Symmetry in Graphs of Polar Equations

It is sometimes helpful to analyze polar equations for certain properties that predict symmetry in the graph of the equation. The following rules guarantee the existence of symmetry in the graph.

However, the graphs of some polar equations exhibit symmetry even though the rules do not predict it.

1. If replacing $\theta$ by $-\theta$ in the equation is symmetric with respect to the line containing the polar axis (the $y$-axis in the rectangular coordinate system).
2. If replacing $\theta$ by $\pi - \theta$ in the equation is symmetric with respect to the pole.
3. If replacing $r$ by $-r$ in the equation is symmetric with respect to the line containing the polar axis.

Example

Identify the symmetry of and graph $r = \cos 2\theta$.

See students' graphs.

Identify the symmetry of and graph each polar equation on polar grid paper.

1. $r = 1 + \cos \theta$ (cardioid)
2. $r = 3 \sin 3\theta$ (rose)
3. $r = 1 + 2 \cos \theta$ (limaçon)
4. $r = 2 + 2 \sin \theta$ (cardioid)
5. $r = 0.5\theta$ (spiral of Archimedes)
6. $r^2 = 16 \cos 2\theta$ (lemniscate)

Graph each system of polar equations. Solve the system using algebra and trigonometry. Assume $0 \leq \theta < 2\pi$.

7. $r = 1 + 2 \sin \theta$
   $r = 2 + \sin \theta$
   $(3, \frac{\pi}{2})$
8. $r = 1 + \cos \theta$
   $r = 3 \cos \theta$
   $(1.5, \frac{\pi}{3}); (1.5, \frac{5\pi}{3})$

9. Design  Mikaela is designing a border for her stationery. Suppose she uses a rose curve. Determine an equation for designing a rose that has 8 petals with each petal 4 units long.
   Sample answer: $r = 4 \sin 4\theta$

Graph each polar equation. Identify the type of curve each represents.

1. $r = 1 + \cos \theta$
2. $r = 3 \sin 3\theta$
3. $r = 1 + 2 \cos \theta$
4. $r = 2 + 2 \sin \theta$
5. $r = 0.5\theta$
6. $r^2 = 16 \cos 2\theta$
7. $r = 1 + 2 \sin \theta$
   $r = 2 + \sin \theta$
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