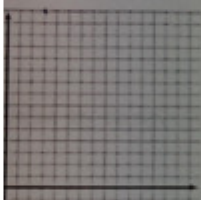
quadratic equations, find vertex, AOS, min/max

d the vertex (by hand). Show work. Graph the parabola.

$f(x) = x^2 - 6x + 8$



AOS: $x = 3$

Vertex: $(3, -1)$

Y - Int: $(0, 8)$

x	y
2	
3	
4	

$a = 1$
$b = -6$
$c = 8$

$x = \frac{-b}{2a}$

$x = \frac{6}{2(1)}$

$x = 3$

$f(3) = 3^2 - 6(3) + 8$
$= 9 - 18 + 8$
$= -9 + 8$
$f(3) = -1$

Vrite a quadratic function that has a minimum value. $y = -3x^2 + 2x + 5$

ate the vertex: $y = 4(x-2)^2 - 7$

$a = 4$
$h = 2$
$k = -7$

3. $(2, -7)$

tate whether the parabola is opening Down or Up

a) down $f(x) = -2x^2 - 3x + 4$

c) up $f(x) = 2x^2 - 3x - 4$

b) up $y = 2x^2 - 3x + 4$

d) down $y = -2x^2 - 3x - 4$

tate whether the function is quadratic (Yes) or not (No)

a) No $f(x) = 3x + 4$

c) No $f(x) = 2^x - 10x + 5$

b) Yes $f(x) = x^2 - 9$

d) No $f(x) = 3x^3 + x^2 - 2x - 11$

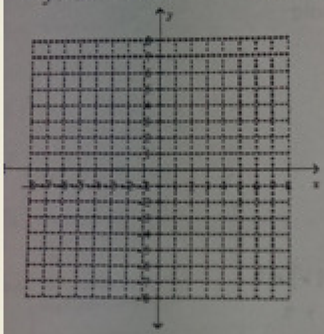
* looking for the highest exponent to be 2.

Skip 1

Graph and solve quadratic inequalities

5. $y \le -x^2+4x-1$ Vertex:________ Another Point:________ Dashed or Solid?

if $y > 2x^2-5x+1$ Vertex: ________ Another Point: ________ Dashed or Solid?



tor equations in all forms, including GCF, Special Cases, and Grouping

Use factoring to SOLVE: $x^2-13x+42=0$ 6. $x=6$ or $x=7$

$(x-7)(x-6)=0$

$x-7=0$ or $x-6=0$

$x=7$ or $x=6$

Use factoring to SOLVE: $4x^2-15x=-9$ 7. $x=\frac{3}{4}$ or $x=3$

$4x^2-15x+9=0$

$(4x-3)(x-3)=0$

$4x-3=0$ or $x-3=0$

$4x=3$ $x=3$

$x=\frac{3}{4}$

Use factoring to SOLVE: $30x^3-12x=9x^2$ 8. ________

skip

lve quadratic equations by using square roots (including complex roots)

Solve using square roots: $\frac{1}{2}(x+6)^2-20=0$ 9. $x=-6\pm2\sqrt{10}$

$+20 \quad +20$

$\frac{1}{2}(x+6)^2=20$

$(x+6)^2=40$

$x+6=\pm\sqrt{40}$

$x=-6\pm2\sqrt{10}$

side $\sqrt{40}=\sqrt{4}\cdot\sqrt{10}$

$=2\sqrt{10}$

Solve using square roots: $4x^2+96=0$ 10. $x=\pm2i\sqrt{6}$

side $\sqrt{-24}=\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{6}$

$=i\,2\sqrt{6}$

$\frac{4x^2}{4}=\frac{-96}{4}$

$x^2=-24$

$x=\pm\sqrt{-24}$

Solve for the discriminant, use to find number and type of solutions
Solve quadratic equations using the quadratic formula

13. a) Find the discriminant. b) Tell the number and type of solutions. c) Then, solve usi the quadratic formula. $0 = 2x^2 + 4x - 5$ (you must show work to get credit!)

$a=2 \quad b=4 \quad c=-5$

discriminant $= b^2 - 4ac$
$= 4^2 - 4(2)(-5)$
$= 16 + 40$
$= 56$

quadratic formula:
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$x = \frac{-4 \pm \sqrt{56}}{2(2)}$
$x = \frac{-4 \pm 2\sqrt{14}}{4} = \frac{-2 \pm \sqrt{14}}{2}$

13. a. 56

13. b. 2 Real Solu

13. c. $x = \frac{-2 \pm \sqrt{14}}{2}$

14. a) Find the discriminant. b) Tell the number and type of solutions. c) Then, solve the quadratic formula. $0 = x^2 - 8x + 20$ (you must show work to get credit!)

$a=1 \quad b=-8 \quad c=20$

discriminant $= b^2 - 4ac$
$= (-8)^2 - 4(1)(20)$
$= 64 - 80$
$= -16$

quadratic formula:
$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-5)}}{2(1)}$
$x = \frac{8 \pm \sqrt{-16}}{2} = \frac{8 \pm 4i}{2} = 4 \pm 2i$

14. a. -16

14. b. 2 imaginar

14. c. $4 \pm 2i$

6. **For Exercises 1–12, complete parts a–c for each quadratic equation.**
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solutions by using the Quadratic Formula.

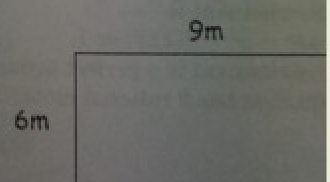
complex

1. $p^2 + 12p = -4$ 2. $9x^2 - 6x + 1 = 0$ 3. $2x^2 - 7x - 4 = 0$

4. $x^2 + 4x - 4 = 0$ 5. $5x^2 - 36x + 7 = 0$ 6. $4x^2 - 4x + 11 = 0$

15. Tamika wants to double the area of her garden by increasing the length and width by the same amo will be the dimensions of her garden then?

skip



(4 of these on final)

Solve each equation by using the method of your choice. Find exact solutions.

16. $7x^2 - 5x = 0$

17. $4x^2 - 9 = 0$

18. $3x^2 + 8x = 3$

19. $x^2 - 21 = 4x$

20. $3x^2 - 13x + 4 = 0$

21. $15x^2 + 22x = -8$

22. $x^2 - 6x + 3 = 0$

23. $x^2 - 14x + 53 = 0$

24. $3x^2 = -54$

25. $25x^2 - 20x - 6 = 0$

26. $4x^2 - 4x + 17 = 0$

27. $8x - 1 = 4x^2$

Absolute Value Equations

Solve: $|x - 4| = 24$

One way: $+(x - 4) = 24$

$x - 4 = 24$

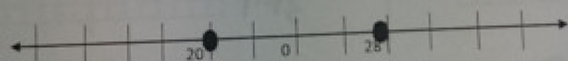
$x = 28$

Second way: $-(x - 4) = 24$

$-x + 4 = 24$

$x = -20$

Graphically we would represent this as shown below:



Sentence	Meaning	Solution
$\lvert x\rvert = a$	The distance from x to 0 is exactly "a" units	$x = a$ or $x = -a$
$\lvert x\rvert > a$	The distance from x to 0 is greater than "a" units.	$x < -a$ or $x > a$
$\lvert x\rvert < a$	The distance from x to 0 is less than "a" units	$-a < x < a$

Station 3 Problems (skip)

Solve and graph the following:

1. $|2x + 5| = 3$

$2x+5=3$ or $2x+5=-3$

$2x=-2$ $\quad$ $2x=-8$

$x=-1$ $\quad$ $x=-4$

2. $|5 - x| + 4 = 10$

$|5-x| = 6$

$5-x=6$ or $5-x=-6$

$-x=1$ or $-x=-11$

$x=-1$ or $x=11$

3. $-2|x + 6| = -$

$|x+6|$

$x+6$

x

4. $|2x - 3| \le 5$

$2x-3 \le 5$ and $2x-3 \ge -5$

$2x \le 8$

$x \le 8$ and $2x \ge -2$

$x \ge -1$

5. $|11 - 3x| + 6 > 10$

$-6 \quad -6$

$|11-3x| > 4$

$11-3x>4$ or $11-3x<-4$

$-3x>-7$ $\quad$ $-3x<-15$

$x<\frac{7}{3}$ or $x>5$

6. For $y = 3|7-2x|+5$, which set describes x when $y < 8$?

a. $\{x|3 < x < 4\}$

b. $\{x|3 < x < 10\}$

c. $\{x|x < 3 \text{ or } x > 4\}$

d. $\{x|x < 3 \text{ or } x > 10\}$

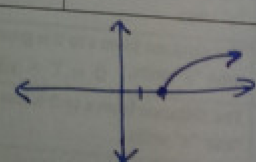
COMPLETE EACH SECTION:

3. Solve the following equations for x. Graph the solution on a number line.

a. $\|2x-3\| = 3.7$	b. $\frac{1}{2}\|2+x\| - 10 = 5$
c. $3\|2x-1\| \le 9$ skip	d. $2\|x+4\| - 5 > 11$ skip

. What is the domain of $\sqrt{x-2}$?

graph →



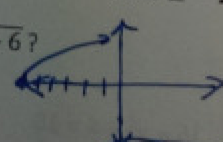
$a=1$
$h=2$
$K=0$

domai… value… 2.

A. $x \ge 0$ B. $x \le 2$ C. $x \ge -2$ D. $x \ge 2$

6. What is the domain of $f(x) = \sqrt{x+6}$?

domain is all x values greater than −6



domain is all greater than

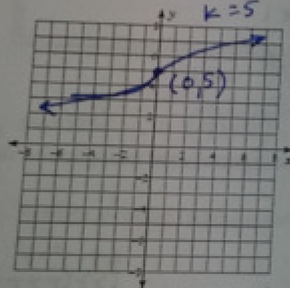
A. $x \ge 0$ B. $x \le -6$ C. $x \ge -6$ D. $x \le 6$

Pair 3	$y = \lvert x \rvert + 5$	
	$y = \lvert x \rvert - 5$	

1) $y = \sqrt[3]{x} + 5$

a=1
h=0
k=5

inflection point (0,5)



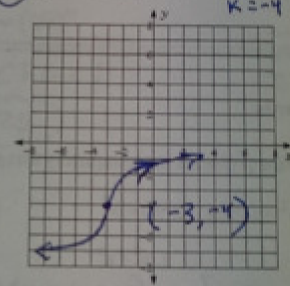
Domain: ℝ

Range: ℝ

2) $y = -4 + \sqrt[3]{x+3}$

a=1
h=-3
k=-4

infl
po
(-



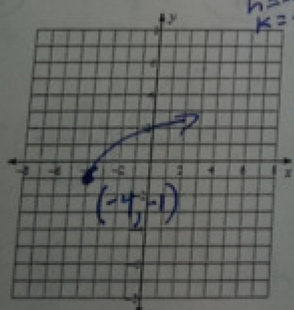
Domain: ℝ

Range: ℝ

3) $y = \sqrt{x+4} - 1$

a=1
h=-4
k=-1

endpoint (-4,-1)

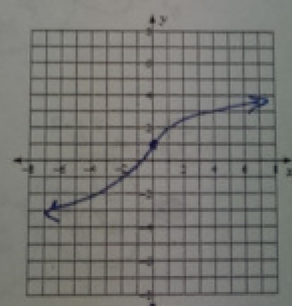


Domain: $x \geq -4$

Range: $x \geq -1$

4) $y = \sqrt[3]{x} + 1$

a=1
h=0
k=1

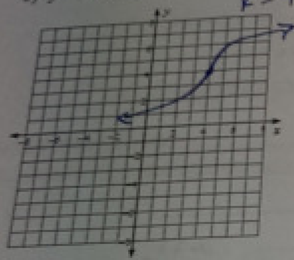


Domain: ℝ

Range: ℝ

5) $y=2\sqrt[3]{x-4}+4$

$a=2$
$h=4$
$k=4$

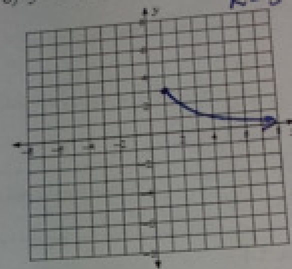


Domain: $\mathbb{R}$

Range: $\mathbb{R}$

6) $y=-2\sqrt{x-1}+3$

$a=-2$ (branch goes down when a is negati
$h=1$
$k=3$



Domain: $x \geq 1$

Range: $y \leq 3$

aws of Exponents

Write the following problems out completely using only the definition of exponents and multiplica ivision.

xample: $x^3 \cdot x^2 = (x \cdot x \cdot x) \cdot (x \cdot x) = x^5$

$a^4 \cdot a^3$

$(a \cdot a \cdot a)(a \cdot a \cdot a) = a^7$

2. $n^5 \cdot n^4$

$(n \cdot n \cdot n \cdot n \cdot n) \cdot (n \cdot n \cdot n \cdot n) = n^9$

3. $x^2 \cdot x^9$

$(x \cdot x) \cdot (x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) = x$

4. $y \cdot y$

$(y) \cdot ($

mple: $\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = \frac{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x}} = \frac{x^3}{1} = x^3$ ←Notice how anything divided by its

$= y$

6. $\frac{a^7}{a^3} = a^4$

7. $\frac{m^6}{m^8} = m^{-2} = \frac{1}{m^2}$

8. $\frac{x^5}{x^6}$

$= x^2 y^{-2} =$

x^2

10. $\frac{a^3 b}{a^8 b^3} = a^{-5} b^{-2}$

$= \frac{1}{}$

11. $\frac{2x^5}{6x^6} = \frac{1}{3x}$

12. $\frac{x^{11}}{x^{18} y^{11}}$

Example:

Find $49^{\frac{3}{2}}$

Solution: We can rewrite the problem.

$$49^{\frac{3}{2}} = (49^3)^{\frac{1}{2}} = \sqrt{49^3} = \left(\sqrt{49}\right)^3 = 7^3 = 343$$

The Rational Exponent Theorem

For any real number a, root n, and exponent m, the following is always true…

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

use the "PIB" chart

1. Find $125^{\frac{4}{3}}$ $= \left(\sqrt[3]{125}\right)^4 = 5^4 = 625$

2. Rewrite $x^{\frac{5}{6}}$ using radical notation. $\sqrt[6]{x^5}$

Simplify the following expressions.

a. $x^{\frac{1}{2}} \cdot x^{\frac{3}{4}} = x^{\frac{1}{2}+\frac{3}{4}} = x^{\frac{2}{4}+\frac{3}{4}} = x^{\frac{5}{4}}$

b. $\left(a^{\frac{1}{3}}\right)^2 = a^{\frac{2}{3}}$

c. $\dfrac{b^{\frac{3}{4}}}{b^{\frac{1}{3}}} = b^{\frac{3}{4}-\frac{1}{3}} = b^{\frac{9}{12}-\frac{4}{12}} = b^{\frac{5}{12}}$

d. $8^{-\frac{2}{3}} = \dfrac{1}{8^{\frac{2}{3}}} = \dfrac{1}{\left(\sqrt[3]{8}\right)^2} = \dfrac{1}{2^2} = \dfrac{1}{4}$

e. Skip Simplify $\dfrac{4x^{\frac{2}{3}}y^4}{16x^2y^{\frac{5}{6}}}$

f. Skip Simplify $\dfrac{\left(2x^{\frac{1}{2}}y^6\right)^{\frac{2}{3}}}{4x^{\frac{5}{4}}y^{\frac{9}{4}}}$

g. Simplify $\sqrt{a} \cdot \sqrt[3]{a^2}$

$a^{\frac{1}{2}} \cdot a^{\frac{2}{3}}$

$a^{\frac{1}{2}+\frac{2}{3}} = a^{\frac{3}{6}+\frac{4}{6}} = a^{\frac{7}{6}} = \sqrt[6]{a^7}$

Skills Practice

Rational Exponents

Using Properties of Rational Exponents

Use the properties of rational exponents to simplify the expression.

a. $x^{2/3} \cdot x^{1/9} = x^{(2/3+1/9)} = x^{(6/9+1/9)} = x^{7/9}$

b. $(2^3x^6)^{1/3} = (2^3)^{1/3} \cdot (x^6)^{1/3} = 2^{(3 \cdot 1/3)} \cdot x^{(6 \cdot 1/3)} = 2^1 \cdot x^2 = 2x^2$

c. $\dfrac{x^{3/4}}{x} = x^{(3/4-1)} = x^{(3/4-4/4)} = x^{-1/4} = \dfrac{1}{x^{1/4}}$

6.2 Intermediate Algebra Rational Exponents

Write each expression in radical form.

1. $3^{\frac{1}{6}} = \sqrt[6]{3}$

2. $8^{\frac{1}{5}} = \left(\sqrt[5]{8}\right)^1$

3. $12^{\frac{2}{3}} = \left(\sqrt[3]{12}\right)^2$

4. $(s^3)^{\frac{3}{5}} = \sqrt[5]{s^9}$

Write each radical using rational exponents.

5. $\sqrt{51}$ $= 51^{\frac{1}{2}}$

6. $\sqrt[3]{37}$ $= 37^{\frac{1}{3}}$

7. $\sqrt[4]{15^3}$ $15^{\frac{3}{4}}$

8. $\sqrt[3]{6xy^2}$ $= (6^{\frac{1}{3}} x^{\frac{1}{3}} y^{\frac{2}{3}})$

Evaluate each expression.

9. $32^{\frac{1}{5}}$ $= 2$

10. $81^{\frac{1}{4}}$

11. $27^{-\frac{1}{3}}$ $= \frac{1}{3}$

12. $4^{-\frac{1}{2}}$

13. $16^{\frac{3}{2}}$ $= 64$

14. $(-243)^{\frac{4}{5}}$

15. $27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}$ $= 27^{\frac{6}{3}} = 27^2 = 729$

16. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

Simplify each expression.

17. $c^{\frac{12}{5}} \cdot c^{\frac{3}{5}}$ $= c^{\frac{12}{5} + \frac{3}{5}} =$

18. $m^{\frac{2}{9}} \cdot m^{\frac{16}{9}}$

9. $\left(q^{\frac{1}{2}}\right)^3$ $= q^{\frac{3}{2}}$

20. $p^{-\frac{1}{5}}$

1. $x^{-\frac{6}{11}}$ $= \frac{1}{x^{\frac{6}{11}}} = \sqrt[11]{x^6}$

22. $\frac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$

$(a^{2/3}\, a^{1/5})^{3/4} =$

$(a^{\frac{10}{15}} \cdot a^{\frac{3}{15}})^{\frac{3}{4}}$

Solving Radical Equations and Inequalities

Solve each equation.

1. $\sqrt{x+6} = 7$

$x+6 = 49$

$x = 43$

2. $\sqrt{5x} = 10$

$5x = 100$

$x = 20$

3. $\sqrt{2x+5} = \sqrt{3x-1}$

$2x+5 = 3x-1$

$6 = x$

4. $\sqrt{x+4} = 3\sqrt{x}$

$x+4 = 9x$

$4 = 8x$

$\frac{1}{2} = x$

9. $4(x-3)^{\frac{1}{2}} = 8$

$(x-3)^{\frac{1}{2}} = 2$

$x-3 = 4$

$x = 7$

10. $4(x-12)^{\frac{1}{3}} = -16$

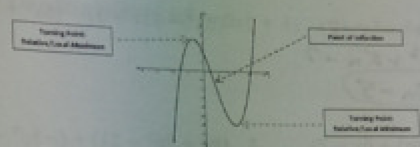
$(x-12)^{\frac{1}{3}} = -4$

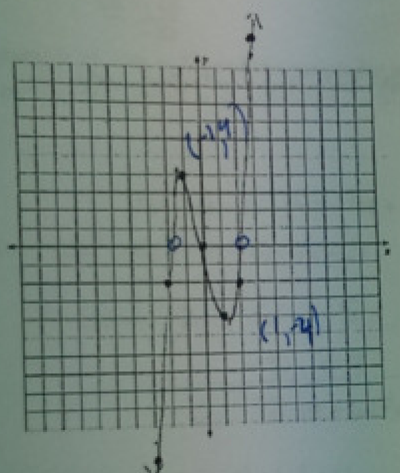
$x-12 = 64$

$x = 76$

A cubic function is a function that can be written in the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$. Graphs of cubic functions have interesting shapes.

Let's take a look at the graph of $y = x^3 - 2x^2 - 4x + 2$.





Domain: $\mathbb{R}$

Range: $\mathbb{R}$

relative minimum: (1,-4)

relative maximum: (-1,4)

increasing interval(s): skip

decreasing interval(s): skip

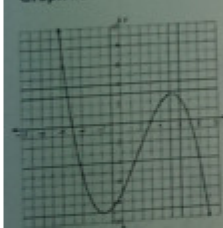
x-intercept(s): (-1.7,0)

y-intercept: (0,0)

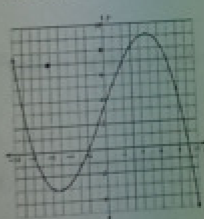
ind the S

skip

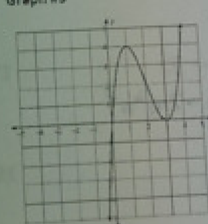
Graph #1



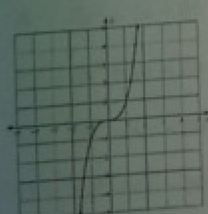
Graph #2



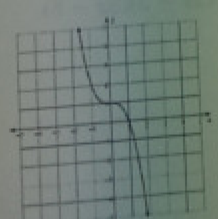
Graph #3



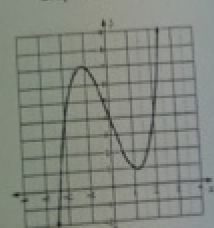
Graph #4



Graph #5



Graph #6



Polynomial Operations Review

Add/Subtract the Polynomials:

1. $(6n^3 + 3n^2 + 2) + (-2n^3 + 5n - 7)$

   $6n^3 + 3n^2 + 2 - 2n^3 + 5n - 7$

   $4n^3 + 3n^2 + 5n - 5$

2. $(9x^2 - 11x + 3) - (6x^2 + 3x - 12)$

3. $(-7y^7 - 2y^4 + y) - (-4y^7 + 6y - 2)$

   $-7y^7 - 2y^4 + y - 4y^7 - 6y + 2$

   $-11y^7 - 2y^4 - 5y + 2$

4. $(6p + 2p^3 - 11) + (-3p^3 - p + 4)$

5. $(-8u^2 + 5u - 2) - (3u^2 + 6u - 2)$

   $-8u^2 + 5u - 2 - 3u^2 - 6u + 2$

   $-11u^2 - 1u$

6. $(4m^3 + 2m^2 - 4m) + (m^3 - 5m^2 - m)$

7. $(-4a + 2a^3) - (-3a^3 + 9a^2 - 7a)$

   $-4a + 2a^3 + 3a^3 - 9a^2 + 7a$

   $5a^3 + 3a - 9a^2$

8. $(7b + 3b^2 - 8b^3) - (1 + 4b - 5b^2)$

Multiplying Polynomials

12. $-5(2c^2 - d^2)$

    $-10c^2 + 5d^2$

13. $x^2(2x + 9)$

14. $2q(3pq + 4q^4)$

    $6pq^2 + 8q^5$

15. $8w(hk^2 + 10h^3m^4 - 6k^5w^3)$

16. $m^2n^3(-4m^2n^2 - 2mnp - 7)$

    $-4m^4n^5 - 2m^3n^4p - 7m^2n^3$

17. $-3s^2y(-2s^4y^2 + 3sy^3 + 4)$

18. $(c + 2)(c + 8)$

    $c^2 + 10c + 16$

19. $(z - 7)(z + 4)$

20. $(a - 5)^2$

    $(a-5)(a-5)$

    $a^2 - 10a + 25$

21. $(2x - 3)(3x - 5)$

Example 2 Use long division to find $(x^3 - 8x^2 + 4x - 9) \div (x - 4)$.

$$
\begin{array}{r}
x^2 - 4x - 12 \\
x - 4 \overline{) x^3 - 8x^2 + 4x - 9} \\
\underline{(-) x^3 - 4x^2 } \\
-4x^2 + 4x \\
\underline{(-) -4x^2 + 16x } \\
-12x - 9 \\
\underline{(-) -12x + 48} \\
-57
\end{array}
$$

The quotient is $x^2 - 4x - 12$, and the remainder is -57.

Therefore $\frac{x^3 - 8x^2 + 4x - 9}{x - 4} = x^2 - 4x - 12 - \frac{57}{x - 4}$.

1. Divide $(2x^4 + 6x^3 + 5x - 6) \div (x + 2)$

$$
\begin{array}{r|rrrr}
-2 & 2 & 6 & 5 & -6 \\
 & & -4 & -4 & -2 \\
\hline
 & 2 & 2 & 1 & -8
\end{array}
$$

$2x^2 + 2x + 1 + \frac{-8}{x+}$

2. Divide: $(3x^4 + 2x^3 - 8x + 6) \div (x^2 - 1)$

skip

3. Divide: skip

$(6x^3 - 11x^2 - 47x - 20) \div (2x + 1)$

A $3x^2 - 7x - 20$

B $3x^2 + 7x - 20$

C $3x^2 - 4x - 20$

D $3x^2 + 4x - 20$

	…ply the sum, -3, by r: $-3 \cdot 1 = -3$. Write the product under the next coefficient and add: $5 + (-3) = 2$.	$\begin{array}{r\|rrrr} 1 & 2 & -5 & 5 & -2 \\ & & 2 & -3 & \\ \hline & 2 & -3 & 2 & \end{array}$
Step 5	Multiply the sum, 2, by r: $2 \cdot 1 = 2$. Write the product under the next coefficient and add: $-2 + 2 = 0$. The remainder is 0.	$\begin{array}{r\|rrrr} 1 & 2 & -5 & 5 & -2 \\ & & 2 & -3 & 2 \\ \hline & 2 & -3 & 2 & 0 \end{array}$

Thus, $(2x^3 - 5x^2 + 5x - 2) \div (x - 1) = 2x^2 - 3x + 2$.

se Synthetic Division below to find the remaining factors (factor down to quad

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

13. $x^3 + 2x^2 - x - 2; x + 1$

14. $x^3 + x^2 - 5x + 3; x - 1$

15. $x^3 + 3x^2 - 4x - 12; x + 3$

16. $x^3 - 6x^2 + 11x - 6; x - 3$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -1 & -2 \\ & & -1 & -1 & 2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$x^2 + x - 2$

$\boxed{(x+2)(x-1)}$

remaining factors

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -5 & 3 \\ & & 1 & 2 & -3 \\ \hline & 1 & 2 & -3 & 0 \end{array}$$

$x^2 + 2x - 3$

$\boxed{(x+3)(x-1)}$