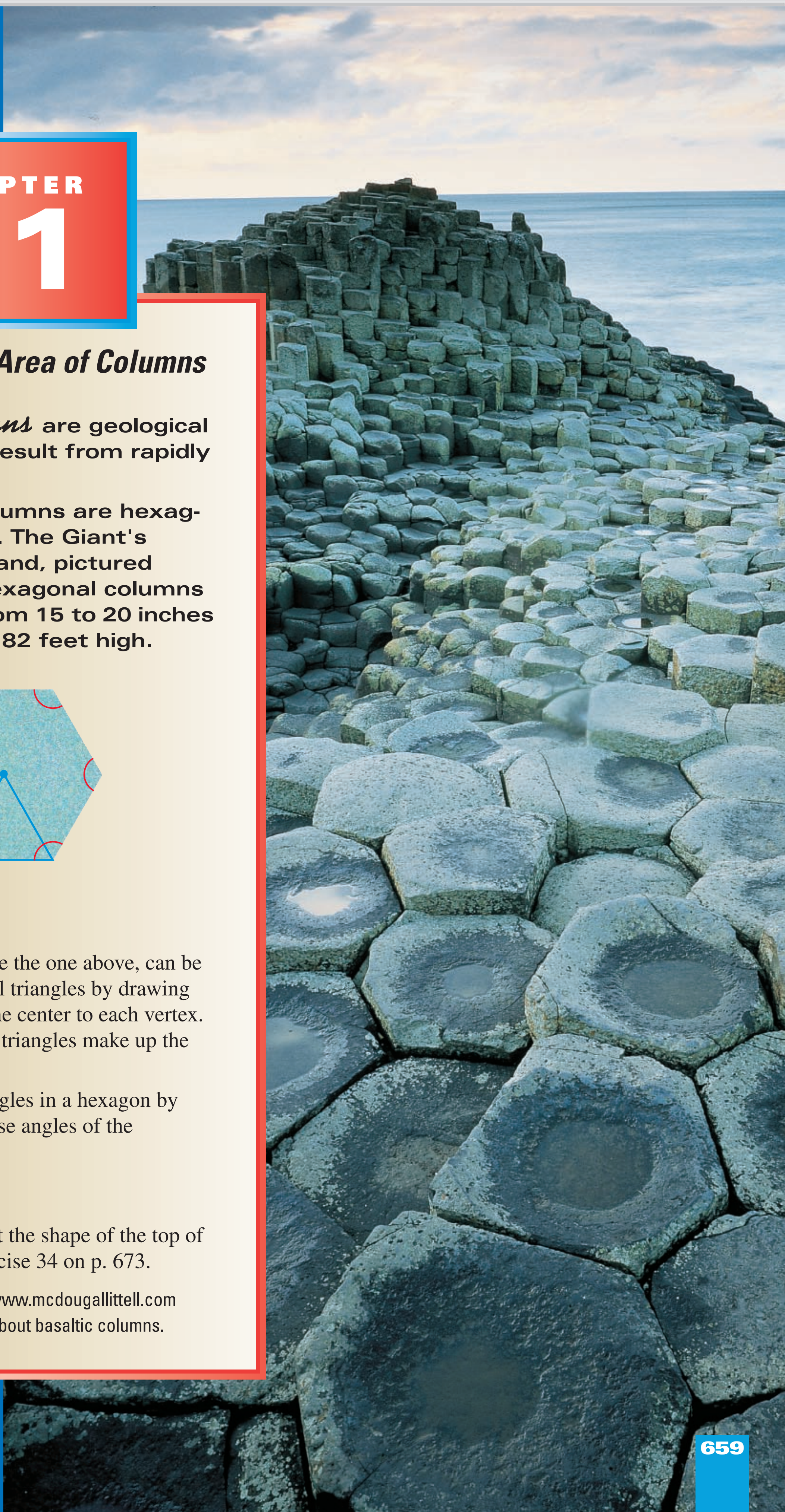
Think & Discuss

1. A regular hexagon, like the one above, can be divided into equilateral triangles by drawing segments to connect the center to each vertex. How many equilateral triangles make up the hexagon?
2. Find the sum of the angles in a hexagon by adding together the base angles of the equilateral triangles.

Learn More About It

You will learn more about the shape of the top of a basaltic column in Exercise 34 on p. 673.

INTERNET APPLICATION LINK Visit www.mcdougallittell.com for more information about basaltic columns.



CHAPTER 11

Study Guide

PREVIEW

What's the chapter about?

Chapter 11 is about **areas of polygons and circles**. In Chapter 11, you'll learn

- how to find angle measures and areas of polygons.
- how to compare perimeters and areas of similar figures.
- how to find the circumference and area of a circle and to find other measures related to circles.

KEY VOCABULARY

Review

- polygon, p. 322
- *n*-gon, p. 322
- convex polygon, p. 323
- regular polygon, p. 323
- similar polygons, p. 473
- trigonometric ratio, p. 558
- circle, p. 595
- center of a circle, p. 595
- radius of a circle, p. 595
- measure of an arc, p. 603

New

- apothem of a polygon, p. 670
- central angle of a regular polygon, p. 671
- circumference, p. 683
- arc length, p. 683
- sector of a circle, p. 692
- probability, p. 699
- geometric probability, p. 699

PREPARE

Are you ready for the chapter?

SKILL REVIEW Do these exercises to review key skills that you'll apply in this chapter. See the given **reference page** if there is something you don't understand.

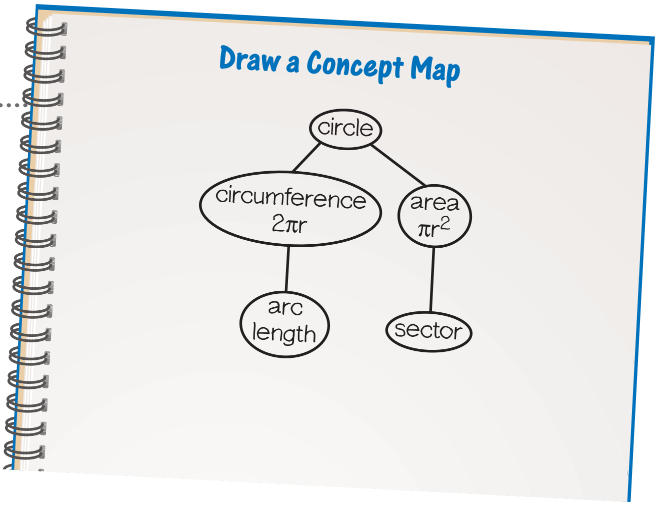
1. Find the area of a triangle with height 8 in. and base 12 in. (Review p. 51)
2. In $\triangle ABC$, $m\angle A = 57°$ and $m\angle C = 79°$. Find the measure of $\angle B$ and the measure of an exterior angle at each vertex. (Review pp. 196–197)
3. If $\triangle DEF \sim \triangle XYZ$, $DF = 8$, and $XZ = 12$, find each ratio.

   a. $\dfrac{XY}{DE}$ **b.** $\dfrac{\text{Perimeter of } \triangle DEF}{\text{Perimeter of } \triangle XYZ}$ (Review pp. 475, 480)
4. A right triangle has sides of length 20, 21, and 29. Find the measures of the acute angles of the triangle to the nearest tenth. (Review pp. 567–568)

STUDY STRATEGY

Here's a study strategy!

A *concept map* is a diagram that highlights the connections between ideas. Drawing a concept map for a chapter can help you focus on the important ideas and on how they are related.



11.1

Angle Measures in Polygons

What *you should learn*

GOAL 1 Find the measures of interior and exterior angles of polygons.

GOAL 2 Use measures of angles of polygons to solve **real-life** problems.

Why *you should learn it*

▼ To solve **real-life** problems, such as finding the measures of the interior angles of a home plate marker of a softball field in **Example 4**.

REAL LIFE

GOAL 1 MEASURES OF INTERIOR AND EXTERIOR ANGLES

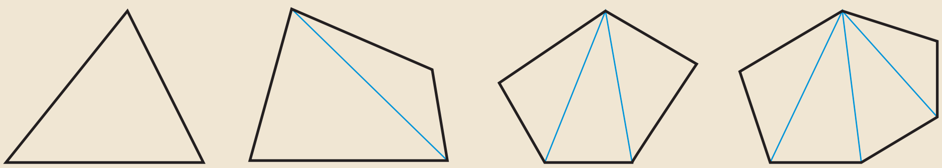
You have already learned that the name of a polygon depends on the number of sides in the polygon: triangle, quadrilateral, pentagon, hexagon, and so forth. The sum of the measures of the interior angles of a polygon also depends on the number of sides.

In Lesson 6.1, you found the sum of the measures of the interior angles of a quadrilateral by dividing the quadrilateral into two triangles. You can use this triangle method to find the sum of the measures of the interior angles of any convex polygon with n sides, called an n-gon.

ACTIVITY Developing Concepts

Investigating the Sum of Polygon Angle Measures

Draw examples of 3-sided, 4-sided, 5-sided, and 6-sided convex polygons. In each polygon, draw all the diagonals from one vertex. Notice that this divides each polygon into triangular regions.



Triangle **Quadrilateral** **Pentagon** **Hexagon**

Complete the table below. What is the pattern in the sum of the measures of the interior angles of any convex n-gon?

Polygon	Number of sides	Number of triangles	Sum of measures of interior angles
Triangle	3	1	$1 \cdot 180° = 180°$
Quadrilateral	?	?	$2 \cdot 180° = 360°$
Pentagon	?	?	?
Hexagon	?	?	?
⋮	⋮	⋮	⋮
n-gon	n	?	?

STUDENT HELP

Look Back
For help with *regular* polygons, see p. 323.

You may have found in the activity that the sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$. This relationship can be used to find the measure of each interior angle in a *regular* n-gon, because the angles are all congruent. Exercises 43 and 44 ask you to write proofs of the following results.

THEOREMS ABOUT INTERIOR ANGLES

THEOREM 11.1 ***Polygon Interior Angles Theorem***

The sum of the measures of the interior angles of a convex n-gon is $(n - 2) \cdot 180°$.

COROLLARY TO THEOREM 11.1

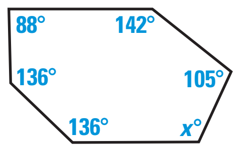
The measure of each interior angle of a regular n-gon is

$$\frac{1}{n} \cdot (n - 2) \cdot 180°, \text{ or } \frac{(n-2) \cdot 180°}{n}.$$

EXAMPLE 1 *Finding Measures of Interior Angles of Polygons*

Using Algebra

Find the value of x in the diagram shown.



SOLUTION

The sum of the measures of the interior angles of any hexagon is $(6 - 2) \cdot 180° = 4 \cdot 180° = 720°$.

Add the measures of the interior angles of the hexagon.

$136° + 136° + 88° + 142° + 105° + x° = 720°$	**The sum is 720°.**
$607 + x = 720$	**Simplify.**
$x = 113$	**Subtract 607 from each side.**

▶ The measure of the sixth interior angle of the hexagon is 113°.

EXAMPLE 2 *Finding the Number of Sides of a Polygon*

The measure of each interior angle of a regular polygon is 140°. How many sides does the polygon have?

SOLUTION

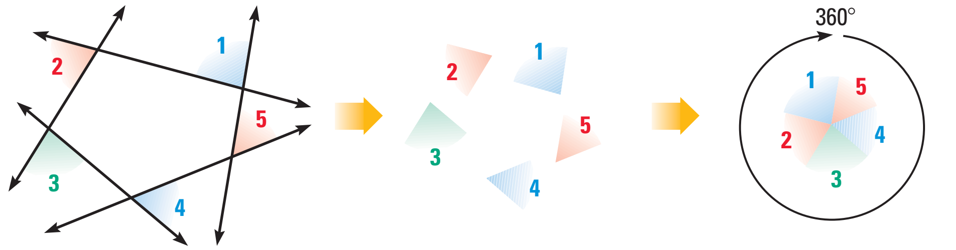
$\frac{1}{n} \cdot (n - 2) \cdot 180° = 140°$	**Corollary to Theorem 11.1**
$(n - 2) \cdot 180 = 140n$	**Multiply each side by *n*.**
$180n - 360 = 140n$	**Distributive property**
$40n = 360$	**Addition and subtraction properties of equality**
$n = 9$	**Divide each side by 40.**

▶ The polygon has 9 sides. It is a regular nonagon.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

The diagrams below show that the sum of the measures of the *exterior* angles of any convex polygon is 360°. You can also find the measure of each exterior angle of a *regular* polygon. Exercises 45 and 46 ask for proofs of these results.

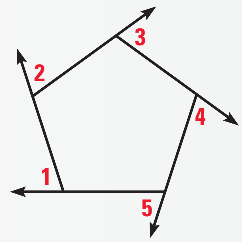


1. Shade one exterior angle at each vertex.
2. Cut out the exterior angles.
3. Arrange the exterior angles to form 360°.

THEOREMS ABOUT EXTERIOR ANGLES

THEOREM 11.2 ***Polygon Exterior Angles Theorem***

The sum of the measures of the exterior angles of a convex polygon, one angle at each vertex, is 360°.



COROLLARY TO THEOREM 11.2

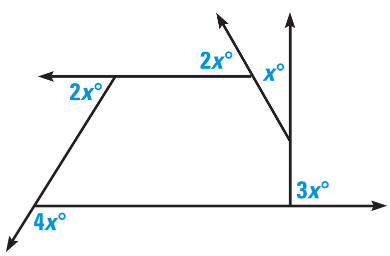
The measure of each exterior angle of a regular n-gon is $\frac{1}{n} \cdot 360°$, or $\frac{360°}{n}$.

Using Algebra

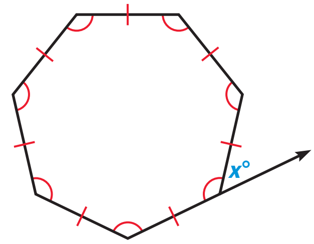
EXAMPLE 3 *Finding the Measure of an Exterior Angle*

Find the value of x in each diagram.

a.



b.



SOLUTION

a. $2x° + x° + 3x° + 4x° + 2x° = 360°$ **Use Theorem 11.2.**

$12x = 360$ **Combine like terms.**

$x = 30$ **Divide each side by 12.**

b. $x° = \frac{1}{7} \cdot 360°$ **Use $n = 7$ in the Corollary to Theorem 11.2.**

≈ 51.4 **Use a calculator.**

▶ The measure of each exterior angle of a regular heptagon is about 51.4°.

GOAL 2 USING ANGLE MEASURES IN REAL LIFE

You can use Theorems 11.1 and 11.2 and their corollaries to find angle measures.

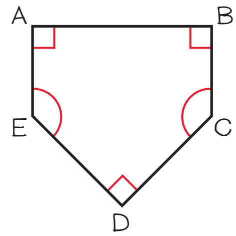
EXAMPLE 4 *Finding Angle Measures of a Polygon*

SOFTBALL A home plate marker for a softball field is a pentagon. Three of the interior angles of the pentagon are right angles. The remaining two interior angles are congruent. What is the measure of each angle?

SOLUTION

PROBLEM SOLVING STRATEGY

DRAW A SKETCH Sketch and label a diagram for the home plate marker. It is a nonregular pentagon. The right angles are $\angle A$, $\angle B$, and $\angle D$. The remaining angles are congruent. So $\angle C \cong \angle E$. The sum of the measures of the interior angles of the pentagon is 540°.



VERBAL MODEL **Sum of measures of interior angles** = **3** • **Measure of each right angle** + **2** • **Measure of $\angle C$ and $\angle E$**

LABELS

Sum of measures of interior angles = **540** (degrees)

Measure of each right angle = **90** (degrees)

Measure of $\angle C$ and $\angle E$ = x (degrees)

REASONING

$540 = 3 \cdot 90 + 2x$ Write the equation.

$540 = 270 + 2x$ Simplify.

$270 = 2x$ Subtract 270 from each side.

$135 = x$ Divide each side by 2.

▶ So, the measure of each of the two congruent angles is 135°.

FOCUS ON PEOPLE

REAL LIFE **JOAN JOYCE** set a number of softball pitching records from 1956–1975. She delivered 40 pitches to slugger Ted Williams during an exhibition game in 1962. Williams only connected twice, for one foul ball and one base hit.

EXAMPLE 5 *Using Angle Measures of a Regular Polygon*

SPORTS EQUIPMENT If you were designing the home plate marker for some new type of ball game, would it be possible to make a home plate marker that is a regular polygon with each interior angle having a measure of (**a**) 135°? (**b**) 145°?

SOLUTION

a. Solve the equation $\frac{1}{n} \cdot (n - 2) \cdot 180° = 135°$ for n. You get $n = 8$.

▶ Yes, it would be possible. A polygon can have 8 sides.

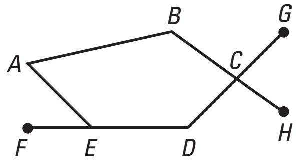
b. Solve the equation $\frac{1}{n} \cdot (n - 2) \cdot 180° = 145°$ for n. You get $n \approx 10.3$.

▶ No, it would not be possible. A polygon cannot have 10.3 sides.

GUIDED PRACTICE

Vocabulary Check ✓

1. Name an *interior angle* and an *exterior angle* of the polygon shown at the right.



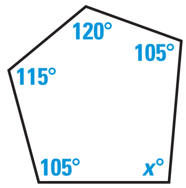
Concept Check ✓

2. How many exterior angles are there in an n-gon? Are they all considered when using the Polygon Exterior Angles Theorem? Explain.

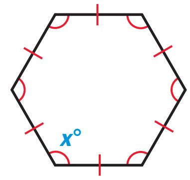
Skill Check ✓

Find the value of *x*.

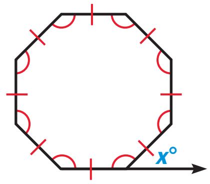
3.



4.



5.



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 823.

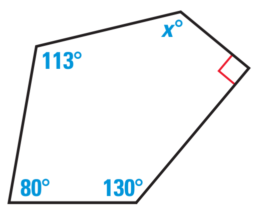
SUMS OF ANGLE MEASURES **Find the sum of the measures of the interior angles of the convex polygon.**

6. 10-gon **7.** 12-gon **8.** 15-gon **9.** 18-gon

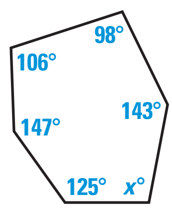
10. 20-gon **11.** 30-gon **12.** 40-gon **13.** 100-gon

ANGLE MEASURES **In Exercises 14–19, find the value of *x*.**

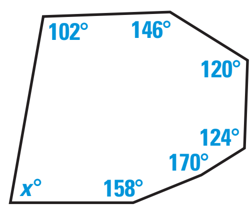
14.



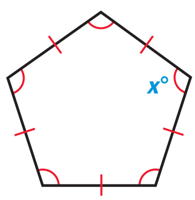
15.



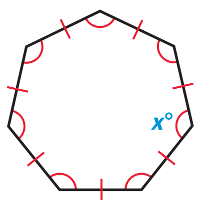
16.



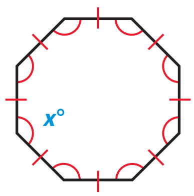
17.



18.



19.



20. A convex quadrilateral has interior angles that measure 80°, 110°, and 80°. What is the measure of the fourth interior angle?

21. A convex pentagon has interior angles that measure 60°, 80°, 120°, and 140°. What is the measure of the fifth interior angle?

DETERMINING NUMBER OF SIDES **In Exercises 22–25, you are given the measure of each interior angle of a regular *n*-gon. Find the value of *n*.**

22. 144° **23.** 120° **24.** 140° **25.** 157.5°

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 6–16, 20, 21
Example 2: Exs. 17–19, 22–28
Example 3: Exs. 29–38
Example 4: Exs. 39, 40, 49, 50
Example 5: Exs. 51–54

CONSTRUCTION **Use a compass, protractor, and ruler to check the results of Example 2 on page 662.**

26. Draw a large angle that measures 140°. Mark congruent lengths on the sides of the angle.

27. From the end of one of the congruent lengths in Exercise 26, draw the second side of another angle that measures 140°. Mark another congruent length along this new side.

28. Continue to draw angles that measure 140° until a polygon is formed. Verify that the polygon is regular and has 9 sides.

DETERMINING ANGLE MEASURES **In Exercises 29–32, you are given the number of sides of a regular polygon. Find the measure of each exterior angle.**

29. 12 **30.** 11 **31.** 21 **32.** 15

DETERMINING NUMBER OF SIDES **In Exercises 33–36, you are given the measure of each exterior angle of a regular *n*-gon. Find the value of *n*.**

33. 60° **34.** 20° **35.** 72° **36.** 10°

37. A convex hexagon has exterior angles that measure 48°, 52°, 55°, 62°, and 68°. What is the measure of the exterior angle of the sixth vertex?

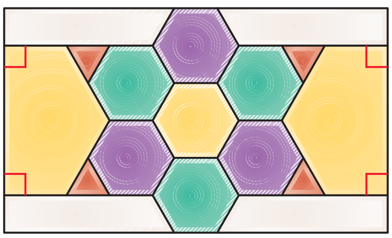
38. What is the measure of each exterior angle of a regular decagon?

FOCUS ON APPLICATIONS

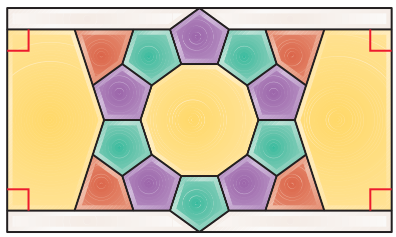
REAL LIFE **STAINED GLASS** is tinted glass that has been cut into shapes and arranged to form a picture or design. The pieces of glass are held in place by strips of lead.

STAINED GLASS WINDOWS **In Exercises 39 and 40, the purple and green pieces of glass are in the shape of regular polygons. Find the measure of each interior angle of the red and yellow pieces of glass.**

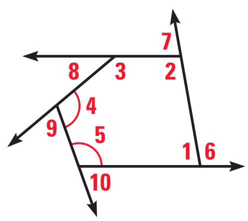
39.



40.

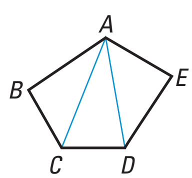


41. FINDING MEASURES OF ANGLES In the diagram at the right, $m\angle 2 = 100°$, $m\angle 8 = 40°$, $m\angle 4 = m\angle 5 = 110°$. Find the measures of the other labeled angles and explain your reasoning.



42. *Writing* Explain why the sum of the measures of the interior angles of any two *n*-gons with the same number of sides (two octagons, for example) is the same. Do the *n*-gons need to be regular? Do they need to be similar?

43. **PROOF** Use *ABCDE* to write a paragraph proof to prove Theorem 11.1 for pentagons.



44. **PROOF** Use a paragraph proof to prove the Corollary to Theorem 11.1.

45. **PROOF** Use this plan to write a paragraph proof of Theorem 11.2.
Plan for Proof In a convex n-gon, the sum of the measures of an interior angle and an adjacent exterior angle at any vertex is 180°. Multiply by n to get the sum of all such sums at each vertex. Then subtract the sum of the interior angles derived by using Theorem 11.1.

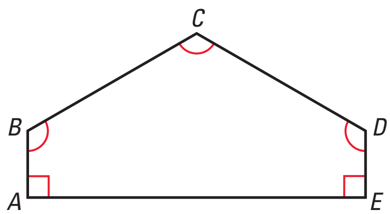
46. **PROOF** Use a paragraph proof to prove the Corollary to Theorem 11.2.

TECHNOLOGY In Exercises 47 and 48, use geometry software to construct a polygon. At each vertex, extend one of the sides of the polygon to form an exterior angle.

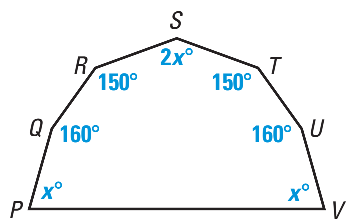
47. Measure each exterior angle and verify that the sum of the measures is 360°.

48. Move any vertex to change the shape of your polygon. What happens to the measures of the exterior angles? What happens to their sum?

49. **HOUSES** Pentagon *ABCDE* is an outline of the front of a house. Find the measure of each angle.



50. **TENTS** Heptagon *PQRSTUV* is an outline of a camping tent. Find the unknown angle measures.

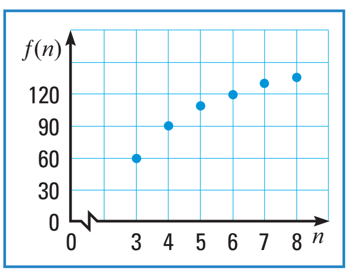


POSSIBLE POLYGONS Would it be possible for a regular polygon to have interior angles with the angle measure described? Explain.

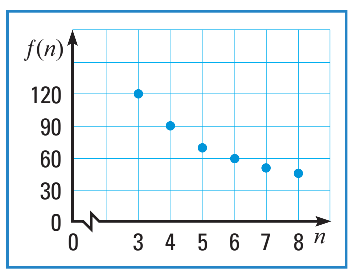
51. 150° **52.** 90° **53.** 72° **54.** 18°

USING ALGEBRA In Exercises 55 and 56, you are given a function and its graph. In each function, n is the number of sides of a polygon and $f(n)$ is measured in degrees. How does the function relate to polygons? What happens to the value of $f(n)$ as n gets larger and larger?

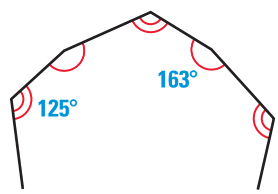
55. $f(n) = \dfrac{180n - 360}{n}$



56. $f(n) = \dfrac{360}{n}$



57. **LOGICAL REASONING** You are shown part of a convex n-gon. The pattern of congruent angles continues around the polygon. Use the Polygon Exterior Angles Theorem to find the value of n.



Test Preparation

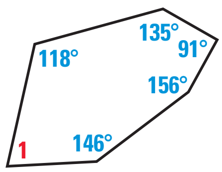
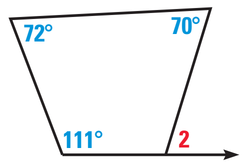
QUANTITATIVE COMPARISON **In Exercises 58–61, choose the statement that is true about the given quantities.**

Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the given information.

	Column A	Column B
58.	The sum of the interior angle measures of a decagon	The sum of the interior angle measures of a 15-gon
59.	The sum of the exterior angle measures of an octagon	$8(45°)$
60.	$m\angle 1$ 	$m\angle 2$ 
61.	Number of sides of a polygon with an exterior angle measuring 72°	Number of sides of a polygon with an exterior angle measuring 144°

★ **Challenge**

62. Polygon $STUVWXYZ$ is a regular octagon. Suppose sides $\overline{ST}$ and $\overline{UV}$ are extended to meet at a point R. Find the measure of $\angle TRU$.

MIXED REVIEW

FINDING AREA **Find the area of the triangle described.** **(Review 1.7 for 11.2)**

63. base: 11 inches; height: 5 inches

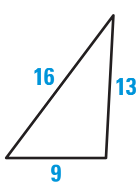
64. base: 43 meters; height: 11 meters

65. vertices: $A(2, 0)$, $B(7, 0)$, $C(5, 15)$

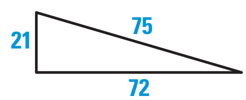
66. vertices: $D(-3, 3)$, $E(3, 3)$, $F(-7, 11)$

VERIFYING RIGHT TRIANGLES **Tell whether the triangle is a right triangle.** **(Review 9.3)**

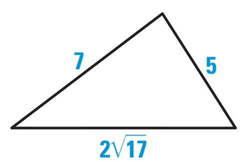
67.



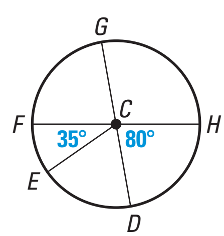
68.



69.



FINDING MEASUREMENTS **$\overline{GD}$ and $\overline{FH}$ are diameters of circle C. Find the indicated arc measure.** **(Review 10.2)**



70. $m\widehat{DH}$

71. $m\widehat{ED}$

72. $m\widehat{EH}$

73. $m\widehat{EHG}$

11.2 Areas of Regular Polygons

What *you should learn*

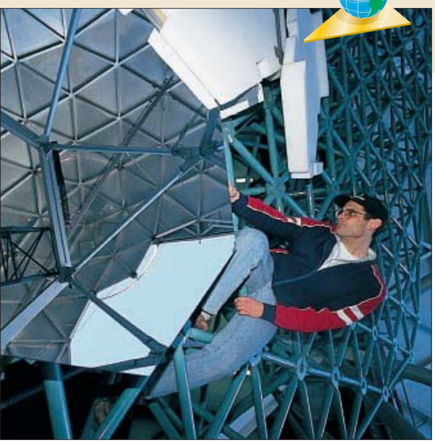
GOAL 1 Find the area of an equilateral triangle.

GOAL 2 Find the area of a regular polygon, such as the area of a dodecagon in **Example 4**.

Why *you should learn it*

▼ To solve **real-life** problems, such as finding the area of a hexagonal mirror on the Hobby-Eberly Telescope in **Exs. 45 and 46**.

REAL LIFE



GOAL 1 FINDING THE AREA OF AN EQUILATERAL TRIANGLE

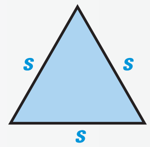
The area of *any* triangle with base length b and height h is given by $A = \frac{1}{2}bh$. The following formula for equilateral triangles, however, uses only the side length.

THEOREM

THEOREM 11.3 ***Area of an Equilateral Triangle***

The area of an equilateral triangle is one fourth the square of the length of the side times $\sqrt{3}$.

$$A = \frac{1}{4}\sqrt{3}s^2$$



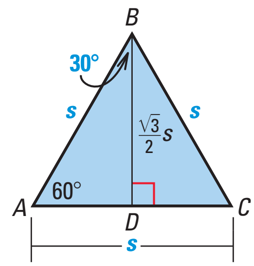
EXAMPLE 1 ***Proof of Theorem 11.3***

Prove Theorem 11.3. Refer to the figure below.

SOLUTION

GIVEN ▶ $\triangle ABC$ is equilateral.

PROVE ▶ Area of $\triangle ABC$ is $A = \frac{1}{4}\sqrt{3}s^2$.



Paragraph Proof Draw the altitude from B to side $\overline{AC}$. Then $\triangle ABD$ is a 30°-60°-90° triangle. From Lesson 9.4, the length of $\overline{BD}$, the side opposite the 60° angle in $\triangle ABD$, is $\frac{\sqrt{3}}{2}s$. Using the formula for the area of a triangle,

$$A = \frac{1}{2}bh = \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right) = \frac{1}{4}\sqrt{3}s^2.$$

EXAMPLE 2 ***Finding the Area of an Equilateral Triangle***

Find the area of an equilateral triangle with 8 inch sides.

SOLUTION

Use $s = 8$ in the formula from Theorem 11.3.

$$A = \frac{1}{4}\sqrt{3}s^2 = \frac{1}{4}\sqrt{3}(8^2) = \frac{1}{4}\sqrt{3}(64) = \frac{1}{4}(64)\sqrt{3} = 16\sqrt{3} \text{ square inches}$$

▶ Using a calculator, the area is about 27.7 square inches.

STUDENT HELP

Study Tip

Be careful with radical signs. Notice in Example 1 that $\sqrt{3}s^2$ and $\sqrt{3s^2}$ do not mean the same thing.

GOAL 2 FINDING THE AREA OF A REGULAR POLYGON

You can use equilateral triangles to find the area of a regular hexagon.

ACTIVITY
Developing Concepts

Investigating the Area of a Regular Hexagon

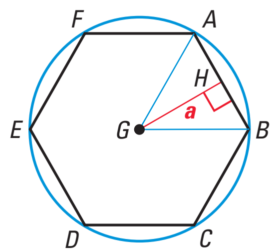
Use a protractor and ruler to draw a regular hexagon. Cut out your hexagon. Fold and draw the three lines through opposite vertices. The point where these lines intersect is the *center* of the hexagon.

1. How many triangles are formed? What kind of triangles are they?
2. Measure a side of the hexagon. Find the area of one of the triangles. What is the area of the entire hexagon? Explain your reasoning.

Think of the hexagon in the activity above, or another regular polygon, as inscribed in a circle.

The **center of the polygon** and **radius of the polygon** are the center and radius of its circumscribed circle, respectively.

The distance from the center to any side of the polygon is called the **apothem of the polygon.** The apothem is the height of a triangle between the center and two consecutive vertices of the polygon.



Hexagon *ABCDEF* with center *G*, radius *GA*, and apothem *GH*

STUDENT HELP

Study Tip
In a regular polygon, the length of each side is the same. If this length is s and there are n sides, then the perimeter P of the polygon is $n \cdot s$, or $P = ns$.

As in the activity, you can find the area of any regular n-gon by dividing the polygon into congruent triangles.

$$A = \boxed{\text{area of one triangle}} \cdot \boxed{\text{number of triangles}}$$

$$= \left(\frac{1}{2} \cdot \text{apothem} \cdot \text{side length } s\right) \cdot \text{number of sides}$$

$$= \frac{1}{2} \cdot \text{apothem} \cdot \text{number of sides} \cdot \text{side length } s$$

$$= \frac{1}{2} \cdot \text{apothem} \cdot \text{perimeter of polygon}$$

The number of congruent triangles formed will be the same as the number of sides of the polygon.

This approach can be used to find the area of any regular polygon.

THEOREM

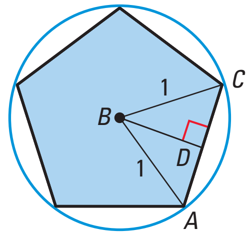
THEOREM 11.4 ***Area of a Regular Polygon***

The area of a regular n-gon with side length s is half the product of the apothem a and the perimeter P, so $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$.

A **central angle of a regular polygon** is an angle whose vertex is the center and whose sides contain two consecutive vertices of the polygon. You can divide 360° by the number of sides to find the measure of each central angle of the polygon.

EXAMPLE 3 *Finding the Area of a Regular Polygon*

A regular pentagon is inscribed in a circle with radius 1 unit. Find the area of the pentagon.



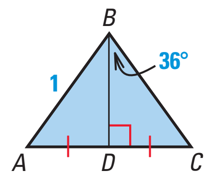
SOLUTION

To apply the formula for the area of a regular pentagon, you must find its apothem and perimeter.

The measure of central $\angle ABC$ is $\frac{1}{5} \cdot 360°$, or $72°$.

In isosceles triangle $\triangle ABC$, the altitude to base $\overline{AC}$ also bisects $\angle ABC$ and side $\overline{AC}$. The measure of $\angle DBC$, then, is 36°. In right triangle $\triangle BDC$, you can use trigonometric ratios to find the lengths of the legs.

$$\cos 36° = \frac{BD}{BC} = \frac{BD}{1} = BD \qquad \sin 36° = \frac{DC}{BC} = \frac{DC}{1} = DC$$



▶ So, the pentagon has an apothem of $a = BD = \cos 36°$ and a perimeter of $P = 5(AC) = 5(2 \cdot DC) = 10 \sin 36°$. The area of the pentagon is

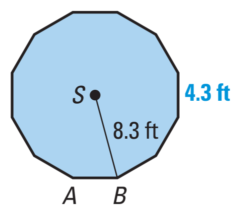
$$A = \frac{1}{2}aP = \frac{1}{2}(\cos 36°)(10 \sin 36°) \approx 2.38 \text{ square units.}$$

STUDENT HELP

Look Back
For help with trigonometric ratios, see p. 558.

EXAMPLE 4 *Finding the Area of a Regular Dodecagon*

PENDULUMS The enclosure on the floor underneath the Foucault Pendulum at the Houston Museum of Natural Sciences in Houston, Texas, is a regular dodecagon with a side length of about 4.3 feet and a radius of about 8.3 feet. What is the floor area of the enclosure?



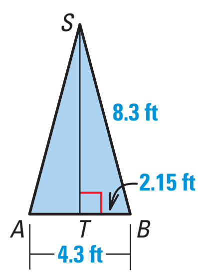
SOLUTION

A dodecagon has 12 sides. So, the perimeter of the enclosure is

$$P \approx 12(4.3) = 51.6 \text{ feet.}$$

In $\triangle SBT$, $BT = \frac{1}{2}(BA) = \frac{1}{2}(4.3) = 2.15$ feet. Use the Pythagorean Theorem to find the apothem ST.

$$a = \sqrt{8.3^2 - 2.15^2} \approx 8 \text{ feet}$$



▶ So, the floor area of the enclosure is

$$A = \frac{1}{2}aP \approx \frac{1}{2}(8)(51.6) = 206.4 \text{ square feet.}$$

FOCUS ON APPLICATIONS

REAL LIFE

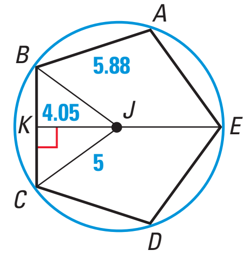
FOUCAULT PENDULUMS swing continuously in a straight line. Watching the pendulum, though, you may think its path shifts. Instead, it is Earth and you that are turning. The floor under this pendulum in Texas rotates fully about every 48 hours.

INTERNET **APPLICATION LINK**
www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check ✓

In Exercises 1–4, use the diagram shown.



1. Identify the *center* of polygon *ABCDE*.

2. Identify the *radius* of the polygon.

3. Identify a *central angle* of the polygon.

4. Identify a segment whose length is the *apothem*.

Concept Check ✓

5. In a regular polygon, how do you find the measure of each central angle?

Skill Check ✓

6. What is the area of an equilateral triangle with 3 inch sides?

STOP SIGN The stop sign shown is a regular octagon. Its perimeter is about 80 inches and its height is about 24 inches.

7. What is the measure of each central angle?

8. Find the apothem, radius, and area of the stop sign.

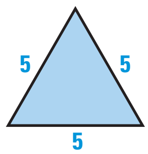
PRACTICE AND APPLICATIONS

STUDENT HELP

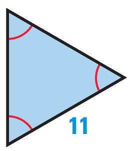
Extra Practice to help you master skills is on p. 823.

FINDING AREA Find the area of the triangle.

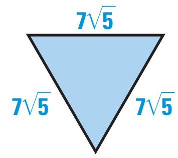
9.



10.



11.

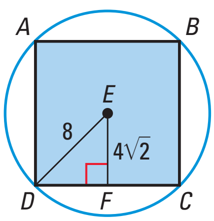


MEASURES OF CENTRAL ANGLES Find the measure of a central angle of a regular polygon with the given number of sides.

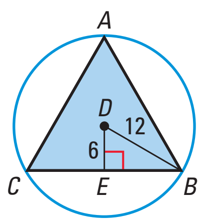
12. 9 sides **13.** 12 sides **14.** 15 sides **15.** 180 sides

FINDING AREA Find the area of the inscribed regular polygon shown.

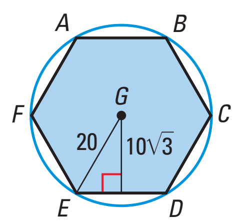
16.



17.

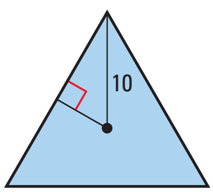


18.

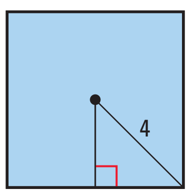


PERIMETER AND AREA Find the perimeter and area of the regular polygon.

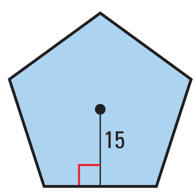
19.



20.



21.



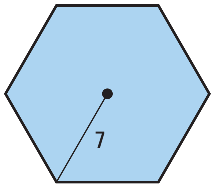
STUDENT HELP

HOMEWORK HELP

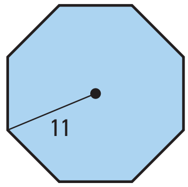
Example 1: Exs. 9–11, 17, 19, 25, 33
Example 2: Exs. 9–11, 17, 19, 25, 33
Example 3: Exs. 12–24, 26, 34
Example 4: Exs. 34, 45–49

PERIMETER AND AREA **In Exercises 22–24, find the perimeter and area of the regular polygon.**

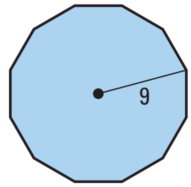
22.



23.



24.



25. AREA Find the area of an equilateral triangle that has a height of 15 inches.

26. AREA Find the area of a regular dodecagon (or 12-gon) that has 4 inch sides.

LOGICAL REASONING **Decide whether the statement is *true* or *false*. Explain your choice.**

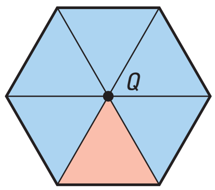
27. The area of a regular polygon of fixed radius r increases as the number of sides increases.

28. The apothem of a regular polygon is always less than the radius.

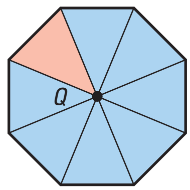
29. The radius of a regular polygon is always less than the side length.

AREA **In Exercises 30–32, find the area of the regular polygon. The area of the portion shaded in red is given. Round answers to the nearest tenth.**

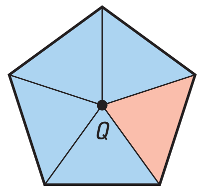
30. Area $= 16\sqrt{3}$



31. Area $= 4 \tan 67.5°$



32. Area $= \tan 54°$

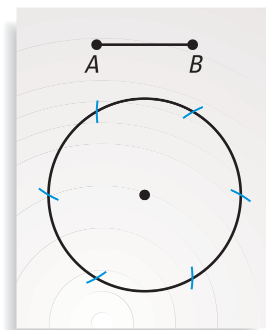


33. USING THE AREA FORMULAS Show that the area of a regular hexagon is six times the area of an equilateral triangle with the same side length.

$\left(\textit{Hint}\text{: Show that for a hexagon with side lengths } s,\ \frac{1}{2}aP = 6 \cdot \left(\frac{1}{4}\sqrt{3}\,s^2\right).\right)$

34. BASALTIC COLUMNS Suppose the top of one of the columns along the Giant's Causeway (see p. 659) is in the shape of a regular hexagon with a diameter of 18 inches. What is its apothem?

CONSTRUCTION **In Exercises 35–39, use a straightedge and a compass to construct a regular hexagon and an equilateral triangle.**



35. Draw $\overline{AB}$ with a length of 1 inch. Open the compass to 1 inch and draw a circle with that radius.

36. Using the same compass setting, mark off equal parts along the circle.

37. Connect the six points where the compass marks and circle intersect to draw a regular hexagon.

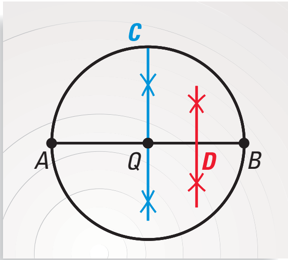
38. What is the area of the hexagon?

39. *Writing* Explain how you could use this construction to construct an equilateral triangle.

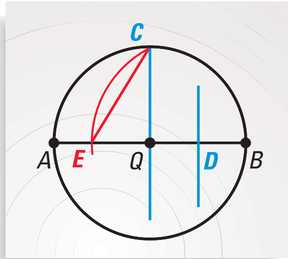
STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell for help with construction in Exs. 40–44.

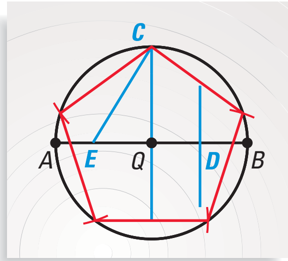
CONSTRUCTION **In Exercises 40–44, use a straightedge and a compass to construct a regular pentagon as shown in the diagrams below.**



Exs. 40, 41



Ex. 42



Exs. 43, 44

40. Draw a circle with center Q. Draw a diameter $\overline{AB}$. Construct the perpendicular bisector of $\overline{AB}$ and label its intersection with the circle as point C.

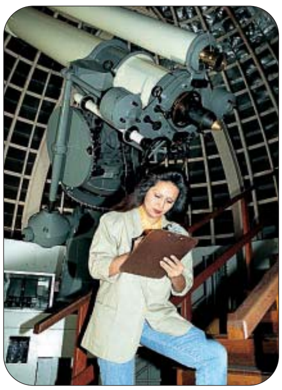
41. Construct point D, the midpoint of $\overline{QB}$.

42. Place the compass point at D. Open the compass to the length DC and draw an arc from C so it intersects $\overline{AB}$ at a point, E. Draw $\overline{CE}$.

43. Open the compass to the length CE. Starting at C, mark off equal parts along the circle.

44. Connect the five points where the compass marks and circle intersect to draw a regular pentagon. What is the area of your pentagon?

FOCUS ON CAREERS



ASTRONOMERS use physics and mathematics to study the universe, including the sun, moon, planets, stars, and galaxies.

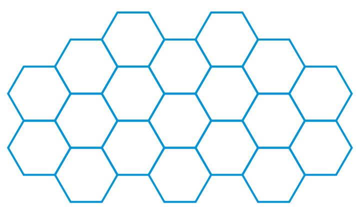
CAREER LINK
www.mcdougallittell.com

TELESCOPES **In Exercises 45 and 46, use the following information.**
The Hobby-Eberly Telescope in Fort Davis, Texas, is the largest optical telescope in North America. The primary mirror for the telescope consists of 91 smaller mirrors forming a hexagon shape. Each of the smaller mirror parts is itself a hexagon with side length 0.5 meter.

45. What is the apothem of one of the smaller mirrors?

46. Find the perimeter and area of one of the smaller mirrors.

TILING **In Exercises 47–49, use the following information.**
You are tiling a bathroom floor with tiles that are regular hexagons, as shown. Each tile has 6 inch sides. You want to choose different colors so that no two adjacent tiles are the same color.



47. What is the minimum number of colors that you can use?

48. What is the area of each tile?

49. The floor that you are tiling is rectangular. Its width is 6 feet and its length is 8 feet. At least how many tiles of each color will you need?

Test Preparation

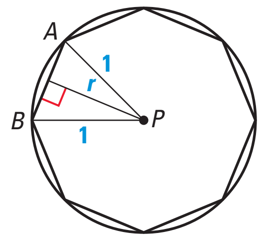
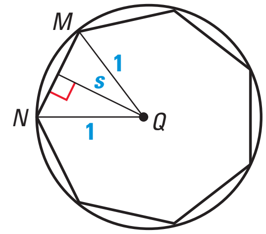
QUANTITATIVE COMPARISON **In Exercises 50–52, choose the statement that is true about the given quantities.**

Ⓐ The quantity in column A is greater.

Ⓑ The quantity in column B is greater.

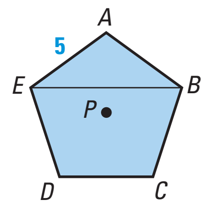
Ⓒ The two quantities are equal.

Ⓓ The relationship cannot be determined from the given information.

	Column A	Column B
		
50.	$m\angle APB$	$m\angle MQN$
51.	Apothem r	Apothem s
52.	Perimeter of octagon with center P	Perimeter of heptagon with center Q

★ **Challenge**

53. USING DIFFERENT METHODS Find the area of $ABCDE$ by using two methods. First, use the formula $A = \frac{1}{2}aP$, or $A = \frac{1}{2}a \cdot ns$. Second, add the areas of the smaller polygons. Check that both methods yield the same area.



EXTRA CHALLENGE

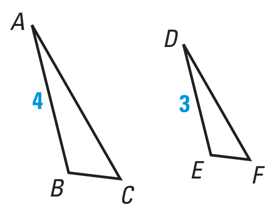
www.mcdougallittell.com

MIXED REVIEW

SOLVING PROPORTIONS **Solve the proportion.** **(Review 8.1 for 11.3)**

54. $\frac{x}{6} = \frac{11}{12}$ **55.** $\frac{20}{4} = \frac{15}{x}$ **56.** $\frac{12}{x+7} = \frac{13}{x}$ **57.** $\frac{x+6}{9} = \frac{x}{11}$

USING SIMILAR POLYGONS **In the diagram shown, $\triangle ABC \sim \triangle DEF$. Use the figures to determine whether the statement is true.** **(Review 8.3 for 11.3)**

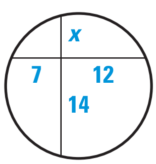


58. $\frac{AC}{BC} = \frac{DF}{EF}$ **59.** $\frac{DF}{AC} = \frac{EF + DE + DF}{BC + AB + AC}$

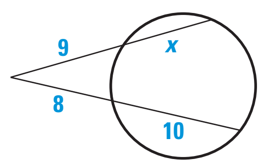
60. $\angle B \cong \angle E$ **61.** $\overline{BC} \cong \overline{EF}$

FINDING SEGMENT LENGTHS **Find the value of x.** **(Review 10.5)**

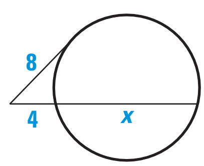
62.



63.



64.



11.3 Perimeters and Areas of Similar Figures

What you should learn

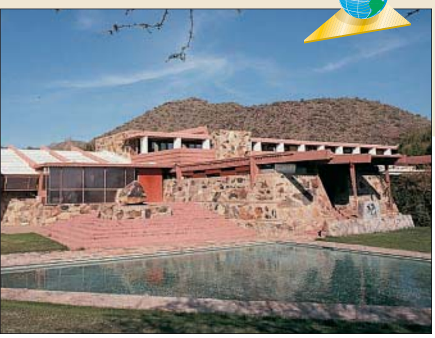
GOAL 1 Compare perimeters and areas of similar figures.

GOAL 2 Use perimeters and areas of similar figures to solve **real-life** problems, as applied in **Example 2**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the area of the walkway around a polygonal pool in **Exs. 25–27**.

REAL LIFE



Frank Lloyd Wright included this triangular pool and walkway in his design of *Taliesin West* in Scottsdale, Arizona.

GOAL 1 COMPARING PERIMETER AND AREA

For any polygon, the *perimeter of the polygon* is the sum of the lengths of its sides and the *area of the polygon* is the number of square units contained in its interior.

In Lesson 8.3, you learned that if two polygons are *similar*, then the ratio of their perimeters is the same as the ratio of the lengths of their corresponding sides. In Activity 11.3 on page 676, you may have discovered that the ratio of the areas of two similar polygons is *not* this same ratio, as shown in Theorem 11.5. Exercise 22 asks you to write a proof of this theorem for rectangles.

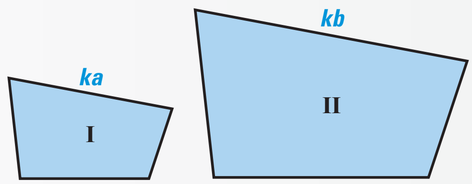
THEOREM

THEOREM 11.5 ***Areas of Similar Polygons***

If two polygons are similar with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their areas is $a^2:b^2$.

$$\frac{\text{Side length of Quad. I}}{\text{Side length of Quad. II}} = \frac{a}{b}$$

$$\frac{\text{Area of Quad. I}}{\text{Area of Quad. II}} = \frac{a^2}{b^2}$$



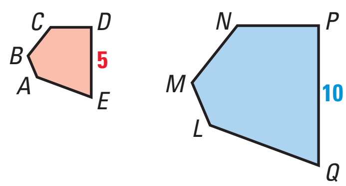
Quad. I ~ **Quad.** II

EXAMPLE 1 *Finding Ratios of Similar Polygons*

Pentagons *ABCDE* and *LMNPQ* are similar.

a. Find the ratio (red to blue) of the perimeters of the pentagons.

b. Find the ratio (red to blue) of the areas of the pentagons.



SOLUTION

The ratio of the lengths of corresponding sides in the pentagons is $\frac{5}{10} = \frac{1}{2}$, or $1:2$.

a. The ratio of the perimeters is also $1:2$. So, the perimeter of pentagon *ABCDE* is half the perimeter of pentagon *LMNPQ*.

b. Using Theorem 11.5, the ratio of the areas is $1^2:2^2$, or $1:4$. So, the area of pentagon *ABCDE* is one fourth the area of pentagon *LMNPQ*.

STUDENT HELP

Study Tip
The ratio "*a* to *b*," for example, can be written using a fraction bar $\left(\frac{a}{b}\right)$ or a colon ($a:b$).

GOAL 2 USING PERIMETER AND AREA IN REAL LIFE

EXAMPLE 2 *Using Areas of Similar Figures*

COMPARING COSTS You are buying photographic paper to print a photo in different sizes. An 8 inch by 10 inch sheet of the paper costs \$.42. What is a reasonable cost for a 16 inch by 20 inch sheet?

SOLUTION

Because the ratio of the lengths of the sides of the two rectangular pieces of paper is $1:2$, the ratio of the areas of the pieces of paper is $1^2:2^2$, or $1:4$. Because the cost of the paper should be a function of its area, the larger piece of paper should cost about four times as much, or \$1.68.

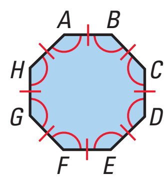
EXAMPLE 3 *Finding Perimeters and Areas of Similar Polygons*

OCTAGONAL FLOORS A trading pit at the Chicago Board of Trade is in the shape of a series of regular octagons. One octagon has a side length of about 14.25 feet and an area of about 980.4 square feet. Find the area of a smaller octagon that has a perimeter of about 76 feet.

SOLUTION

All regular octagons are similar because all corresponding angles are congruent and the corresponding side lengths are proportional.

Draw and label a sketch.



Find the ratio of the side lengths of the two octagons, which is the same as the ratio of their perimeters.

$$\frac{\text{perimeter of } ABCDEFGH}{\text{perimeter of } JKLMNPQR} = \frac{a}{b} \approx \frac{76}{8(14.25)} = \frac{76}{114} = \frac{2}{3}$$

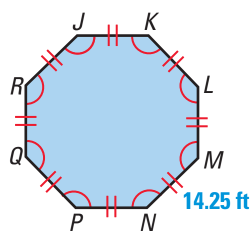
Calculate the area of the smaller octagon. Let A represent the area of the smaller octagon. The ratio of the areas of the smaller octagon to the larger is $a^2:b^2 = 2^2:3^2$, or $4:9$.

$\frac{A}{980.4} = \frac{4}{9}$ **Write proportion.**

$9A = 980.4 \cdot 4$ **Cross product property**

$A = \frac{3921.6}{9}$ **Divide each side by 9.**

$A \approx 435.7$ **Use a calculator.**



▶ The area of the smaller octagon is about 435.7 square feet.

FOCUS ON APPLICATIONS

CHICAGO BOARD OF TRADE
Commodities such as grains, coffee, and financial securities are exchanged at this marketplace. Associated traders stand on the descending steps in the same "pie-slice" section of an octagonal pit. The different levels allow buyers and sellers to see each other as orders are yelled out.

APPLICATION LINK
www.mcdougallittell.com

GUIDED PRACTICE

Vocabulary Check ✓

1. If two polygons are *similar* with the lengths of corresponding sides in the ratio of $a:b$, then the ratio of their perimeters is __?__ and the ratio of their areas is __?__.

Concept Check ✓

Tell whether the statement is *true* or *false*. Explain.

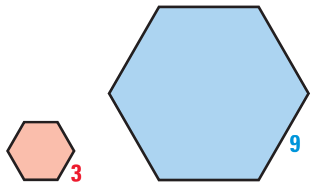
2. Any two regular polygons with the same number of sides are similar.

3. Doubling the side length of a square doubles the area.

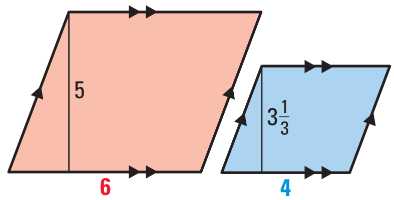
Skill Check ✓

In Exercises 4 and 5, the red and blue figures are similar. Find the ratio (red to blue) of their perimeters and of their areas.

4.



5.



6. **PHOTOGRAPHY** Use the information from Example 2 on page 678 to find a reasonable cost for a sheet of 4 inch by 5 inch photographic paper.

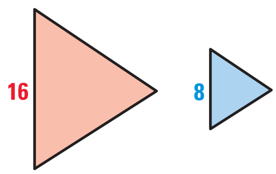
PRACTICE AND APPLICATIONS

STUDENT HELP

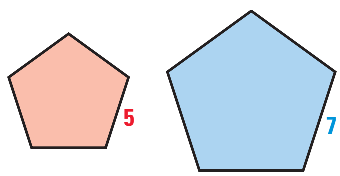
Extra Practice to help you master skills is on p. 823.

FINDING RATIOS **In Exercises 7–10, the polygons are similar. Find the ratio (red to blue) of their perimeters and of their areas.**

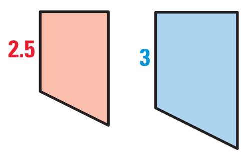
7.



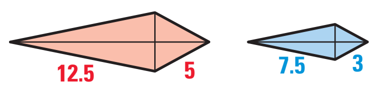
8.



9.



10.



LOGICAL REASONING **In Exercises 11–13, complete the statement using *always*, *sometimes*, or *never*.**

11. Two similar hexagons __?__ have the same perimeter.

12. Two rectangles with the same area are __?__ similar.

13. Two regular pentagons are __?__ similar.

14. **HEXAGONS** The ratio of the lengths of corresponding sides of two similar hexagons is 2:5. What is the ratio of their areas?

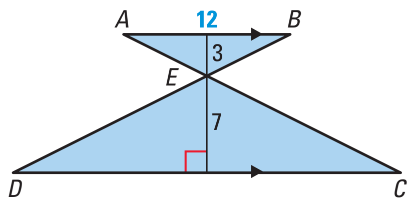
15. **OCTAGONS** A regular octagon has an area of 49 m^2. Find the scale factor of this octagon to a similar octagon that has an area of 100 m^2.

STUDENT HELP

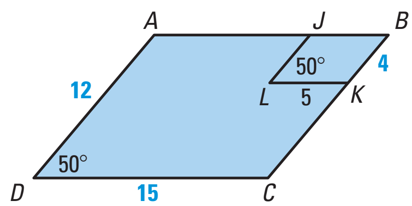
HOMEWORK HELP
Example 1: Exs. 7–10, 14–18
Example 2: Exs. 23, 24
Example 3: Exs. 25–28

16. RIGHT TRIANGLES $\triangle ABC$ is a right triangle whose hypotenuse $\overline{AC}$ is 8 inches long. Given that the area of $\triangle ABC$ is 13.9 square inches, find the area of similar triangle $\triangle DEF$ whose hypotenuse $\overline{DF}$ is 20 inches long.

17. FINDING AREA Explain why $\triangle CDE$ is similar to $\triangle ABE$. Find the area of $\triangle CDE$.



18. FINDING AREA Explain why $\square JBKL \sim \square ABCD$. The area of $\square JBKL$ is 15.3 square inches. Find the area of $\square ABCD$.



STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with scale factors in Exs. 19–21.

19. SCALE FACTOR Regular pentagon $ABCDE$ has a side length of $6\sqrt{5}$ centimeters. Regular pentagon $QRSTU$ has a perimeter of 40 centimeters. Find the ratio of the perimeters of $ABCDE$ to $QRSTU$.

20. SCALE FACTOR A square has a perimeter of 36 centimeters. A smaller square has a side length of 4 centimeters. What is the ratio of the areas of the larger square to the smaller one?

21. SCALE FACTOR A regular nonagon has an area of 90 square feet. A similar nonagon has an area of 25 square feet. What is the ratio of the perimeters of the first nonagon to the second?

22. PROOF Prove Theorem 11.5 for rectangles.

RUG COSTS Suppose you want to be sure that a large rug is priced fairly. The price of a small rug (29 inches by 47 inches) is \$79 and the price of the large rug (4 feet 10 inches by 7 feet 10 inches) is \$299.

23. What are the areas of the two rugs? What is the ratio of the areas?

24. Compare the rug costs. Do you think the large rug is a good buy? Explain.

FOCUS ON APPLICATIONS

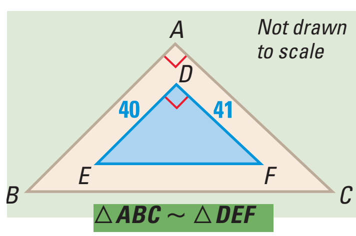
REAL LIFE **FORT JEFFERSON** is in the Dry Tortugas National Park 70 miles west of Key West, Florida. The fort has been used as a prison, a navy base, a seaplane port, and an observation post.

TRIANGULAR POOL In Exercises 25–27, use the following information. The pool at *Taliesin West* (see page 677) is a right triangle with legs of length 40 feet and 41 feet.

25. Find the area of the triangular pool, $\triangle DEF$.

26. The walkway bordering the pool is 40 inches wide, so the scale factor of the similar triangles is about 1.3 : 1. Find AB.

27. Find the area of $\triangle ABC$. What is the area of the walkway?



28. FORT JEFFERSON The outer wall of Fort Jefferson, which was originally constructed in the mid-1800s, is in the shape of a hexagon with an area of about 466,170 square feet. The length of one side is about 477 feet. The inner courtyard is a similar hexagon with an area of about 446,400 square feet. Calculate the length of a corresponding side in the inner courtyard to the nearest foot.

Test Preparation

29. MULTI-STEP PROBLEM Use the following information about similar triangles $\triangle ABC$ and $\triangle DEF$.

The scale factor of $\triangle ABC$ to $\triangle DEF$ is 15 : 2.

The area of $\triangle ABC$ is $25x$. The area of $\triangle DEF$ is $x - 5$.

The perimeter of $\triangle ABC$ is $8 + y$. The perimeter of $\triangle DEF$ is $3y - 19$.

a. Use the scale factor to find the ratio of the area of $\triangle ABC$ to the area of $\triangle DEF$.

b. Write and solve a proportion to find the value of x.

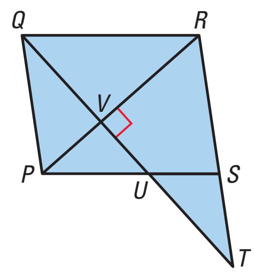
c. Use the scale factor to find the ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle DEF$.

d. Write and solve a proportion to find the value of y.

e. *Writing* Explain how you could find the value of z if $AB = 22.5$ and the length of the corresponding side $\overline{DE}$ is $13z - 10$.

★ Challenge

Use the figure shown at the right. *PQRS* is a parallelogram.



30. Name three pairs of similar triangles and explain how you know that they are similar.

31. The ratio of the area of $\triangle PVQ$ to the area of $\triangle RVT$ is 9 : 25, and the length RV is 10. Find PV.

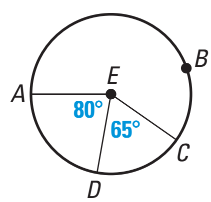
32. If VT is 15, find VQ, VU, and UT.

33. Find the ratio of the areas of each pair of similar triangles that you found in Exercise 30.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

FINDING MEASURES In Exercises 34–37, use the diagram shown at the right. (Review 10.2 for 11.4)



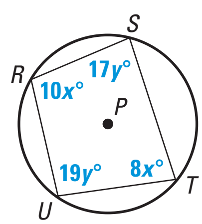
34. Find $m\overset{\frown}{AD}$.

35. Find $m\angle AEC$.

36. Find $m\overset{\frown}{AC}$.

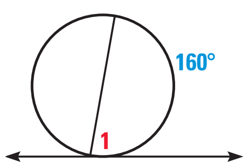
37. Find $m\overset{\frown}{ABC}$.

38. USING AN INSCRIBED QUADRILATERAL In the diagram shown at the right, quadrilateral $RSTU$ is inscribed in circle P. Find the values of x and y, and use them to find the measures of the angles of $RSTU$. (Review 10.3)

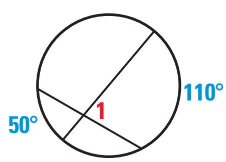


FINDING ANGLE MEASURES Find the measure of ∠1. (Review 10.4 for 11.4)

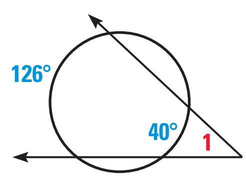
39.



40.



41.



QUIZ 1

Self-Test for Lessons 11.1–11.3

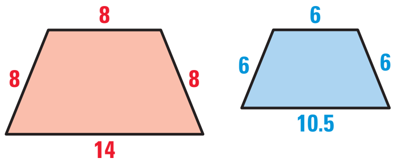
1. Find the sum of the measures of the interior angles of a convex 20-gon. **(Lesson 11.1)**

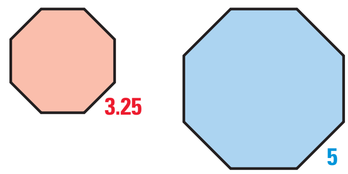
2. What is the measure of each exterior angle of a regular 25-gon? **(Lesson 11.1)**

3. Find the area of an equilateral triangle with a side length of 17 inches. **(Lesson 11.2)**

4. Find the area of a regular nonagon with an apothem of 9 centimeters. **(Lesson 11.2)**

In Exercises 5 and 6, the polygons are similar. Find the ratio (red to blue) of their perimeters and of their areas. **(Lesson 11.3)**

5. 

6. 

7. **CARPET** You just carpeted a 9 foot by 12 foot room for \$480. The carpet is priced by the square foot. About how much would you expect to pay for the same carpet in another room that is 21 feet by 28 feet? **(Lesson 11.3)**

MATH & History

History of Approximating Pi

APPLICATION LINK
www.mcdougallittell.com

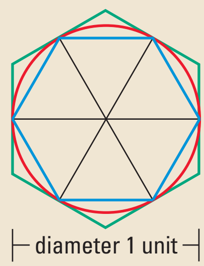
THEN

THOUSANDS OF YEARS AGO, people first noticed that the circumference of a circle is the product of its diameter and a value that is a little more than three. Over time, various methods have been used to find better approximations of this value, called π (*pi*).

1. In the third century B.C., Archimedes approximated the value of π by calculating the perimeters of inscribed and circumscribed regular polygons of a circle with diameter 1 unit. Copy the diagram and follow the steps below to use his method.

   - Find the perimeter of the **inscribed** hexagon in terms of the length of the diameter of the circle.
   - Draw a radius of the **circumscribed** hexagon. Find the length of one side of the hexagon. Then find its perimeter.
   - Write an inequality that approximates the value of π:

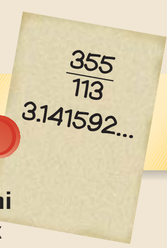
$$\text{perimeter of inscribed hexagon} < \pi < \text{perimeter of circumscribed hexagon}$$



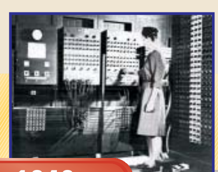
NOW

MATHEMATICIANS use computers to calculate the value of π to billions of decimal places.

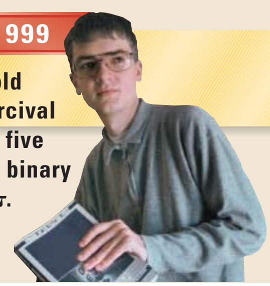
200s B.C.
Archimedes uses perimeters of polygons.



A.D. 400s
Tsu Chung Chi finds π to six decimal places.



1949
ENIAC computer finds π to 2037 decimal places.



1999
17 year old Colin Percival finds the five trillionth binary digit of π.

11.4 Circumference and Arc Length

What you should learn

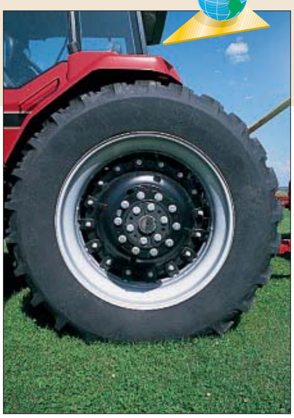
GOAL 1 Find the circumference of a circle and the length of a circular arc.

GOAL 2 Use circumference and arc length to solve **real-life** problems such as finding the distance around a track in **Example 5**.

Why you should learn it

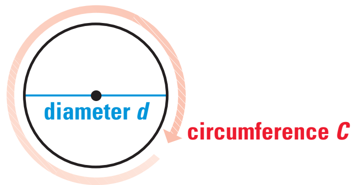
▼ To solve **real-life** problems, such as finding the number of revolutions a tire needs to make to travel a given distance in **Example 4** and **Exs. 39–41**.

REAL LIFE



GOAL 1 FINDING CIRCUMFERENCE AND ARC LENGTH

The **circumference** of a circle is the distance around the circle. For all circles, the ratio of the circumference to the diameter is the same. This ratio is known as π, or *pi*.



THEOREM

THEOREM 11.6 ***Circumference of a Circle***

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.

EXAMPLE 1 ***Using Circumference***

Find (**a**) the circumference of a circle with radius 6 centimeters and (**b**) the radius of a circle with circumference 31 meters. Round decimal answers to two decimal places.

SOLUTION

a. $C = 2\pi r$

$= 2 \cdot \pi \cdot 6$

$= 12\pi$ **Use a calculator.**

≈ 37.70

▶ So, the circumference is about 37.70 centimeters.

b. $C = 2\pi r$

$31 = 2\pi r$

$\frac{31}{2\pi} = r$ **Use a calculator.**

$4.93 \approx r$

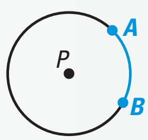
▶ So, the radius is about 4.93 meters.

An **arc length** is a portion of the circumference of a circle. You can use the measure of the arc (in degrees) to find its length (in linear units).

COROLLARY

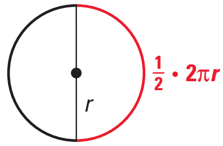
ARC LENGTH COROLLARY

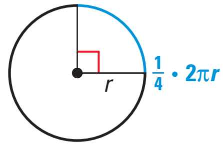
In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.



$$\frac{\text{Arc length of } \widehat{AB}}{2\pi r} = \frac{m\widehat{AB}}{360^\circ}, \text{ or Arc length of } \widehat{AB} = \frac{m\widehat{AB}}{360^\circ} \cdot 2\pi r$$

The length of a **semicircle** is one half the circumference, and the length of a **$90°$ arc** is one quarter of the circumference.

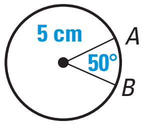




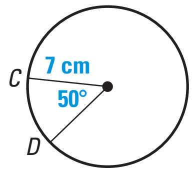
EXAMPLE 2 *Finding Arc Lengths*

Find the length of each arc.

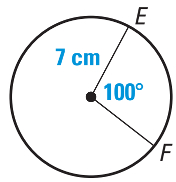
a.



b.



c.



SOLUTION

a. Arc length of $\overset{\frown}{AB} = \frac{50°}{360°} \cdot 2\pi(5) \approx 4.36$ centimeters

b. Arc length of $\overset{\frown}{CD} = \frac{50°}{360°} \cdot 2\pi(7) \approx 6.11$ centimeters

c. Arc length of $\overset{\frown}{EF} = \frac{100°}{360°} \cdot 2\pi(7) \approx 12.22$ centimeters

....

In parts (a) and (b) in Example 2, note that the arcs have the same measure, but different lengths because the circumferences of the circles are not equal.

STUDENT HELP

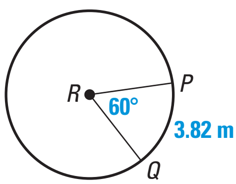
Study Tip

Throughout this chapter, you should use the π key on a calculator, then round decimal answers to two decimal places unless instructed otherwise.

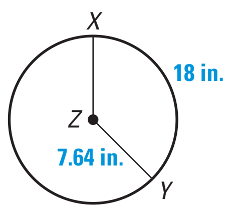
EXAMPLE 3 *Using Arc Lengths*

Find the indicated measure.

a. Circumference



b. $m\overset{\frown}{XY}$



SOLUTION

a.

$$\frac{\text{Arc length of } \overset{\frown}{PQ}}{2\pi r} = \frac{m\overset{\frown}{PQ}}{360°}$$

$$\frac{3.82}{2\pi r} = \frac{60°}{360°}$$

$$\frac{3.82}{2\pi r} = \frac{1}{6}$$

$$3.82(6) = 2\pi r$$

$$22.92 = 2\pi r$$

▶ So, $C = 2\pi r \approx 22.92$ meters.

b.

$$\frac{\text{Arc length of } \overset{\frown}{XY}}{2\pi r} = \frac{m\overset{\frown}{XY}}{360°}$$

$$\frac{18}{2\pi(7.64)} = \frac{m\overset{\frown}{XY}}{360°}$$

$$360° \cdot \frac{18}{2\pi(7.64)} = m\overset{\frown}{XY}$$

$$135° \approx m\overset{\frown}{XY}$$

▶ So, $m\overset{\frown}{XY} \approx 135°$.

STUDENT HELP

INTERNET

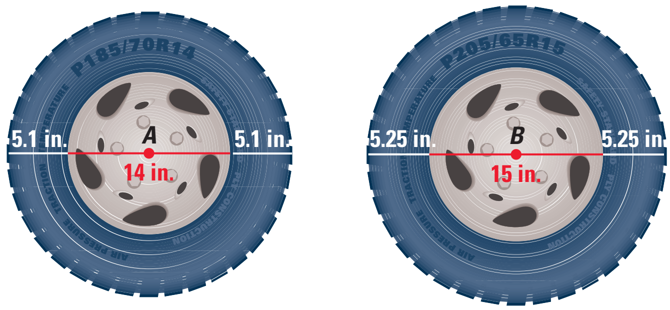
HOMEWORK HELP

Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 CIRCUMFERENCE CIRCUMFERENCES

EXAMPLE 4 *Comparing Circumferences*

TIRE REVOLUTIONS Tires from two different automobiles are shown below. How many revolutions does each tire make while traveling 100 feet? Round decimal answers to one decimal place.



SOLUTION

Tire A has a diameter of $14 + 2(5.1)$, or 24.2 inches. Its circumference is $\pi(24.2)$, or about 76.03 inches.

Tire B has a diameter of $15 + 2(5.25)$, or 25.5 inches. Its circumference is $\pi(25.5)$, or about 80.11 inches.

Divide the distance traveled by the tire circumference to find the number of revolutions made. First convert 100 feet to 1200 inches.

Tire A: $\dfrac{100 \text{ ft}}{76.03 \text{ in.}} = \dfrac{1200 \text{ in.}}{76.03 \text{ in.}}$

≈ 15.8 revolutions

Tire B: $\dfrac{100 \text{ ft}}{80.11 \text{ in.}} = \dfrac{1200 \text{ in.}}{80.11 \text{ in.}}$

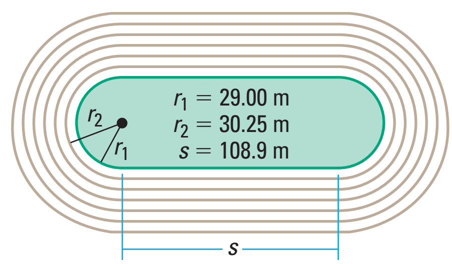
≈ 15.0 revolutions

EXAMPLE 5 *Finding Arc Length*

TRACK The track shown has six lanes. Each lane is 1.25 meters wide. There is a 180° arc at each end of the track. The radii for the arcs in the first two lanes are given.

a. Find the distance around Lane 1.

b. Find the distance around Lane 2.



SOLUTION

The track is made up of two semicircles and two straight sections with length s. To find the total distance around each lane, find the sum of the lengths of each part. Round decimal answers to one decimal place.

a. Distance $= 2s + 2\pi r_1$

$= 2(108.9) + 2\pi(29.00)$

≈ 400.0 meters

b. Distance $= 2s + 2\pi r_2$

$= 2(108.9) + 2\pi(30.25)$

≈ 407.9 meters

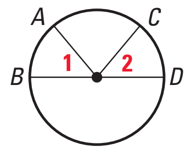
FOCUS ON PEOPLE

JACOB HEILVEIL was born in Korea and now lives in the United States. He was the top American finisher in the 10,000 meter race at the 1996 Paralympics held in Atlanta, Georgia.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is the difference between *arc measure* and *arc length*?

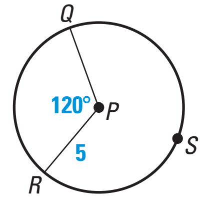


Concept Check ✓

2. In the diagram, $\overline{BD}$ is a diameter and $\angle 1 \cong \angle 2$. Explain why $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ have the same length.

Skill Check ✓

In Exercises 3–8, match the measure with its value.



A. $\frac{10}{3}\pi$ **B.** 10π **C.** $\frac{20}{3}\pi$

D. 10 **E.** 5π **F.** 120°

3. $m\overset{\frown}{QR}$ **4.** Diameter of $\odot P$

5. Length of $\overset{\frown}{QSR}$ **6.** Circumference of $\odot P$

7. Length of $\overset{\frown}{QR}$ **8.** Length of semicircle of $\odot P$

Is the statement *true* or *false*? If it is false, provide a counterexample.

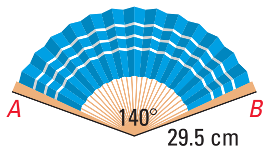
9. Two arcs with the same measure have the same length.

10. If the radius of a circle is doubled, its circumference is multiplied by 4.

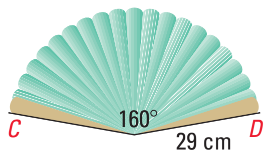
11. Two arcs with the same length have the same measure.

FANS **Find the indicated measure.**

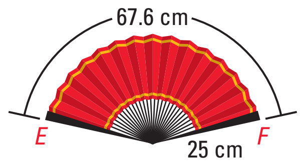
12. Length of $\overset{\frown}{AB}$



13. Length of $\overset{\frown}{CD}$



14. $m\overset{\frown}{EF}$



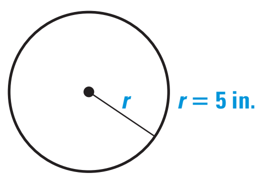
PRACTICE AND APPLICATIONS

STUDENT HELP

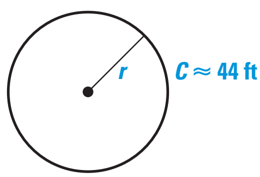
Extra Practice to help you master skills is on p. 824.

USING CIRCUMFERENCE **In Exercises 15 and 16, find the indicated measure.**

15. Circumference



16. Radius



17. Find the circumference of a circle with diameter 8 meters.

18. Find the circumference of a circle with radius 15 inches. (Leave your answer in terms of π.)

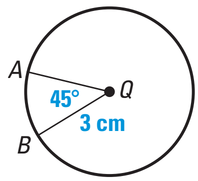
19. Find the radius of a circle with circumference 32 yards.

STUDENT HELP

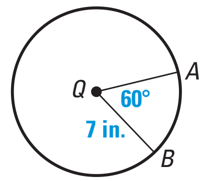
HOMEWORK HELP
Example 1: Exs. 15–19
Example 2: Exs. 20–23
Example 3: Exs. 24–29
Example 4: Exs. 39–41
Example 5: Exs. 42–46

FINDING ARC LENGTHS In Exercises 20–22, find the length of $\widehat{AB}$.

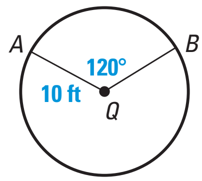
20.



21.



22.

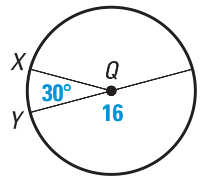


23. FINDING VALUES Complete the table.

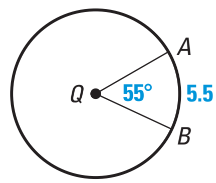
Radius	?	3	0.6	3.5	?	$3\sqrt{3}$
$m\widehat{AB}$	45°	30°	?	192°	90°	?
Length of $\widehat{AB}$	3π	?	0.4π	?	2.55π	3.09π

FINDING MEASURES Find the indicated measure.

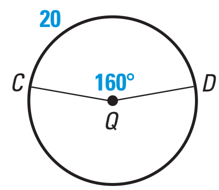
24. Length of $\widehat{XY}$



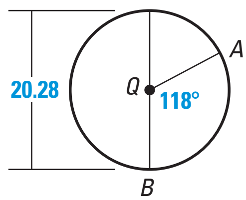
25. Circumference



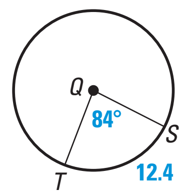
26. Radius



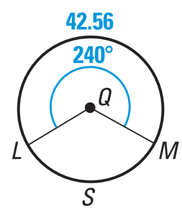
27. Length of $\widehat{AB}$



28. Circumference

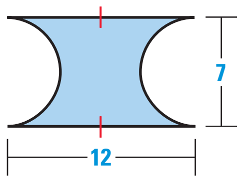


29. Radius

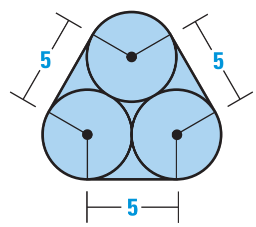


CALCULATING PERIMETERS In Exercises 30–32, the region is bounded by circular arcs and line segments. Find the perimeter of the region.

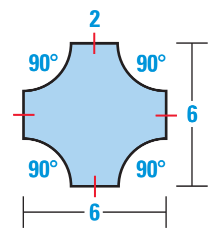
30.



31.

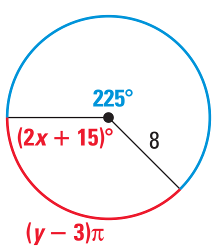


32.

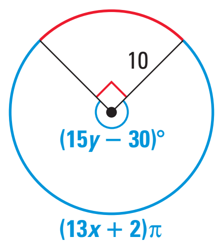


USING ALGEBRA Find the values of x and y.

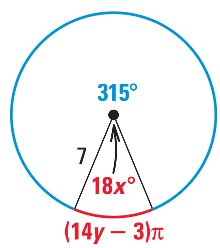
33.



34.



35.



USING ALGEBRA **Find the circumference of the circle whose equation is given. (Leave your answer in terms of π.)**

36. $x^2 + y^2 = 9$ **37.** $x^2 + y^2 = 28$ **38.** $(x + 1)^2 + (y - 5)^2 = 4$

AUTOMOBILE TIRES **In Exercises 39–41, use the table below. The table gives the rim diameters and sidewall widths of three automobile tires.**

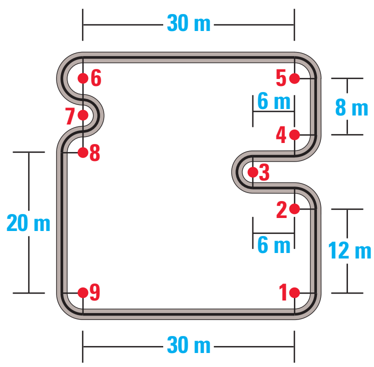
	Rim diameter	Sidewall width
Tire A	15 in.	4.60 in.
Tire B	16 in.	4.43 in.
Tire C	17 in.	4.33 in.

39. Find the diameter of each automobile tire.

40. How many revolutions does each tire make while traveling 500 feet?

41. A student determines that the circumference of a tire with a rim diameter of 15 inches and a sidewall width of 5.5 inches is 64.40 inches. Explain the error.

GO-CART TRACK **Use the diagram of the go-cart track for Exercises 42 and 43. Turns 1, 2, 4, 5, 6, 8, and 9 all have a radius of 3 meters. Turns 3 and 7 each have a radius of 2.25 meters.**

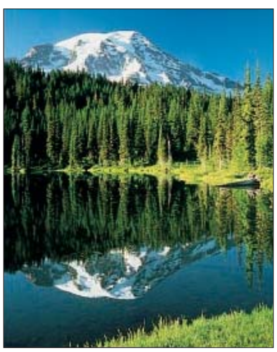


42. Calculate the length of the track.

43. How many laps do you need to make to travel 1609 meters (about 1 mile)?

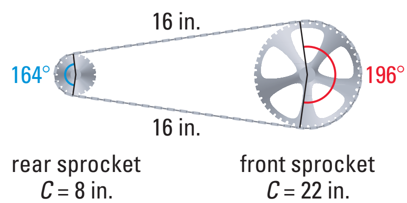
44. **MOUNT RAINIER** In Example 5 on page 623 of Lesson 10.4, you calculated the measure of the arc of Earth's surface seen from the top of Mount Rainier. Use that information to calculate the distance in miles that can be seen looking in one direction from the top of Mount Rainier.

FOCUS ON APPLICATIONS



REAL LIFE **MT. RAINIER**, at 14,410 ft high, is the tallest mountain in Washington State.

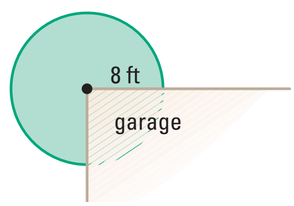
BICYCLES **Use the diagram of a bicycle chain for a fixed gear bicycle in Exercises 45 and 46.**



45. The chain travels along the front and rear sprockets. The circumference of each sprocket is given. About how long is the chain?

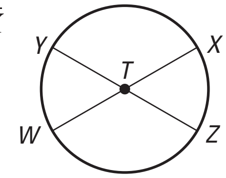
46. On a chain, the teeth are spaced in $\frac{1}{2}$ inch intervals. How many teeth are there on this chain?

47. **ENCLOSING A GARDEN** Suppose you have planted a circular garden adjacent to one of the corners of your garage, as shown at the right. If you want to fence in your garden, how much fencing do you need?



Test Preparation

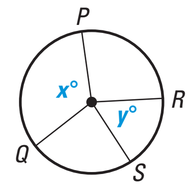
48\. **MULTIPLE CHOICE** In the diagram shown, $\overline{YZ}$ and $\overline{WX}$ each measure 8 units and are diameters of $\odot T$. If $\overset{\frown}{YX}$ measures 120°, what is the length of $\overset{\frown}{XZ}$?



Ⓐ $\frac{2}{3}\pi$ Ⓑ $\frac{4}{3}\pi$ Ⓒ $\frac{8}{3}\pi$

Ⓓ 4π Ⓔ 8π

49\. **MULTIPLE CHOICE** In the diagram shown, the ratio of the length of $\overset{\frown}{PQ}$ to the length of $\overset{\frown}{RS}$ is 2 to 1. What is the ratio of x to y?



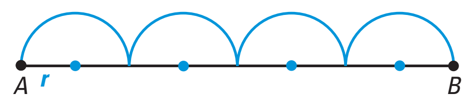
Ⓐ 4 to 1 Ⓑ 2 to 1 Ⓒ 1 to 1

Ⓓ 1 to 2 Ⓔ 1 to 4

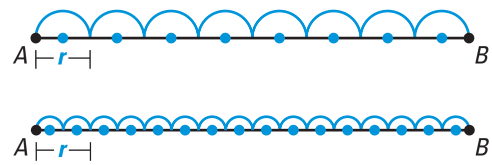
★ **Challenge**

CALCULATING ARC LENGTHS Suppose $\overline{AB}$ is divided into four congruent segments and semicircles with radius r are drawn.

50\. What is the sum of the four arc lengths if the radius of each arc is r?



51\. Imagine that $\overline{AB}$ is divided into n congruent segments and that semicircles are drawn. What would the sum of the arc lengths be for 8 segments? 16 segments? n segments? Does the number of segments matter?



EXTRA CHALLENGE

www.mcdougallittell.com

MIXED REVIEW

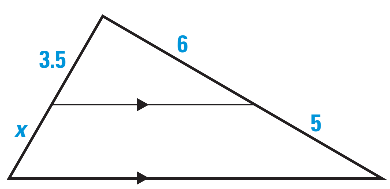
FINDING AREA In Exercises 52–55, the radius of a circle is given. Use the formula $A = \pi r^2$ to calculate the area of the circle. (Review 1.7 for 11.5)

52\. $r = 9$ ft 53. $r = 3.3$ in. 54. $r = \frac{27}{5}$ cm 55. $r = 4\sqrt{11}$ m

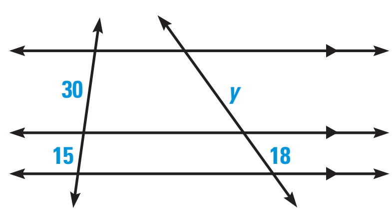
56\. **USING ALGEBRA** Line n_1 has the equation $y = \frac{2}{3}x + 8$. Line n_2 is parallel to n_1 and passes through the point $(9, -2)$. Write an equation for n_2. (Review 3.6)

USING PROPORTIONALITY THEOREMS In Exercises 57 and 58, find the value of the variable. (Review 8.6)

57\.



58\.



CALCULATING ARC MEASURES You are given the measure of an inscribed angle of a circle. Find the measure of its intercepted arc. (Review 10.3)

59\. 48° 60. 88° 61. 129° 62. 15.5°

11.5 Areas of Circles and Sectors

What you should learn

GOAL 1 Find the area of a circle and a sector of a circle.

GOAL 2 Use areas of circles and sectors to solve **real-life** problems, such as finding the area of a boomerang in **Example 6**.

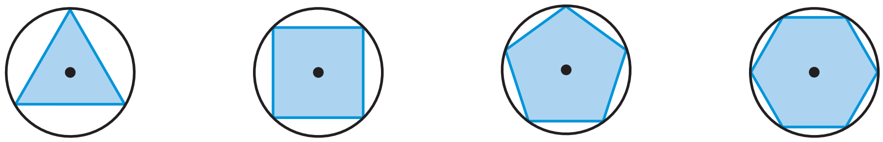
Why you should learn it

▼ To solve **real-life** problems, such as finding the area of portions of tree trunks that are used to build Viking ships in **Exs. 38 and 39**.

REAL LIFE

GOAL 1 AREAS OF CIRCLES AND SECTORS

The diagrams below show regular polygons inscribed in circles with radius r. Exercise 42 on page 697 demonstrates that as the number of sides increases, the area of the polygon approaches the value πr^2.

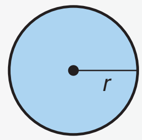


3-gon 4-gon 5-gon 6-gon

THEOREM

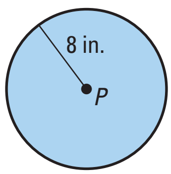
THEOREM 11.7 ***Area of a Circle***

The area of a circle is π times the square of the radius, or $A = \pi r^2$.

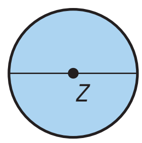


EXAMPLE 1 ***Using the Area of a Circle***

a. Find the area of $\odot P$.



b. Find the diameter of $\odot Z$.



Area of $\odot Z$ = 96 cm^2

SOLUTION

a. Use $r = 8$ in the area formula.

$$A = \pi r^2$$
$$= \pi \cdot 8^2$$
$$= 64\pi$$
$$\approx 201.06$$

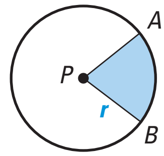
▶ So, the area is 64π, or about 201.06, square inches.

b. The diameter is twice the radius.

$$A = \pi r^2$$
$$96 = \pi r^2$$
$$\frac{96}{\pi} = r^2$$
$$30.56 \approx r^2$$
$$5.53 \approx r \qquad \text{Find the square roots.}$$

▶ The diameter of the circle is about 2(5.53), or about 11.06, centimeters.

A **sector of a circle** is the region bounded by two radii of the circle and their intercepted arc. In the diagram, sector APB is bounded by $\overline{AP}$, $\overline{BP}$, and $\overset{\frown}{AB}$. The following theorem gives a method for finding the area of a sector.



THEOREM

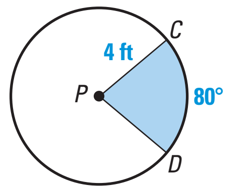
THEOREM 11.8 ***Area of a Sector***

The ratio of the area A of a sector of a circle to the area of the circle is equal to the ratio of the measure of the intercepted arc to 360°.

$$\frac{A}{\pi r^2} = \frac{m\overset{\frown}{AB}}{360^\circ}, \text{ or } A = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot \pi r^2$$

EXAMPLE 2 *Finding the Area of a Sector*

Find the area of the sector shown at the right.



SOLUTION

Sector CPD intercepts an arc whose measure is 80°. The radius is 4 feet.

$$A = \frac{m\overset{\frown}{CD}}{360^\circ} \cdot \pi r^2 \qquad \text{Write the formula for the area of a sector.}$$

$$= \frac{80^\circ}{360^\circ} \cdot \pi \cdot 4^2 \qquad \text{Substitute known values.}$$

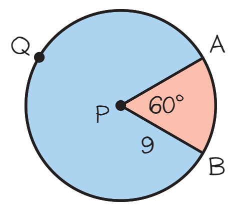
$$\approx 11.17 \qquad \text{Use a calculator.}$$

▶ So, the area of the sector is about 11.17 square feet.

EXAMPLE 3 *Finding the Area of a Sector*

A and B are two points on a $\odot P$ with radius 9 inches and $m\angle APB = 60^\circ$. Find the areas of the sectors formed by $\angle APB$.

SOLUTION



Draw a diagram of $\odot P$ and $\angle APB$. Shade the sectors.

Label a point Q on the major arc.

Find the measures of the minor and major arcs.

Because $m\angle APB = 60^\circ$, $m\overset{\frown}{AB} = 60^\circ$ and $m\overset{\frown}{AQB} = 360^\circ - 60^\circ = 300^\circ$.

Use the formula for the area of a sector.

$$\text{Area of small sector} = \frac{60^\circ}{360^\circ} \cdot \pi \cdot 9^2 = \frac{1}{6} \cdot \pi \cdot 81 \approx 42.41 \text{ square inches}$$

$$\text{Area of larger sector} = \frac{300^\circ}{360^\circ} \cdot \pi \cdot 9^2 = \frac{5}{6} \cdot \pi \cdot 81 \approx 212.06 \text{ square inches}$$

STUDENT HELP

INTERNET

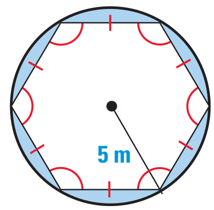
HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

GOAL 2 USING AREAS OF CIRCLES AND REGIONS

You may need to divide a figure into different regions to find its area. The regions may be polygons, circles, or sectors. To find the area of the entire figure, add or subtract the areas of the separate regions as appropriate.

EXAMPLE 4 *Finding the Area of a Region*

Find the area of the shaded region shown at the right.



SOLUTION

The diagram shows a regular hexagon inscribed in a circle with radius 5 meters. The shaded region is the part of the circle that is outside of the hexagon.

$$\text{Area of shaded region} = \boxed{\text{Area of circle}} - \boxed{\text{Area of hexagon}}$$

$$= \pi r^2 - \frac{1}{2}aP$$

$$= \pi \cdot 5^2 - \frac{1}{2} \cdot \left(\frac{5}{2}\sqrt{3}\right) \cdot (6 \cdot 5)$$

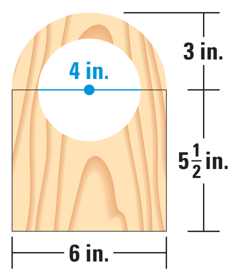
The apothem of a hexagon is $\frac{1}{2}$ • side length • $\sqrt{3}$.

$$= 25\pi - \frac{75}{2}\sqrt{3}$$

▶ So, the area of the shaded region is $25\pi - \frac{75}{2}\sqrt{3}$, or about 13.59 square meters.

EXAMPLE 5 *Finding the Area of a Region*

WOODWORKING You are cutting the front face of a clock out of wood, as shown in the diagram. What is the area of the front of the case?



SOLUTION

The front of the case is formed by a rectangle and a sector, with a circle removed. Note that the intercepted arc of the sector is a semicircle.

$$\text{Area} = \boxed{\text{Area of rectangle}} + \boxed{\text{Area of sector}} - \boxed{\text{Area of circle}}$$

$$= 6 \cdot \frac{11}{2} + \frac{180^\circ}{360^\circ} \cdot \pi \cdot 3^2 - \pi \cdot \left(\frac{1}{2} \cdot 4\right)^2$$

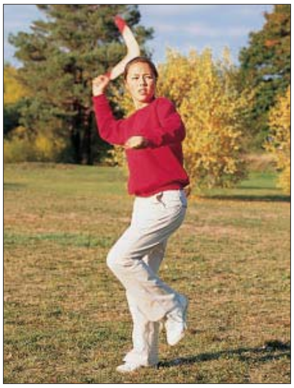
$$= 33 + \frac{1}{2} \cdot \pi \cdot 9 - \pi \cdot (2)^2$$

$$= 33 + \frac{9}{2}\pi - 4\pi$$

$$\approx 34.57$$

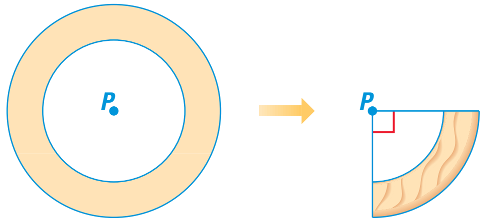
▶ The area of the front of the case is about 34.57 square inches.

FOCUS ON APPLICATIONS



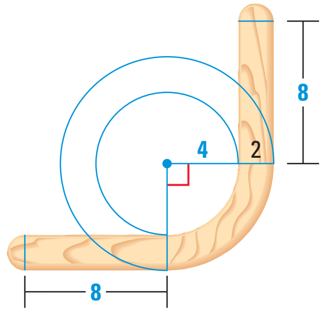
BOOMERANGS are slightly curved at the ends and travel in an arc when thrown. Small boomerangs used for sport make a full circle and return to the thrower.

Complicated shapes may involve a number of regions. In Example 6, the curved region is a portion of a ring whose edges are formed by concentric circles. Notice that the area of a portion of the ring is the difference of the areas of two sectors.



EXAMPLE 6 *Finding the Area of a Boomerang*

BOOMERANGS Find the area of the boomerang shown. The dimensions are given in inches. Give your answer in terms of π and to two decimal places.

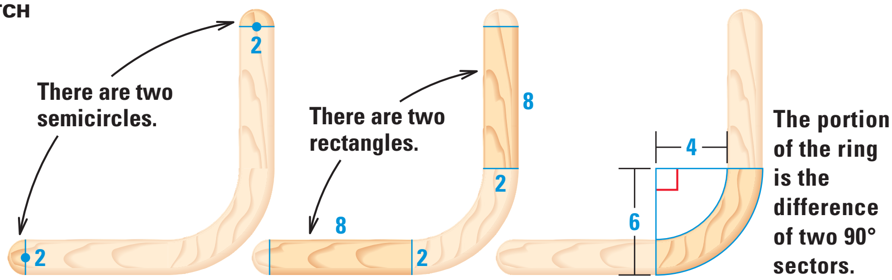


SOLUTION

Separate the boomerang into different regions. The regions are two semicircles (at the ends), two rectangles, and a portion of a ring. Find the area of each region and add these areas together.

PROBLEM SOLVING STRATEGY

DRAW AND LABEL A SKETCH Draw and label a sketch of each region in the boomerang.



VERBAL MODEL

$$\boxed{\text{Area of boomerang}} = 2 \cdot \boxed{\text{Area of semicircle}} + 2 \cdot \boxed{\text{Area of rectangle}} + \boxed{\text{Area of portion of ring}}$$

LABELS

Area of semicircle $= \frac{1}{2} \cdot \pi \cdot 1^2$ (square inches)

Area of rectangle $= 8 \cdot 2$ (square inches)

Area of portion of ring $= \frac{1}{4} \cdot \pi \cdot 6^2 - \frac{1}{4} \cdot \pi \cdot 4^2$ (square inches)

REASONING

$$\boxed{\text{Area of boomerang}} = 2\left(\frac{1}{2} \cdot \pi \cdot 1^2\right) + 2(8 \cdot 2) + \left(\frac{1}{4} \cdot \pi \cdot 6^2 - \frac{1}{4} \cdot \pi \cdot 4^2\right)$$

$$= 2\left(\frac{1}{2} \cdot \pi \cdot 1\right) + 2 \cdot 16 + \left(\frac{1}{4} \cdot \pi \cdot 36 - \frac{1}{4} \cdot \pi \cdot 16\right)$$

$$= \pi + 32 + (9\pi - 4\pi)$$

$$= 6\pi + 32$$

So, the area of the boomerang is $(6\pi + 32)$, or about 50.85 square inches.

GUIDED PRACTICE

Vocabulary Check ✓

1. Describe the boundaries of a *sector of a circle*.

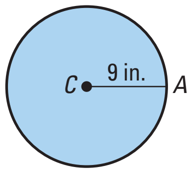
Concept Check ✓

2. In Example 5 on page 693, explain why the expression $\pi \cdot \left(\frac{1}{2} \cdot 4\right)^2$ represents the area of the circle cut from the wood.

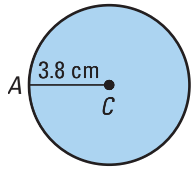
Skill Check ✓

In Exercises 3–8, find the area of the shaded region.

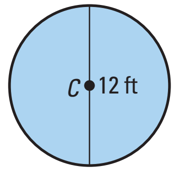
3.



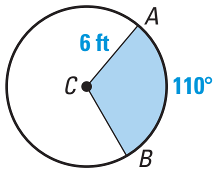
4.



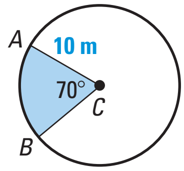
5.



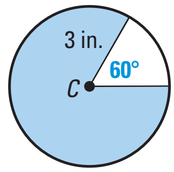
6.



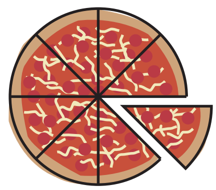
7.



8.



9. **PIECES OF PIZZA** Suppose the pizza shown is divided into 8 equal pieces. The diameter of the pizza is 16 inches. What is the area of one piece of pizza?



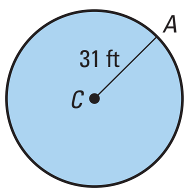
PRACTICE AND APPLICATIONS

STUDENT HELP

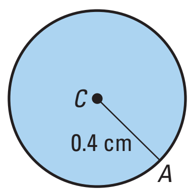
Extra Practice
to help you master skills is on p. 824.

FINDING AREA **In Exercises 10–18, find the area of the shaded region.**

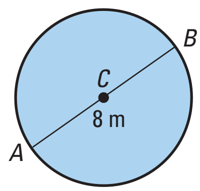
10.



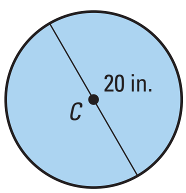
11.



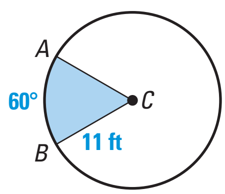
12.



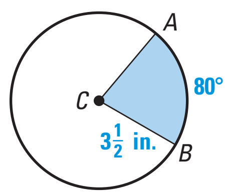
13.



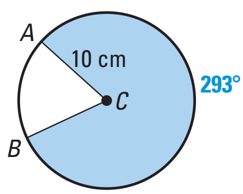
14.



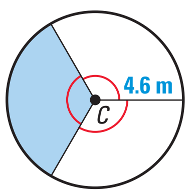
15.



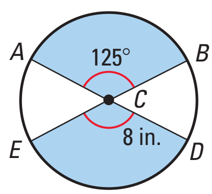
16.



17.



18.



19. **USING AREA** What is the area of a circle with diameter 20 feet?

20. **USING AREA** What is the radius of a circle with area 50 square meters?

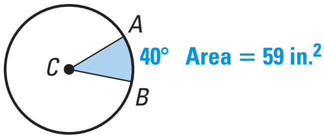
STUDENT HELP

HOMEWORK HELP

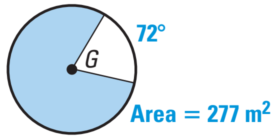
Example 1: Exs. 10–13, 19, 20
Example 2: Exs. 14–18, 21, 22, 29
Example 3: Exs. 14–18, 21, 22, 29
Example 4: Exs. 23–28, 35–37
Example 5: Exs. 23–28, 35–37
Example 6: Exs. 38–40

USING AREA **Find the indicated measure. The area given next to the diagram refers to the shaded region only.**

21. Find the radius of $\odot C$.

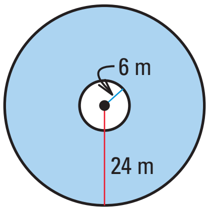


22. Find the diameter of $\odot G$.

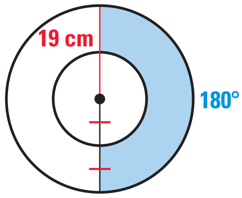


FINDING AREA **Find the area of the shaded region.**

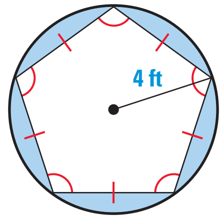
23.



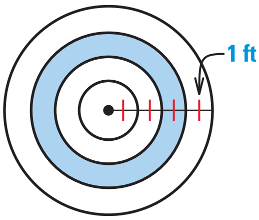
24.



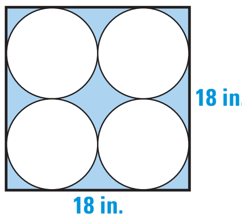
25.



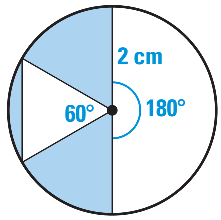
26.



27.



28.



FINDING A PATTERN **In Exercises 29–32, consider an arc of a circle with a radius of 3 inches.**

29. Copy and complete the table. Round your answers to the nearest tenth.

Measure of arc, x	30°	60°	90°	120°	150°	180°
Area of corresponding sector, y	?	?	?	?	?	?

30. **USING ALGEBRA** Graph the data in the table.

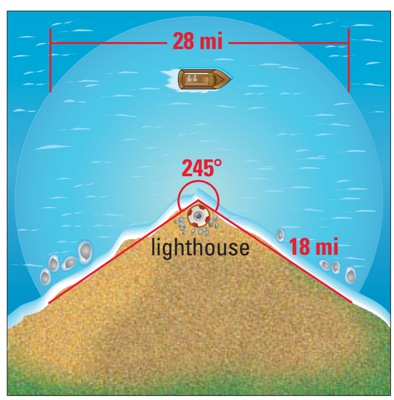
31. **USING ALGEBRA** Is the relationship between x and y linear? Explain.

32. **LOGICAL REASONING** If Exercises 29–31 were repeated using a circle with a 5 inch radius, would the areas in the table change? Would your answer to Exercise 31 change? Explain your reasoning.

FOCUS ON APPLICATIONS

LIGHTHOUSES use special lenses that increase the intensity of the light projected. Some lenses are 8 feet high and 6 feet in diameter.

LIGHTHOUSES **The diagram shows a projected beam of light from a lighthouse.**

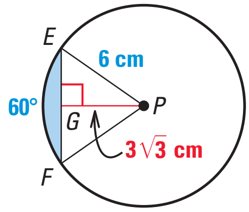


33. What is the area of water that can be covered by the light from the lighthouse?

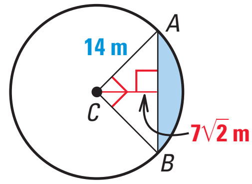
34. Suppose a boat traveling along a straight line is illuminated by the lighthouse for approximately 28 miles of its route. What is the closest distance between the lighthouse and the boat?

USING AREA **In Exercises 35–37, find the area of the shaded region in the circle formed by a chord and its intercepted arc. (*Hint:* Find the difference between the areas of a sector and a triangle.)**

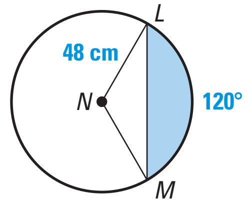
35.



36.



37.



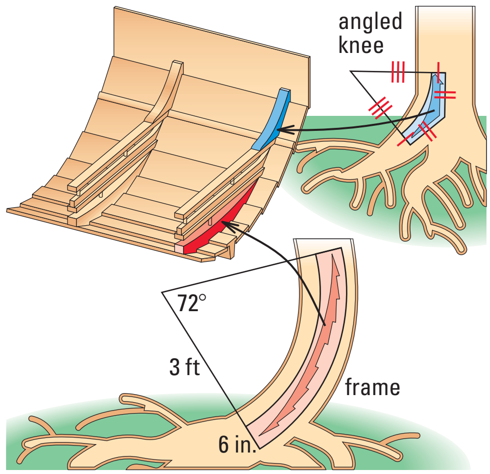
FOCUS ON APPLICATIONS

REAL LIFE **VIKING LONGSHIPS**
The planks in the hull of a longship were cut in a radial pattern from a single green log, providing uniform resiliency and strength.

INTERNET **APPLICATION LINK**
www.mcdougallittell.com

VIKING LONGSHIPS **Use the information below for Exercises 38 and 39.**

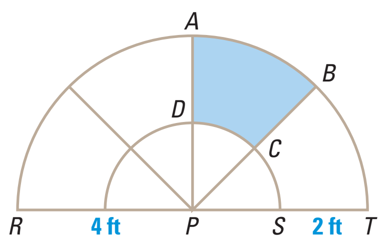
When Vikings constructed *longships*, they cut hull-hugging frames from curved trees. Straight trees provided angled knees, which were used to brace the frames.



38. Find the area of a cross-section of the frame piece shown in red.

39. *Writing* The angled knee piece shown in blue has a cross section whose shape results from subtracting a sector from a kite. What measurements would you need to know to find its area?

40. **WINDOW DESIGN** The window shown is in the shape of a semicircle with radius 4 feet. The distance from S to T is 2 feet, and the measure of $\overparen{AB}$ is 45°. Find the area of the glass in the region $ABCD$.

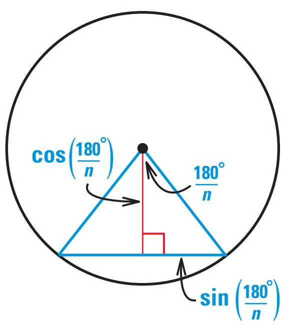


41. **LOGICAL REASONING** Suppose a circle has a radius of 4.5 inches. If you double the radius of the circle, does the area of the circle double as well? What happens to the circle's circumference? Explain.

42. **TECHNOLOGY** The area of a regular n-gon inscribed in a circle with radius 1 unit can be written as

$$A = \frac{1}{2}\left(\cos\left(\frac{180°}{n}\right)\right)\left(2n \cdot \sin\left(\frac{180°}{n}\right)\right).$$

Use a spreadsheet to make a table. The first column is for the number of sides n and the second column is for the area of the n-gon. Fill in your table up to a 16-gon. What do you notice as n gets larger and larger?

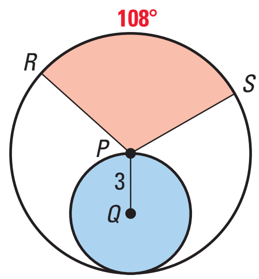


STUDENT HELP

Look Back
to Activity 11.4 on p. 690 for help with spreadsheets.

Test Preparation

⊙Q and ⊙P are tangent. Use the diagram for Exercises 43 and 44.



43. MULTIPLE CHOICE If ⊙Q is cut away, what is the remaining area of ⊙P?

Ⓐ 6π Ⓑ 9π Ⓒ 27π

Ⓓ 60π Ⓔ 180π

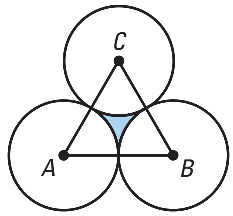
44. MULTIPLE CHOICE What is the area of the region shaded in red?

Ⓐ 0.3 Ⓑ 1.8π Ⓒ 6π

Ⓓ 10.8π Ⓔ 108π

★ Challenge

45. FINDING AREA Find the area between the three congruent tangent circles. The radius of each circle is 6 centimeters. (*Hint:* $\triangle ABC$ is equilateral.)



EXTRA CHALLENGE
www.mcdougallittell.com

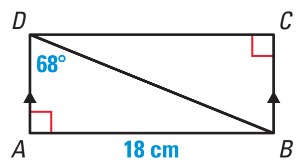
MIXED REVIEW

SIMPLIFYING RATIOS **In Exercises 46–49, simplify the ratio.** (Review 8.1 for 11.6)

46. $\frac{8 \text{ cats}}{20 \text{ cats}}$ **47.** $\frac{6 \text{ teachers}}{32 \text{ teachers}}$ **48.** $\frac{12 \text{ inches}}{63 \text{ inches}}$ **49.** $\frac{52 \text{ weeks}}{143 \text{ weeks}}$

50. The length of the diagonal of a square is 30. What is the length of each side? (Review 9.4)

FINDING MEASURES **Use the diagram to find the indicated measure. Round decimals to the nearest tenth.** (Review 9.6)



51. BD **52.** DC

53. $m\angle DBC$ **54.** BC

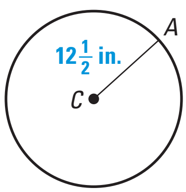
WRITING EQUATIONS **Write the standard equation of the circle with the given center and radius.** (Review 10.6)

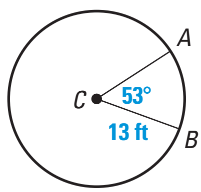
55. center $(-2, -7)$, radius 6 **56.** center $(0, -9)$, radius 10

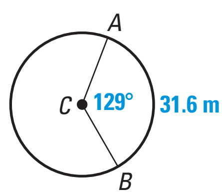
57. center $(-4, 5)$, radius 3.2 **58.** center $(8, 2)$, radius $\sqrt{11}$

FINDING MEASURES **Find the indicated measure.** (Review 11.4)

59. Circumference **60.** Length of $\widehat{AB}$ **61.** Radius







11.6 Geometric Probability

What you should learn

GOAL 1 Find a geometric probability.

GOAL 2 Use geometric probability to solve **real-life** problems, as applied in **Example 2**.

Why you should learn it

▼ Geometric probability is one model for calculating **real-life** probabilities, such as the probability that a bus will be waiting outside a hotel in **Ex. 28**.

REAL LIFE

GOAL 1 Finding a Geometric Probability

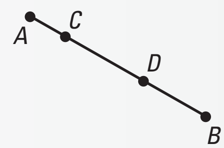
A **probability** is a number from 0 to 1 that represents the chance that an event will occur. Assuming that all outcomes are equally likely, an event with a probability of 0 *cannot* occur. An event with a probability of 1 is *certain* to occur, and an event with a probability of 0.5 is just as likely to occur as not.

In an earlier course, you may have evaluated probabilities by counting the number of favorable outcomes and dividing that number by the total number of possible outcomes. In this lesson, you will use a related process in which the division involves geometric measures such as length or area. This process is called **geometric probability**.

GEOMETRIC PROBABILITY

PROBABILITY AND LENGTH

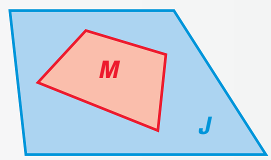
Let $\overline{AB}$ be a segment that contains the segment $\overline{CD}$. If a point K on $\overline{AB}$ is chosen at random, then the probability that it is on $\overline{CD}$ is as follows:



$$P(\text{Point } K \text{ is on } \overline{CD}) = \frac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}}$$

PROBABILITY AND AREA

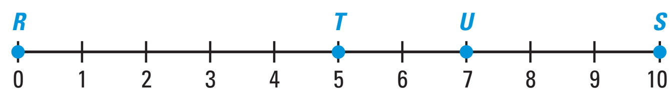
Let J be a region that contains region M. If a point K in J is chosen at random, then the probability that it is in region M is as follows:



$$P(\text{Point } K \text{ is in region } M) = \frac{\text{Area of } M}{\text{Area of } J}$$

EXAMPLE 1 *Finding a Geometric Probability*

Find the probability that a point chosen at random on $\overline{RS}$ is on $\overline{TU}$.



SOLUTION

$$P(\text{Point is on } \overline{TU}) = \frac{\text{Length of } \overline{TU}}{\text{Length of } \overline{RS}} = \frac{2}{10} = \frac{1}{5}$$

▶ The probability can be written as $\frac{1}{5}$, 0.2, or 20%.

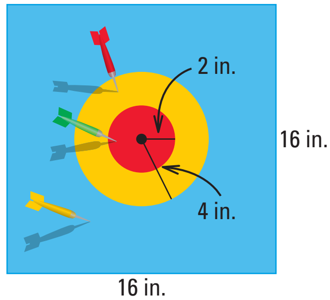
STUDENT HELP

Study Tip
When applying a formula for geometric probability, it is important that every point on the segment or in the region is equally likely to be chosen.

GOAL 2 USING GEOMETRIC PROBABILITY IN REAL LIFE

EXAMPLE 2 *Using Areas to Find a Geometric Probability*

DART BOARD A dart is tossed and hits the dart board shown. The dart is equally likely to land on any point on the dart board. Find the probability that the dart lands in the red region.



SOLUTION

Find the ratio of the area of the red region to the area of the dart board.

$$P(\text{Dart lands in red region}) = \frac{\text{Area of red region}}{\text{Area of dart board}}$$

$$= \frac{\pi(2^2)}{16^2}$$

$$= \frac{4\pi}{256}$$

$$\approx 0.05$$

▶ The probability that the dart lands in the red region is about 0.05, or 5%.

EXAMPLE 3 *Using a Segment to Find a Geometric Probability*

TRANSPORTATION You are visiting San Francisco and are taking a trolley ride to a store on Market Street. You are supposed to meet a friend at the store at 3:00 P.M. The trolleys run every 10 minutes and the trip to the store is 8 minutes. You arrive at the trolley stop at 2:48 P.M. What is the probability that you will arrive at the store by 3:00 P.M.?

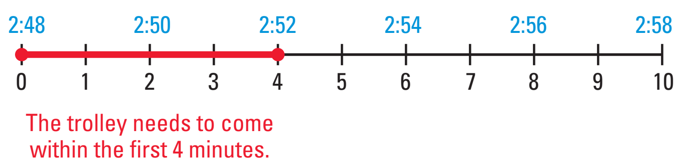
Logical Reasoning

SOLUTION

To begin, find the greatest amount of time you can afford to wait for the trolley and still get to the store by 3:00 P.M.

Because the ride takes 8 minutes, you need to catch the trolley no later than 8 minutes before 3:00 P.M., or in other words by 2:52 P.M.

So, you can afford to wait 4 minutes (2:52 − 2:48 = 4 min). You can use a line segment to model the probability that the trolley will come within 4 minutes.



STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

$$P(\text{Get to store by 3:00}) = \frac{\text{Favorable waiting time}}{\text{Maximum waiting time}} = \frac{4}{10} = \frac{2}{5}$$

▶ The probability is $\frac{2}{5}$, or 40%.

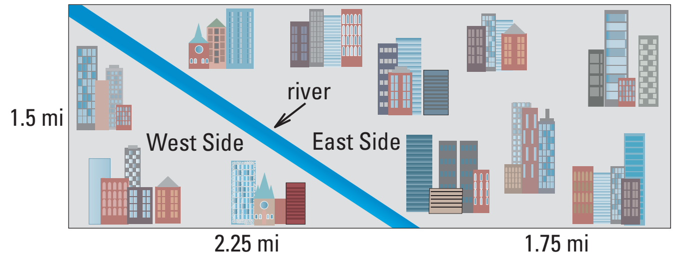
FOCUS ON CAREERS

EMPLOYMENT COUNSELORS help people make decisions about career choices. A counselor evaluates a client's interests and skills and works with the client to locate and apply for appropriate jobs.

CAREER LINK
www.mcdougallittell.com

EXAMPLE 4 *Finding a Geometric Probability*

JOB LOCATION You work for a temporary employment agency. You live on the west side of town and prefer to work there. The work assignments are spread evenly throughout the rectangular region shown. Find the probability that an assignment chosen at random for you is on the west side of town.



SOLUTION

The west side of town is approximately triangular. Its area is $\frac{1}{2} \cdot 2.25 \cdot 1.5$, or about 1.69 square miles. The area of the rectangular region is $1.5 \cdot 4$, or 6 square miles. So, the probability that the assignment is on the west side of town is

$$P(\text{Assignment is on west side}) = \frac{\text{Area of west side}}{\text{Area of rectangular region}} \approx \frac{1.69}{6} \approx 0.28.$$

▶ So, the probability that the work assignment is on the west side is about 28%.

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain how a *geometric probability* is different from a *probability* found by dividing the number of favorable outcomes by the total number of possible outcomes.

Concept Check ✓

Determine whether you would use the *length method* or *area method* to find the geometric probability. Explain your reasoning.

2. The probability that an outcome lies in a triangular region

3. The probability that an outcome occurs within a certain time period

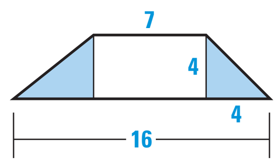
Skill Check ✓

In Exercises 4–7, K is chosen at random on $\overline{AF}$. Find the probability that K is on the indicated segment.

4. $\overline{AB}$ **5.** $\overline{BD}$ **6.** $\overline{BF}$

7. Explain the relationship between your answers to Exercises 4 and 6.

8. Find the probability that a point chosen at random in the trapezoid shown lies in either of the shaded regions.

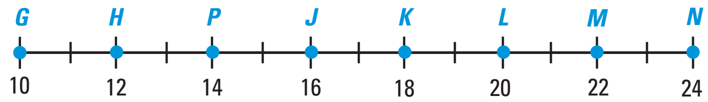


PRACTICE AND APPLICATIONS

STUDENT HELP

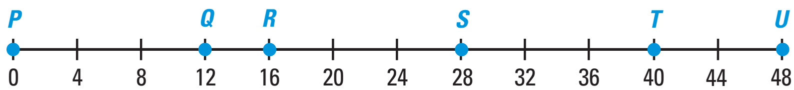
Extra Practice to help you master skills is on p. 824.

PROBABILITY ON A SEGMENT **In Exercises 9–12, find the probability that a point *A*, selected randomly on $\overline{GN}$, is on the given segment.**



9. $\overline{GH}$ **10.** $\overline{JL}$ **11.** $\overline{JN}$ **12.** $\overline{GJ}$

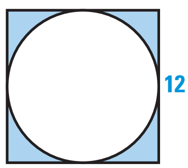
PROBABILITY ON A SEGMENT **In Exercises 13–16, find the probability that a point *K*, selected randomly on $\overline{PU}$, is on the given segment.**



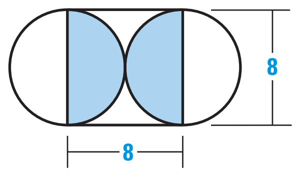
13. $\overline{PQ}$ **14.** $\overline{PS}$ **15.** $\overline{SU}$ **16.** $\overline{PU}$

FINDING A GEOMETRIC PROBABILITY **Find the probability that a randomly chosen point in the figure lies in the shaded region.**

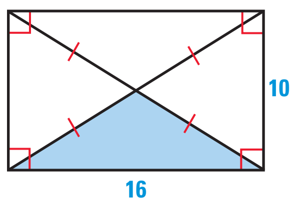
17.



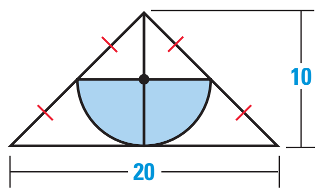
18.



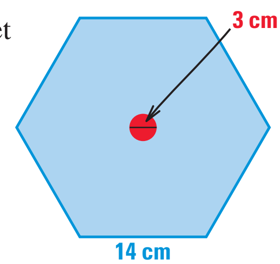
19.



20.



TARGETS **A regular hexagonal shaped target with sides of length 14 centimeters has a circular bull's eye with a diameter of 3 centimeters. In Exercises 21–23, darts are thrown and hit the target at random.**



21. What is the probability that a dart that hits the target will hit the bull's eye?

22. Estimate how many times a dart will hit the bull's eye if 100 darts hit the target.

23. Find the probability that a dart will hit the bull's eye if the bull's eye's radius is doubled.

24. **LOGICAL REASONING** The midpoint of $\overline{JK}$ is M. What is the probability that a randomly selected point on $\overline{JK}$ is closer to M than to J or to K?

25. **LOGICAL REASONING** A circle with radius $\sqrt{2}$ units is circumscribed about a square with side length 2 units. Find the probability that a randomly chosen point will be inside the circle but outside the square.

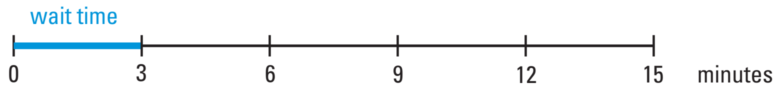
STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 9–16
Example 2: Exs. 17–23, 29–34
Example 3: Exs. 26–28
Example 4: Exs. 40–42

26. **FIRE ALARM** Suppose that your school day begins at 7:30 A.M. and ends at 3:00 P.M. You eat lunch at 11:00 A.M. If there is a fire drill at a random time during the day, what is the probability that it begins before lunch?

27. **PHONE CALL** You are expecting a call from a friend anytime between 6:00 P.M. and 7:00 P.M. Unexpectedly, you have to run an errand for a relative and are gone from 5:45 P.M. until 6:10 P.M. What is the probability that you missed your friend's call?

28. **TRANSPORTATION** Buses arrive at a resort hotel every 15 minutes. They wait for three minutes while passengers get on and get off, and then the buses depart. What is the probability that there is a bus waiting when a hotel guest walks out of the door at a randomly chosen time?



FOCUS ON APPLICATIONS

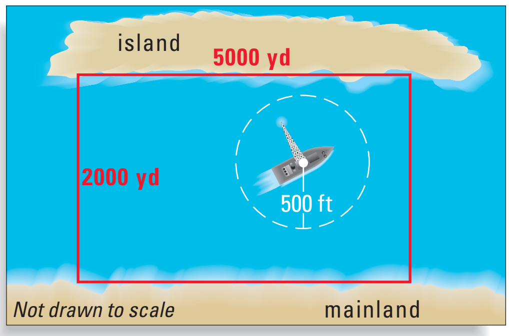
REAL LIFE **SHIP SALVAGE** Searchers for sunken items such as ships, planes, or even a space capsule, use charts, sonar, and video cameras in their search and recovery expeditions.

INTERNET **APPLICATION LINK** www.mcdougallittell.com

SHIP SALVAGE **In Exercises 29 and 30, use the following information.**
A ship is known to have sunk off the coast, between an island and the mainland as shown. A salvage vessel anchors at a random spot in this rectangular region for divers to search for the ship.

29. Find the approximate area of the rectangular region where the ship sank.

30. The divers search 500 feet in all directions from a point on the ocean floor directly below the salvage vessel. Estimate the probability that the divers will find the sunken ship on the first try.



ARCHERY **In Exercises 31–35, use the following information.**
Imagine that an arrow hitting the target shown is equally likely to hit any point on the target. The 10-point circle has a 4.8 inch diameter and each of the other rings is 2.4 inches wide. Find the probability that the arrow hits the region described.

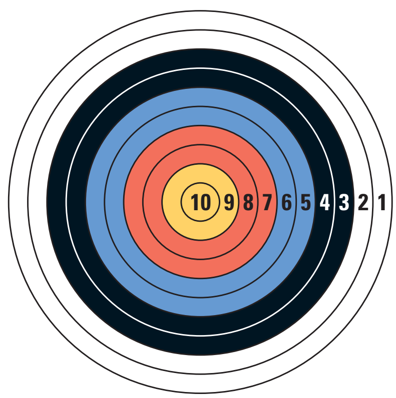
31. The 10-point region

32. The yellow region

33. The white region

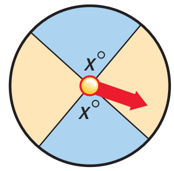
34. The 5-point region

35. **CRITICAL THINKING** Does the geometric probability model hold true when an expert archer shoots an arrow? Explain your reasoning.



36. **USING ALGEBRA** If $0 < y < 1$ and $0 < x < 1$, find the probability that $y < x$. Begin by sketching the graph, and then use the *area method* to find the probability.

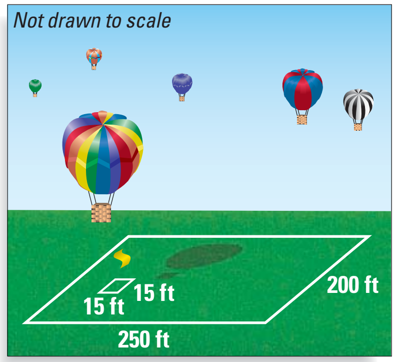
USING ALGEBRA Find the value of x so that the probability of the spinner landing on a blue sector is the value given.



37. $\frac{1}{3}$ **38.** $\frac{1}{4}$ **39.** $\frac{1}{6}$

BALLOON RACE In Exercises 40–42, use the following information.
In a "Hare and Hounds" balloon race, one balloon (the hare) leaves the ground first. About ten minutes later, the other balloons (the hounds) leave. The hare then lands and marks a square region as the target. The hounds each try to drop a marker in the target zone.

40. Suppose that a hound's marker dropped onto a rectangular field that is 200 feet by 250 feet is equally likely to land anywhere in the field. The target region is a 15 foot square that lies in the field. What is the probability that the marker lands in the target region?



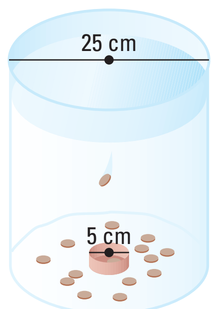
41. If the area of the target region is doubled, how does the probability change?

42. If each side of the target region is doubled, how does the probability change?

Test Preparation

43. MULTI-STEP PROBLEM Use the following information.
You organize a fund-raiser at your school. You fill a large glass jar that has a 25 centimeter diameter with water. You place a dish that has a 5 centimeter diameter at the bottom of the jar. A person donates a coin by dropping it in the jar. If the coin lands in the dish, the person wins a small prize.

a. Calculate the probability that a coin dropped, with an equally likely chance of landing anywhere at the bottom of the jar, lands in the dish.



b. Use the probability in part (a) to estimate the average number of coins needed to win a prize.

c. From past experience, you expect about 250 people to donate 5 coins each. How many prizes should you buy?

d. *Writing* Suppose that instead of the dish, a circle with a diameter of 5 centimeters is painted on the bottom of the jar, and any coin touching the circle wins a prize. Will the probability change? Explain.

★ Challenge

44. USING ALGEBRA Graph the lines $y = x$ and $y = 3$ in a coordinate plane. A point is chosen randomly from within the boundaries $0 < y < 4$ and $0 < x < 4$. Find the probability that the coordinates of the point are a solution of this system of inequalities:

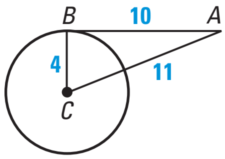
$$y < 3$$
$$y > x$$

EXTRA CHALLENGE
www.mcdougallittell.com

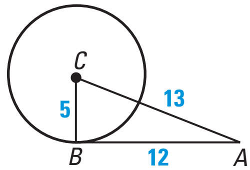
MIXED REVIEW

DETERMINING TANGENCY **Tell whether $\overleftrightarrow{AB}$ is tangent to $\odot C$. Explain your reasoning.** (Review 10.1)

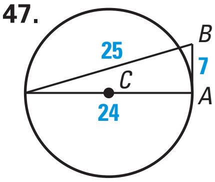
45.



46.



47.



DESCRIBING LINES **In Exercises 48–51, graph the line with the circle $(x - 2)^2 + (y + 4)^2 = 16$. Is the line a *tangent* or a *secant*?** (Review 10.6)

48. $x = -y$

49. $y = 0$

50. $x = 6$

51. $y = x - 1$

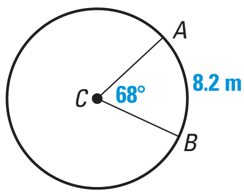
52. LOCUS Find the locus of all points in the coordinate plane that are equidistant from points (3, 2) and (1, 2) and within $\sqrt{2}$ units of the point $(1, -1)$. (Review 10.7)

QUIZ 2

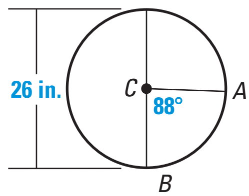
Self-Test for Lessons 11.4–11.6

Find the indicated measure. (Lesson 11.4)

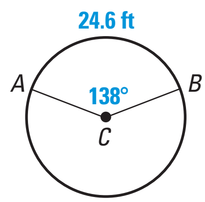
1. Circumference



2. Length of $\overset{\frown}{AB}$

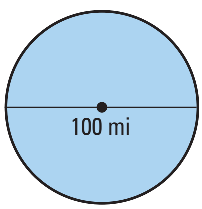


3. Radius

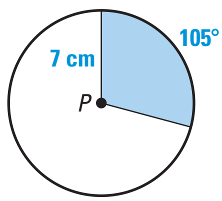


In Exercises 4–6, find the area of the shaded region. (Lesson 11.5)

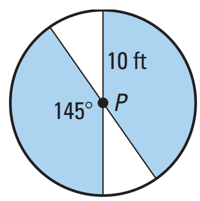
4.



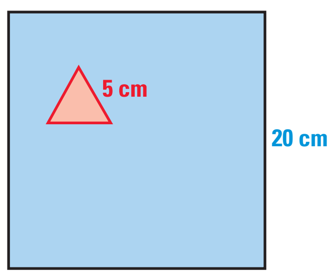
5.



6.



7. TARGETS A square target with 20 cm sides includes a triangular region with equal side lengths of 5 cm. A dart is thrown and hits the target at random. Find the probability that the dart hits the triangle. (Lesson 11.6)



CHAPTER 11

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Find the measures of the interior and exterior angles of polygons. **(11.1)**	Find the measures of angles in real-world objects, such as a home plate marker. **(p. 664)**
Find the areas of equilateral triangles and other regular polygons. **(11.2)**	Solve problems by finding real-life areas, such as the area of a hexagonal mirror in a telescope. **(p. 674)**
Compare perimeters and areas of similar figures. **(11.3)**	Solve real-life problems, such as estimating a reasonable cost for photographic paper. **(p. 678)**
Find the circumference of a circle and the length of an arc of a circle. **(11.4)**	Find real-life distances, such as the distance around a track. **(p. 685)**
Find the areas of circles and sectors. **(11.5)**	Find areas of real-life regions containing circles or parts of circles, such as the area of the front of a case for a clock. **(p. 693)**
Find a geometric probability. **(11.6)**	Estimate the likelihood that an event will occur, such as the likelihood that divers will find a sunken ship on their first dive. **(p. 703)**

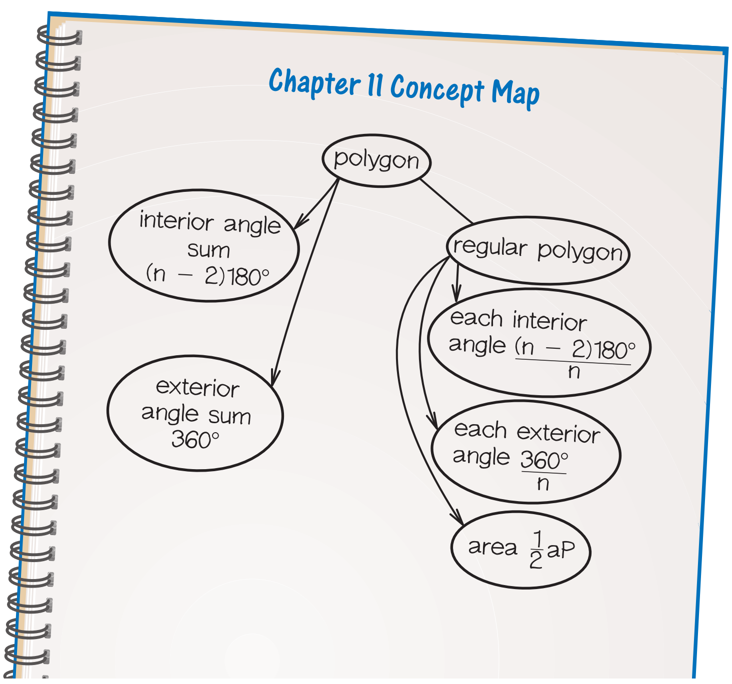
How does Chapter 11 fit into the BIGGER PICTURE of geometry?

The word *geometry* is derived from Greek words meaning "land measurement." The ability to measure angles, arc lengths, perimeters, circumferences, and areas allows you to calculate measurements required to solve problems in the real world. Keep in mind that a region that lies in a plane has two types of measures. The perimeter or circumference of a region is a *one*-dimensional measure that uses units such as centimeters or feet. The area of a region is a *two*-dimensional measure that uses units such as square centimeters or square feet. In the next chapter, you will study a *three*-dimensional measure called *volume*.

STUDY STRATEGY

Did your concept map help you organize your work?

The concept map you made, following the **Study Strategy** on page 660, may include these ideas.



CHAPTER 11

Chapter Review

VOCABULARY

- center of a polygon, p. 670
- radius of a polygon, p. 670
- apothem of a polygon, p. 670
- central angle of a regular polygon, p. 671
- circumference, p. 683
- arc length, p. 683
- sector of a circle, p. 692
- probability, p. 699
- geometric probability, p. 699

11.1 ANGLE MEASURES IN POLYGONS

Examples on pp. 661–664

EXAMPLES If a regular polygon has 15 sides, then the sum of the measures of its interior angles is $(15 - 2) \cdot 180° = 2340°$. The measure of each interior angle is $\frac{1}{15} \cdot 2340° = 156°$. The measure of each exterior angle is $\frac{1}{15} \cdot 360° = 24°$.

In Exercises 1–4, you are given the number of sides of a regular polygon. Find the measure of each interior angle and each exterior angle.

1. 9 **2.** 13 **3.** 16 **4.** 24

In Exercises 5–8, you are given the measure of each interior angle of a regular *n*-gon. Find the value of *n*.

5. 172° **6.** 135° **7.** 150° **8.** 170°

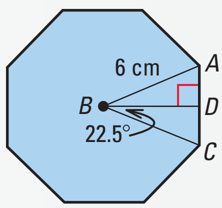
11.2 AREAS OF REGULAR POLYGONS

Examples on pp. 669–671

EXAMPLES The area of an equilateral triangle with sides of length 14 inches is

$A = \frac{1}{4}\sqrt{3}\left(14^2\right) = \frac{1}{4}\sqrt{3}\,(196) = 49\sqrt{3} \approx 84.9 \text{ in.}^2$

In the regular octagon at the right, $m\angle ABC = \frac{1}{8} \cdot 360° = 45°$ and $m\angle DBC = 22.5°$. The apothem BD is $6 \cdot \cos 22.5°$. The perimeter of the octagon is $8 \cdot 2 \cdot DC$, or $16(6 \cdot \sin 22.5°)$. The area of the octagon is



$A = \frac{1}{2}aP = \frac{1}{2}(6 \cos 22.5°) \cdot 16(6 \sin 22.5°) \approx 101.8 \text{ cm}^2.$

9. An equilateral triangle has 12 centimeter sides. Find the area of the triangle.

10. An equilateral triangle has a height of 6 inches. Find the area of the triangle.

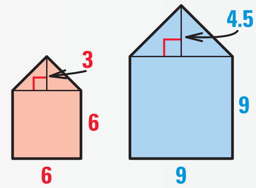
11. A regular hexagon has 5 meter sides. Find the area of the hexagon.

12. A regular decagon has 1.5 foot sides. Find the area of the decagon.

11.3 PERIMETERS AND AREAS OF SIMILAR FIGURES

Examples on pp. 677–678

EXAMPLE The two pentagons at the right are similar. Their corresponding sides are in the ratio $2:3$, so the ratio of their areas is $2^2:3^2 = 4:9$.



$$\frac{\text{Area (smaller)}}{\text{Area (larger)}} = \frac{6 \cdot 6 + \frac{1}{2}(3 \cdot 6)}{9 \cdot 9 + \frac{1}{2}(4.5 \cdot 9)} = \frac{45}{101.25} = \frac{4}{9}$$

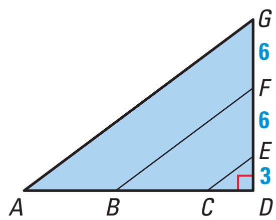
Complete the statement using *always*, *sometimes*, or *never*.

13. If the ratio of the perimeters of two rectangles is $3:5$, then the ratio of their areas is __?__ $9:25$.

14. Two parallelograms are __?__ similar.

15. Two regular dodecagons with perimeters in the ratio 4 to 7 __?__ have areas in the ratio 16 to 49.

In the diagram at the right, $\triangle ADG$, $\triangle BDF$, and $\triangle CDE$ are similar.



16. Find the ratio of the perimeters and of the areas of $\triangle CDE$ and $\triangle BDF$.

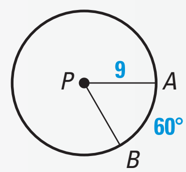
17. Find the ratio of the perimeters and of the areas of $\triangle ADG$ and $\triangle BDF$.

11.4 CIRCUMFERENCE AND ARC LENGTH

Examples on pp. 683–685

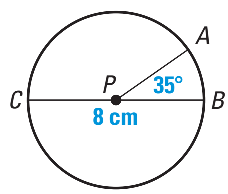
EXAMPLES The circumference of the circle at the right is $C = 2\pi(9) = 18\pi$.

The length of $\widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r = \frac{60°}{360°} \cdot 18\pi = 3\pi$.

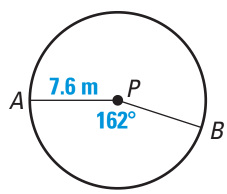


In Exercises 18–20, find the circumference of $\odot P$ and the length of $\widehat{AB}$.

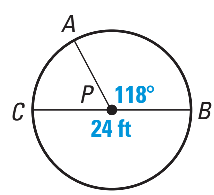
18.



19.



20.



21. Find the radius of a circle with circumference 12 inches.

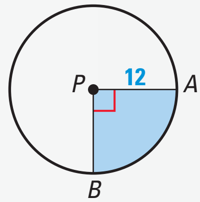
22. Find the diameter of a circle with circumference 15π meters.

11.5 AREAS OF CIRCLES AND SECTORS

Examples on pp. 691–694

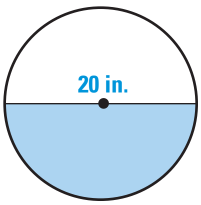
EXAMPLES The area of $\odot P$ at the right is $A = \pi(12^2) = 144\pi$. To find the area A of the shaded sector of $\odot P$, use

$$\frac{A}{\pi r^2} = \frac{m\overparen{AB}}{360°}, \text{ or } A = \frac{m\overparen{AB}}{360°} \cdot \pi r^2 = \frac{90°}{360°} \cdot 144\pi = 36\pi.$$

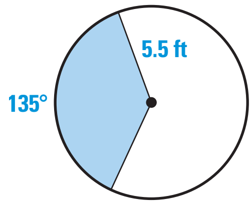


In Exercises 23–26, find the area of the shaded region.

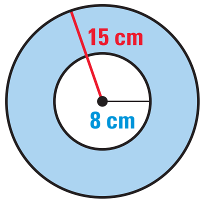
23.



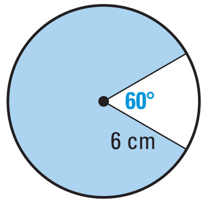
24.



25.



26.



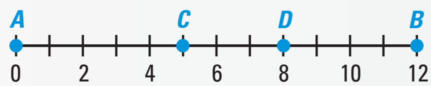
27. What is the area of a circle with diameter 28 feet?

28. What is the radius of a circle with area 40 square inches?

11.6 GEOMETRIC PROBABILITY

Examples on pp. 699–701

EXAMPLES The probability that a randomly chosen point on $\overline{AB}$ is on $\overline{CD}$ is

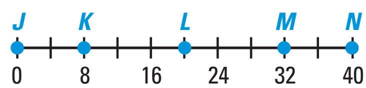


$$P(\text{Point is on } \overline{CD}) = \frac{\text{Length of } \overline{CD}}{\text{Length of } \overline{AB}} = \frac{3}{12} = \frac{1}{4}.$$

Suppose a circular target has radius 12 inches and its bull's eye has radius 2 inches. If a dart that hits the target hits it at a random point, then

$$P(\text{Dart hits bull's eye}) = \frac{\text{Area of bull's eye}}{\text{Area of target}} = \frac{4\pi}{144\pi} = \frac{1}{36}.$$

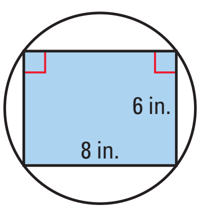
Find the probability that a point *A*, selected randomly on $\overline{JN}$, is on the given segment.



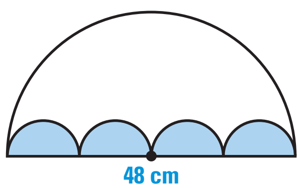
29. $\overline{LM}$ **30.** $\overline{JL}$ **31.** $\overline{KM}$

Find the probability that a randomly chosen point in the figure lies in the shaded region.

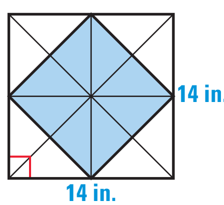
32.



33.

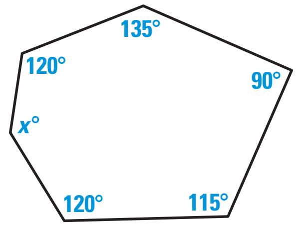


34.



CHAPTER 11 Chapter Test

In Exercises 1 and 2, use the figure at the right.



1. What is the value of x?
2. Find the sum of the measures of the exterior angles, one at each vertex.
3. What is the measure of each interior angle of a regular 30-gon?
4. What is the measure of each exterior angle of a regular 27-gon?

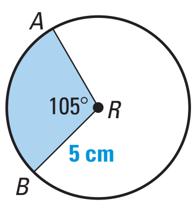
In Exercises 5–8, find the area of the regular polygon to two decimal places.

5. An equilateral triangle with perimeter 30 feet
6. A regular pentagon with apothem 8 inches
7. A regular hexagon with 9 centimeter sides
8. A regular nonagon (9-gon) with radius 1 meter

Rhombus *ABCD* has sides of length 8 centimeters. *EFGH* is a similar rhombus with sides of length 6 centimeters.

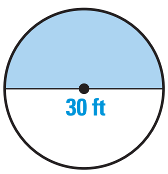
9. Find the ratio of the perimeters of $ABCD$ to $EFGH$. Then find the ratio of their areas.
10. The area of $ABCD$ is 56 square centimeters. Find the area of $EFGH$.

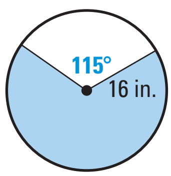
Use the diagram of ⊙*R*.

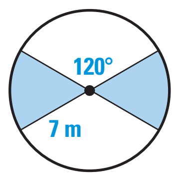


11. Find the circumference and the area of ⊙R.
12. Find the length of $\overset{\frown}{AB}$.
13. Find the area of the sector ARB.

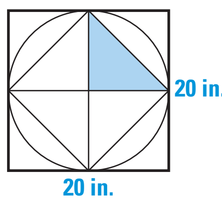
Find the area of the shaded region.

14.


15.


16.


In Exercises 17 and 18, a point is chosen randomly in the 20 inch by 20 inch square at the right.



17. Find the probability that the point is inside the circle.
18. Find the probability that the point is in the shaded area.
19. **WATER-SKIER** A boat that is pulling a water-skier drives in a circle that has a radius of 80 feet. The skier is moving outside the path of the boat in a circle that has a radius of 110 feet. Find the distance traveled by the boat when it has completed a full circle. How much farther has the skier traveled?
20. **WAITING TIME** You are expecting friends to come by your house any time between 6:00 P.M. and 8:00 P.M. Meanwhile, a problem at work has delayed you. If you get home at 6:20 P.M., what is the probability that your friends are already there?