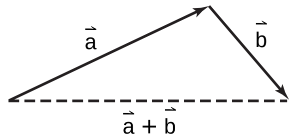
Chapter 8 Vectors and Parametric Equations

8-1 Geometric Vectors

Page 490 Check for Understanding

1. Sample answer:



Draw $\vec{a}$. Then draw $\vec{b}$ so that its initial point (tip) is on the terminal point (tail) of $\vec{a}$. Draw a dashed line from the initial point of $\vec{a}$ to the terminal point of $\vec{b}$. The dashed line is the resultant.

2. Sample answer: A vector has magnitude and direction. A line segment has only length. A vector can be represented by a directed line segment.

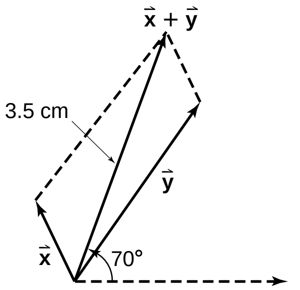
3. Sample answer: the velocities of an airplane and a wind current

4. No, they are opposites.

5-11. Answers may vary slightly.

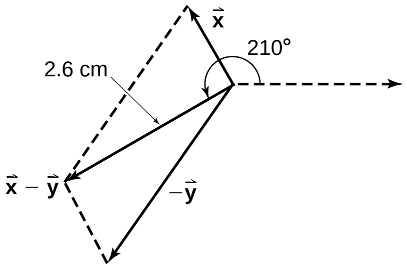
5. 1.2 cm, 120° **6.** 2.9 cm, 55° **7.** 1.4 cm, 20°

8.



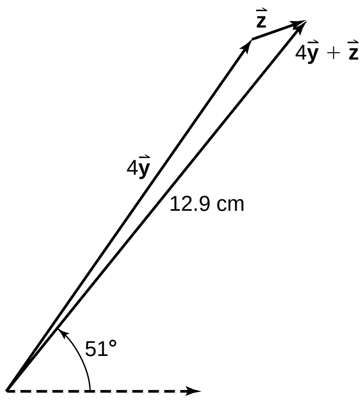
3.5 cm, 70°

9.



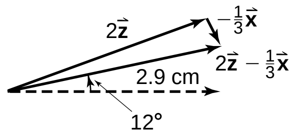
2.6 cm, 210°

10.



12.9 cm, 51°

11.

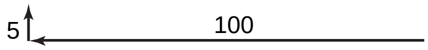


2.9 cm, 12°

12. $h = 2.9 \cos 55°$ $\quad v = 2.9 \sin 55°$

$h \approx 1.66$ cm $\quad v \approx 2.38$ cm

13a.



13b. Use the Pythagorean Theorem.

$c^2 = a^2 + b^2$

$c^2 = (100)^2 + (5)^2$

$c^2 = 10{,}025$

$c = \sqrt{10{,}025}$ or about 100.12 m/s

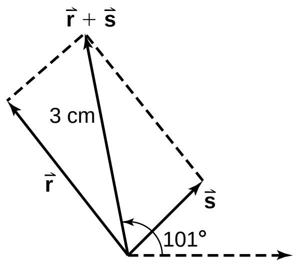
Pages 491–492 Exercises

14. 2.6 cm, 128° **15.** 1.4 cm, 45°

16. 2.1 cm, 14° **17.** 3.0 cm, 340°

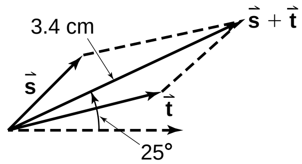
18-30. Answers may vary slightly.

18.



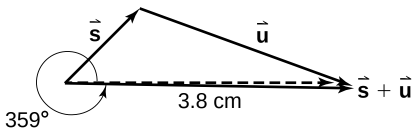
3 cm, 101°

19.



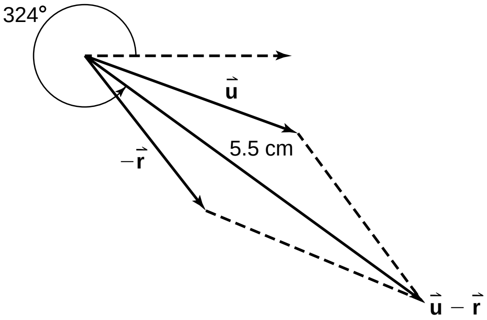
3.4 cm, 25°

20.



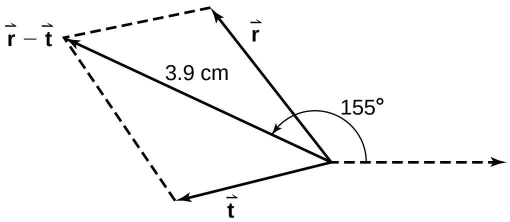
3.8 cm, 359°

21.



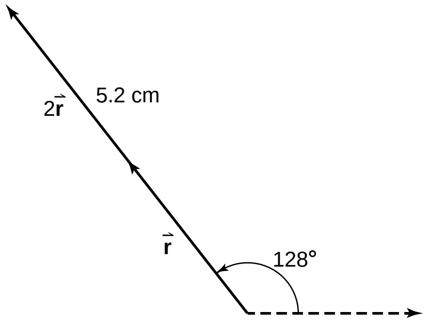
5.5 cm, 324°

22.



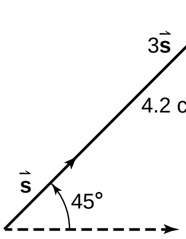
3.9 cm, 155°

23.



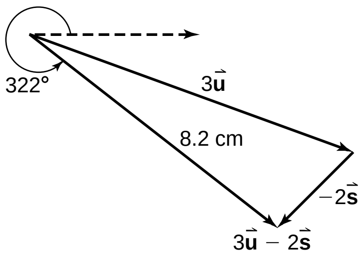
5.2 cm, 128°

24.



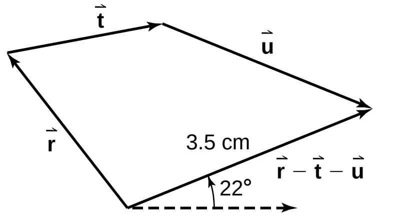
4.2 cm, 45°

25.



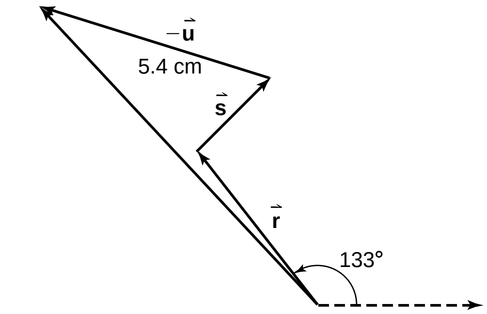
8.2 cm, 322°

26.



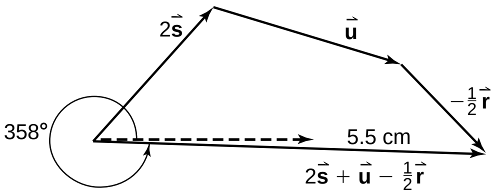
3.5 cm, 22°

27. $\vec{r} + \vec{s} - \vec{u}$



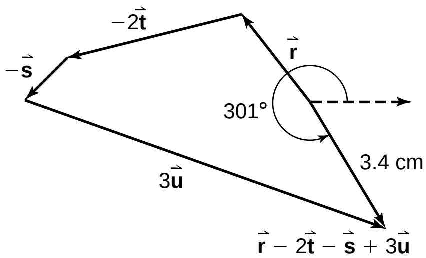
5.4 cm, 133°

28.



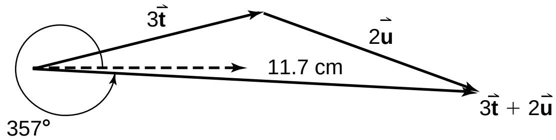
5.5 cm, 358°

29. Draw to scale:



3.4 cm, 301°

30. Draw to scale:



11.7 cm, 357°

31. $h = |2.6 \cos 128°|$ $\quad v = |2.6 \sin 128°|$
$h = 1.60$ cm $\quad v = 2.05$ cm

32. $h = |1.4 \cos 45°|$ $\quad v = |1.4 \sin 45°|$
$h = 0.99$ cm $\quad v = 0.99$ cm

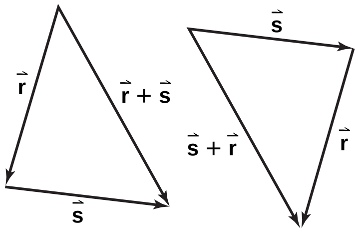
33. $h = |2.1 \cos 14°|$ $\quad v = |2.1 \sin 14°|$
$h = 2.04$ cm $\quad v = 0.51$ cm

34. $h = |3.0 \cos 340°|$ $\quad v = |3.0 \sin 340°|$
$h = 2.82$ cm $\quad v = 1.03$ cm

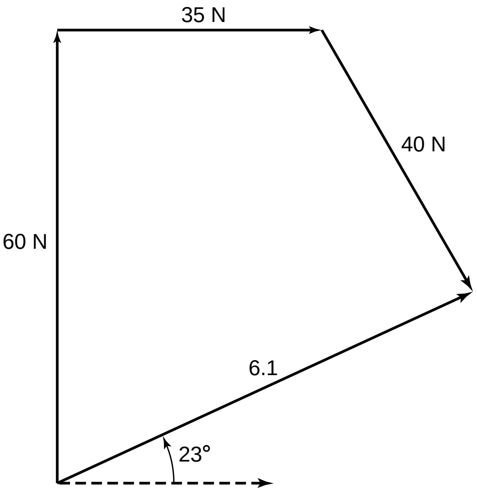
35. $c^2 = a^2 + b^2$
$c^2 = (29.2)^2 + (35.2)^2$
$c^2 = 2091.68$
$c = \sqrt{2091.68}$ or about 45.73 m

36. The difference of the vectors; sample answer: The other diagonal would be the sum of one of the vectors and the opposite of the other vector, so it would be the difference.

37. Yes; sample answer:

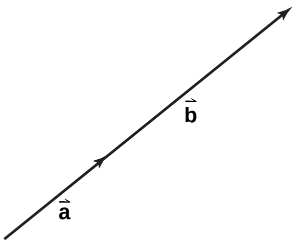


38.

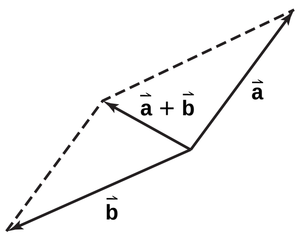


61 N, 23° north of east

39. Sometimes;



$|\vec{a} + \vec{b}| = 5$
$|\vec{a}| + |\vec{b}| = 2.3 + 2.7$
$= 5$



$|\vec{a} + \vec{b}| = 1.5$
$|\vec{a}| + |\vec{b}| = 2.3 + 2.7$
$= 5$

40a. $v = 1.5 \sin 52°$ $\quad h = 1.5 \cos 52°$
$v \approx 1.18$ N $\quad h \approx 0.92$ N

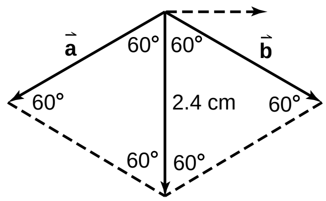
40b. $h = 1.5 \cos 78°$ $\quad v = 1.5 \sin 78°$
$h \approx 0.31$ N $\quad v \approx 1.47$ N

41. $h = 47 \cos 40°$ $\quad v = 47 \sin 40°$
$h \approx 36$ mph $\quad v \approx 30$ mph

42. It is true when $k = 1$ or when $\vec{a}$ is the zero vector.

43. $c^2 = a^2 + b^2$
$c^2 = (50)^2 + (50)^2$
$c = \sqrt{5000}$ or about 71 lb

44.



$\vec{a} + \vec{b} = 24$
equilateral triangle $\vec{a} = \vec{b} = 24$ lb

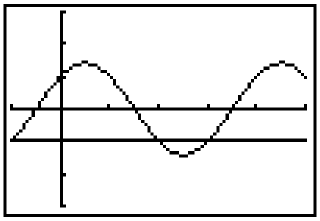
45. The origin is not in the interior of the acute angle.
$d_1 = -d_2$
$d_1 = \frac{x - y + 2}{\sqrt{-1^2 + (-1)^2}}$ or $\frac{x - y + 2}{-\sqrt{2}}$
$d_2 = \frac{y - 5}{\sqrt{0^2 + 1^2}}$ or $y - 5$

$$\frac{x - y + 2}{-\sqrt{2}} = -(y - 5)$$
$$x - y + 2 = \sqrt{2}(y - 5)$$
$$x - y + 2 = \sqrt{2}y - 5\sqrt{2}$$
$$x - y + 2 - \sqrt{2}y + 5\sqrt{2} = 0$$
$$x - (1 + \sqrt{2})y + 2 + 5\sqrt{2} = 0$$

46. $\csc \theta \cos \theta \tan \theta = \frac{1}{\sin \theta} \cdot \cos \theta \cdot \frac{\sin \theta}{\cos \theta}$
$= \frac{\sin \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta}$
$= 1$

47. $\frac{\pi}{4} + \pi n$ where n is an integer

48.



$\left[-\frac{\pi}{2}, \frac{5\pi}{2}\right]$ scl: $\frac{\pi}{2}$ by $[-3, 3]$ scl: 1
$x = \pi, \frac{3\pi}{2}$ for $0 \le x \le 2\pi$

49. $\tan 18°29' = \frac{5}{0.5b}$
$b = \frac{5}{0.5 \tan 18°29'}$
$b = 29.9$ cm

$\sin 18°29' = \frac{5}{h}$
$h = \frac{5}{\sin 18°29'}$
$h = 15.8$ cm

50. v_o = volume of original box
v_n = volume of new box
$v_o = \ell_o \times w_o \times h_o$
$=(w + 1) \times w \times 2w$
$=(w + 1)2w^2$
$= 2w^3 + 2w^2$
$v_n = \ell_n \times w_n \times h_n$
$= (w + 2) \times (w + 1) \times (2w + 2)$
$= (w^2 + 3w + 2)(2w + 2)$
$= 2w^3 + 8w^2 + 10w + 4$
$2w^3 + 8w^2 + 10w + 4 = 160$

w	2	8	10	−156
−1	2	6	4	−160
1	2	10	20	−136
2	2	12	34	−88
3	2	14	52	0

$w_o = 3$
$\ell_o = 2w$
$= 2 \cdot 3$ or 6
$h_o = w + 1$
$= 3 + 1$ or 4
So, the dimensions of the original box are
3 ft × 4 ft × 6 ft

51. $g(x) = \frac{x + 2}{(x - 1)(x + 3)}$
vertical: As x approaches 1 and −3, the expression approaches $+\infty$ or $-\infty$. So, $x = 1$ and $x = -3$ are vertical asymptotes.
horizontal: $y = \frac{x + 2}{x^2 + 2x - 3}$
$$y = \frac{\frac{x}{x^2} + \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{3}{x^2}}$$
$$y = \frac{\frac{1}{x} + \frac{2}{x^2}}{1 + \frac{2}{x} - \frac{3}{x^2}}$$
As x increases positively or negatively, the expression approaches 0. So, $y = 0$ is a horizontal asymptote.

52. Let x, $x + 2$, and $x + 4$ be 3 consecutive odd intergers.
$3x = 2(x + 4) + 3$ $\quad x + 4 = 15$
$3x = 2x + 8 + 3$
$3x - 2x = 11$
$x = 11$
The correct answer 15.

8-2 Algebraic Vectors

Pages 496–497 Check for Understanding

1. Sample answer: $\vec{\mathbf{a}} = \langle 8, 6\rangle$, $\vec{\mathbf{b}} = \langle 6, 8\rangle$; equal vectors have the same magnitude and direction.

2. Use $|\overrightarrow{XY}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ and replace the values for x and y.
$x(-5, -6)$, $y(3, -4)$
$|\overrightarrow{XY}| = \sqrt{[3 - (-5)]^2 + [-4 - (-6)]^2}$
$= \sqrt{(8)^2 + (-2)^2}$
$= \sqrt{64 + 4}$ or $\sqrt{68}$

3. Jacqui is correct. The representation is incorrect. $\langle 2, 0\rangle + \langle 0, -5\rangle$ is not equal to $5\langle 1, 0\rangle + (-2)\langle 0, 1\rangle$. The correct expression is $2\vec{\mathbf{i}} - 5\vec{\mathbf{j}}$.

4. $\overrightarrow{MP} = \langle -3 -2, 4 - (-1)\rangle$ or $\langle -5, 5\rangle$
$|\overrightarrow{MP}| = \sqrt{(-5)^2 + (5)^2}$
$= \sqrt{50}$ or $5\sqrt{2}$ units

5. $\overrightarrow{MP} = \langle 0 - 5, 5 - 6\rangle$ or $\langle -5, -1\rangle$
$|\overrightarrow{MP}| = \sqrt{(-5)^2 + (-1)^2}$
$= \sqrt{26}$ units

6. $\overrightarrow{MP} = \langle 4 - (-19), 0 - 4\rangle$ or $\langle 23, -4\rangle$
$|\overrightarrow{MP}| = \sqrt{(23)^2 + (-4)^2}$
$= \sqrt{545}$ units

7. $\vec{\mathbf{t}} = \vec{\mathbf{u}} + \vec{\mathbf{v}}$
$= \langle -1, 4\rangle + \langle 3, -2\rangle$
$= \langle -1 + 3, 4 + (-2)\rangle$ or $\langle 2, 2\rangle$

8. $\vec{\mathbf{t}} = \frac{1}{2}\vec{\mathbf{u}} - \vec{\mathbf{v}}$
$= \frac{1}{2}\langle -1, 4\rangle - \langle 3, -2\rangle$
$= \langle -\frac{1}{2}, 2\rangle - \langle 3, -2\rangle$
$= \langle -\frac{1}{2} - 3, 2 - (-2)\rangle$ or $\left\langle -3\frac{1}{2}, 4\right\rangle$

9. $\vec{\mathbf{t}} = 4\vec{\mathbf{u}} + 6\vec{\mathbf{v}}$
$= 4\langle -1, 4\rangle + 6\langle 3, -2\rangle$
$= \langle -4, 16\rangle + \langle 18, -12\rangle$
$= \langle -4 + 18, 16 + (-12)\rangle$ or $\langle 14, 4\rangle$

10. $\vec{\mathbf{t}} = -8\vec{\mathbf{u}}$
$= -8\langle -1, 4\rangle$
$= \langle -8(-1), -8(4)\rangle$ or $\langle 8, -32\rangle$

11. $|\langle 8, -6\rangle| = \sqrt{8^2 + (-6)^2}$
$= \sqrt{100}$ or 10
$8\vec{\mathbf{i}} - 6\vec{\mathbf{j}}$

12. $|\langle -7, -5\rangle| = \sqrt{(-7)^2 + (-5)^2}$
$= \sqrt{74}$
$-7\vec{\mathbf{i}} - 5\vec{\mathbf{j}}$

13. Let $\vec{\mathbf{T}}$ represent the force Terrell exerts.
Let $\vec{\mathbf{W}}$ represent the force Mr. Walker exerts.
$|\vec{\mathbf{T}}_x| = 400 \cos 65°$ $\quad |\vec{\mathbf{W}}_x| = 600 \cos 110°$
≈ 169.05 $\quad \approx -205.21$
$|\vec{\mathbf{T}}_y| = 400 \sin 65°$ $\quad |\vec{\mathbf{W}}_y| = 600 \sin 110°$
≈ 362.52 $\quad \approx 563.82$
$\vec{\mathbf{T}} = \langle 169.05, 362.52\rangle$, $\vec{\mathbf{W}} = \langle -205.21, 563.82\rangle$
$\vec{\mathbf{T}} + \vec{\mathbf{W}} = \langle -36.16, 926.34\rangle$
$|\vec{\mathbf{T}} + \vec{\mathbf{W}}| = \sqrt{(-36.16)^2 + (926.34)^2}$
≈ 927 N

Pages 497–499 Exercises

14. $\overrightarrow{YZ} = \langle 2 - 4, 8 - 2\rangle$ or $\langle -2, 6\rangle$
$|\overrightarrow{YZ}| = \sqrt{(-2)^2 + 6^2}$
$= \sqrt{40}$ or $2\sqrt{10}$

15. $\overrightarrow{YZ} = \langle -1 - (-5), 2 - 7\rangle$ or $\langle 4, -5\rangle$
$|\overrightarrow{YZ}| = \sqrt{4^2 + (-5)^2}$
$= \sqrt{41}$

16. $\overrightarrow{YZ} = \langle 1 - (-2), 3 - 5\rangle$ or $\langle 3, -2\rangle$
$|\overrightarrow{YZ}| = \sqrt{3^2 + (-2)^2}$
$= \sqrt{13}$

17. $\overrightarrow{YZ} = \langle 0 - 5, -3 - 4\rangle$ or $\langle -5, -7\rangle$
$|\overrightarrow{YZ}| = \sqrt{(-5)^2 + (-7)^2}$
$= \sqrt{74}$

18. $\overrightarrow{YZ} = \langle 0 - 3, 4 - 1\rangle$ or $\langle -3, 3\rangle$
$|\overrightarrow{YZ}| = \sqrt{(-3)^2 + 3^2}$
$= \sqrt{18}$ or $3\sqrt{2}$

19. $\overrightarrow{YZ} = \langle 1 - (-4), 19 - 12\rangle$ or $\langle 5, 7\rangle$
$|\overrightarrow{YZ}| = \sqrt{5^2 + 7^2}$
$= \sqrt{74}$

20. $\overrightarrow{YZ} = \langle 7 - 5, 6 - 0\rangle$ or $\langle 2, 6\rangle$
$|\overrightarrow{YZ}| = \sqrt{2^2 + 6^2}$
$= \sqrt{40}$ or $2\sqrt{10}$

21. $\overrightarrow{YZ} = \langle 23 - 14, -14 - (-23)\rangle$ or $\langle 9, 9\rangle$
$|\overrightarrow{YZ}| = \sqrt{9^2 + 9^2}$
$= \sqrt{162}$ or $9\sqrt{2}$

22. $\overrightarrow{AB} = \langle 36 - 31, -45 - (-33)\rangle$ or $\langle 5, -12\rangle$
$|\overrightarrow{AB}| = \sqrt{5^2 + (-12)^2}$
$= \sqrt{169}$ or 13

23. $\vec{\mathbf{a}} = \vec{\mathbf{b}} + \vec{\mathbf{c}}$
$= \langle 6, 3\rangle + \langle -4, 8\rangle$
$= \langle 6 + (-4), 3 + 8\rangle$ or $\langle 2, 11\rangle$

24. $\vec{\mathbf{a}} = 2\vec{\mathbf{b}} + \vec{\mathbf{c}}$
$= 2\langle 6, 3\rangle + \langle -4, 8\rangle$
$= \langle 12, 6\rangle + \langle -4, 8\rangle$
$= \langle 12 + (-4), 6 + 8\rangle$ or $\langle 8, 14\rangle$

25. $\vec{\mathbf{a}} = \vec{\mathbf{b}} + 2\vec{\mathbf{c}}$
$= \langle 6, 3\rangle + 2\langle -4, 8\rangle$
$= \langle 6, 3\rangle + \langle -8, 16\rangle$
$= \langle 6 + (-8), 3 + 16\rangle$ or $\langle -2, 19\rangle$

26. $\vec{\mathbf{a}} = 2\vec{\mathbf{b}} + 3\vec{\mathbf{c}}$
$= 2\langle 6, 3\rangle + 3\langle -4, 8\rangle$
$= \langle 12, 6\rangle + \langle -12, 24\rangle$
$= \langle 12 + (-12), 6 + 24\rangle$ or $\langle 0, 30\rangle$

27. $\vec{\mathbf{a}} = -\vec{\mathbf{b}} + 4\vec{\mathbf{c}}$
$= -\langle 6, 3\rangle + 4\langle -4, 8\rangle$
$= \langle -6, -3\rangle + \langle -16, 32\rangle$
$= \langle -6 + (-16), -3 + 32\rangle$ or $\langle -22, 29\rangle$

28. $\vec{\mathbf{a}} = \vec{\mathbf{b}} - 2\vec{\mathbf{c}}$
$= \langle 6, 3\rangle - 2\langle -4, 8\rangle$
$= \langle 6, 3\rangle - \langle -8, 16\rangle$
$= \langle 6 - (-8), 3 - (16)\rangle$ or $\langle 14, -13\rangle$

29. $\vec{\mathbf{a}} = 3\vec{\mathbf{b}}$
$= 3\langle 6, 3\rangle$
$= \langle 3 \cdot 6, 3 \cdot 3\rangle$ or $\langle 18, 9\rangle$

30. $\vec{\mathbf{a}} = -\frac{1}{2}\vec{\mathbf{c}}$
$= -\frac{1}{2}\langle -4, 8\rangle$
$= \left\langle -\frac{1}{2} \cdot (-4), -\frac{1}{2} \cdot 8\right\rangle$ or $\langle 2, -4\rangle$

31. $\vec{\mathbf{a}} = 6\vec{\mathbf{b}} + 4\vec{\mathbf{c}}$
$= 6\langle 6, 3\rangle + 4\langle -4, 8\rangle$
$= \langle 36, 18\rangle + \langle -16, 32\rangle$
$= \langle 36 + (-16), 18 + 32\rangle$ or $\langle 20, 50\rangle$

32. $\vec{\mathbf{a}} = 0.4\vec{\mathbf{b}} - 1.2\vec{\mathbf{c}}$
$= 0.4\langle 6, 3\rangle - 1.2\langle -4, 8\rangle$
$= \langle 2.4, 1.2\rangle - \langle -4.8, 9.6\rangle$
$= \langle 2.4 - (-4.8), 1.2 - 9.6\rangle$ or $\langle 7.2, -8.4\rangle$

33. $\vec{\mathbf{a}} = \frac{1}{3}(2\vec{\mathbf{b}} - 5\vec{\mathbf{c}})$
$= \frac{1}{3}(2\langle 6, 3\rangle - 5\langle -4, 8\rangle)$
$= \frac{1}{3}(\langle 12, 6\rangle - \langle -20, 40\rangle)$
$= \frac{1}{3}(\langle 12 - (-20), 6 - 40\rangle)$
$= \frac{1}{3}\langle 32, -34\rangle$ or $\left\langle \frac{32}{3}, -\frac{34}{3}\right\rangle$

34. $\vec{\mathbf{a}} = (3\vec{\mathbf{b}} + \vec{\mathbf{c}}) + 5\vec{\mathbf{b}}$
$= 3\langle 6, 3\rangle + \langle -4, 8\rangle + 5\langle 6, 3\rangle$
$= \langle 18, 9\rangle + \langle -4, 8\rangle + \langle 30, 15\rangle$
$= \langle 18 + (-4) + 30, 9 + 8 + 15\rangle$ or $\langle 44, 32\rangle$

35. $3\vec{\mathbf{m}} - 2.5\vec{\mathbf{n}} = 3\langle -5, -6\rangle - 2.5\langle 6 - 9\rangle$
$= [-15, -18\rangle - \langle 15, -22.5\rangle$
$= [-15 - (15), -18 - (-22.5\rangle$
$= \langle -30, 4.5\rangle$

36. $|\langle 3, 4\rangle| = \sqrt{3^2 + 4^2}$
$= \sqrt{25}$ or 5
$3\vec{\mathbf{i}} + 4\vec{\mathbf{j}}$

37. $|\langle 2, -3\rangle| = \sqrt{2^2 + (-3)^2}$
$= \sqrt{13}$
$2\vec{\mathbf{i}} - 3\vec{\mathbf{j}}$

38. $|\langle -6, -11\rangle| = \sqrt{(-6)^2 + (-11)^2}$
$= \sqrt{157}$
$-6\vec{\mathbf{i}} - 11\vec{\mathbf{j}}$

39. $|\langle 3.5, 12\rangle| = \sqrt{(3.5)^2 + 12^2}$
$= \sqrt{156.25}$ or 12.5
$3.5\vec{\mathbf{i}} + 12\vec{\mathbf{j}}$

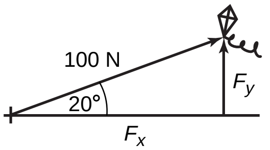
40. $|\langle -4, 1\rangle| = \sqrt{(-4)^2 + 1^2}$
$= \sqrt{17}$
$-4\vec{\mathbf{i}} + \vec{\mathbf{j}}$

41. $|\langle -16, -34\rangle| = \sqrt{(-16)^2 + (-34)^2}$
$= \sqrt{1412}$ or $2\sqrt{353}$
$-16\vec{\mathbf{i}} - 34\vec{\mathbf{j}}$

42. $\overrightarrow{ST} = \langle -4 - (-9), -3 - 2\rangle$ or $\langle 5, -5\rangle$
$5\vec{\mathbf{i}} - 5\vec{\mathbf{j}}$

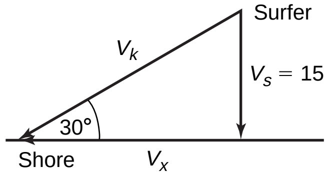
43. Student needs to show that
$(\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2) + \vec{\mathbf{v}}_3 = \vec{\mathbf{v}}_1 + (\vec{\mathbf{v}}_2 + \vec{\mathbf{v}}_3)$
$(\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2) + \vec{\mathbf{v}}_3 = [\langle a, b\rangle + \langle c, d\rangle] + \langle e, f\rangle$
$= \langle a + c, b + d\rangle + \langle e, f\rangle$
$= \langle (a + c) + e, (b + d) + f\rangle$
$= \langle a + (c + e), b + (d + f)\rangle$
$= \langle a, b\rangle + \langle c + e, d + f\rangle$
$= \langle a, b\rangle + [\langle c, d\rangle + \langle e, f\rangle]$
$= \vec{\mathbf{v}}_1 + (\vec{\mathbf{v}}_2 + \vec{\mathbf{v}}_3)$

44a.



44b. $\sin 20° = \frac{|\vec{\mathbf{F}}_y|}{100}$
$|\vec{\mathbf{F}}_y| = 100 \sin 20°$
≈ 34 N

45a.



45b. $\sin 30° = \frac{15}{|\vec{\mathbf{v}}_k|}$

$|\vec{\mathbf{V}}_k| = \frac{15}{\sin 30°}$

≈ 30 mph

46a. Since $\overrightarrow{QR} + \overrightarrow{ST} = 0$, $\overrightarrow{QR} = -\overrightarrow{ST}$.
So, they are opposites.

46b. $\overrightarrow{QR}$ and $\overrightarrow{ST}$ have the same magnitude, but opposite direction. So, they are parallel. Quadrilateral $QRST$ is a parallelogram.

47a. $t = \frac{d}{r}$

$= \frac{150 \text{ m}}{5 \text{ m/s}}$ or 30 s

47b. $d = rt$

$= (1.0 \text{ m/s})(30 \text{ s})$ or 30 m

47c. $|\vec{V}_B + \vec{V}_C| = |\langle 0.5\rangle + \langle 1.0\rangle|$

$= \sqrt{1^2 + 5^2}$

$= \sqrt{26}$ or about 5.1 m/s

48. $\cos\theta = \frac{(x_2 - x_1)}{|\vec{\mathbf{v}}|} \rightarrow (x_2 - x_1) = \vec{\mathbf{v}} \cos\theta$

$\sin\theta = \frac{(y_2 - y_1)}{|\vec{\mathbf{v}}|} \rightarrow (y_2 - y_1) = \vec{\mathbf{v}} \sin\theta$

49. $\overrightarrow{PQ} = \langle -2 - 8, 5 - (-7)\rangle$

$= \langle -10, 12\rangle$

$|\overrightarrow{PQ}| = \sqrt{(-10)^2 + 12^2}$

$= \sqrt{244}$

$\overrightarrow{RS} = \langle 7 - 8, 0 - (-7)\rangle$

$= \langle -1, 7\rangle$

$|\overrightarrow{RS}| = \sqrt{(-1)^2 + 7^2}$

$= \sqrt{50}$

none

50. $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

$d = \frac{|3(-1) - 7(4) - 1|}{\sqrt{3^2 + (-7)^2}}$

$d = \frac{32}{\sqrt{58}}$ or about 4.2

51. $\sin 255° = \sin(225° + 30°)$

$= \sin 225° \cos 30° + \cos 225° \sin 30°$

$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{2}}{2}\right)\frac{1}{2}$

$= -\frac{\sqrt{6} - \sqrt{2}}{4}$

52. $y = A \sin(kx + c)$

A: $|A| = 17$

$A = 17$ or -17

k: $\frac{2\pi}{k} = \frac{\pi}{4}$

$k = 8$

c: $-\frac{c}{k} = -60°$

$-\frac{c}{8} = -60°$

$c = 480°$

$y = \pm 17 \sin(8x + 480°)$

53. Let $a = 400$, $b = 600$, $C = 46.\overline{3}°$

$c^2 = 400^2 + 600^2 - 2(400)(600)\cos 46.\overline{3}°$

$c^2 \approx 18.8578.39$

$c \approx 434$

$P = a + b + c$

$\approx 400 + 600 + 434$

≈ 1434 ft

$s = \frac{1}{2}(a + b + c)$

$s \approx \frac{1}{2}(1434)$ or 717

$k \approx \sqrt{s(s - a)(s - b)(s - c)}$

$k \approx \sqrt{717(717 - 400)(717 - 600)(717 - 434)}$

$k \approx \sqrt{7{,}525{,}766{,}079}$

$k \approx 86{,}751$ sq ft

54. Sample answer:

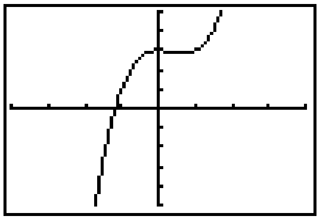
$f(x) = 3x^2 - 2x + 1$			
r	3	−2	1
1	3	1	2
2	3	4	9

An upper bound is 2.

$f(-x) = 3x^2 - 2x + 1$			
r	3	2	1
1	3	5	6
2	3	8	17

A lower bound is −1.

55.



[−4, 4] scl: 1 by [−4, 4] scl: 1
max: (0, 3), min: (0.67, 2.85)

56.

$f(x) = x^2 + 3x + 1$	
x	$f(x)$
−10,000	99,970,001
−1000	997,001
−100	9701
−10	71
0	1
10	131
100	10,301
1000	1,003,001
10,000	100,030,001

$y \rightarrow \infty$ as $x \rightarrow \infty$, $y \rightarrow \infty$ as $x \rightarrow -\infty$

57. $7x + 1 > 7x - 1$

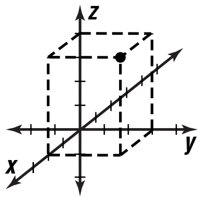
$1 > -1$

This statement is true regardless of the value of x, so it is true for all real values of x.
The correct choice is A.

8-3 Vectors in Three-Dimensional Space

Pages 502–503 Check for Understanding

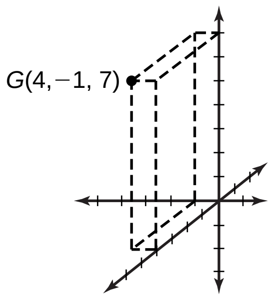
1. Sample answer: sketch a coordinate system with the xy-axes on the horizontal, and the z-axis pointing up. Then, vector $2\vec{\mathbf{i}}$ is two units along the x-axis, vector $3\vec{\mathbf{j}}$ is three units along the y-axis, and vector $4\vec{\mathbf{k}}$ is four units along the z-axis. Draw broken lines to represent three planes.



2. Sample answer: To find the components of the vector, you will need the direction (angle) with the horizontal axis. Using trigonometry, you can obtain the components of the vector.

3. Sample answer: Neither is correct. The sign for the $\vec{\mathbf{j}}$-term must be the same (−), and the coefficient for the $\vec{\mathbf{k}}$-term is 0, so the correct way to express the vector as a sum of unit vectors is $\vec{\mathbf{i}} - 4\vec{\mathbf{j}}$.

4.



$$|\overrightarrow{OG}| = \sqrt{4^2 + (-1)^2 + 7^2} = \sqrt{66}$$

5. $\overrightarrow{RS} = \langle 3 - (-2), 9 - 5, -3 - 8\rangle$ or $\langle 5, 4, -11\rangle$

$|\overrightarrow{RS}| = \sqrt{5^2 + 4^2 + (-11)^2} = \sqrt{162}$ or $9\sqrt{2}$

6. $\overrightarrow{RS} = \langle 10 - 3, -4 - 7, 0 - (-1)\rangle$ or $\langle 7, -11, 1\rangle$

$|\overrightarrow{RS}| = \sqrt{7^2 + (-11)^2 + 1^2} = \sqrt{171}$ or $3\sqrt{19}$

7. $\vec{\mathbf{a}} = 3\vec{\mathbf{f}} + \vec{\mathbf{g}}$

$= 3\langle 1, -3, -8\rangle + \langle 3, 9, -1\rangle$

$= \langle 3, -9, -24\rangle + \langle 3, 9, -1\rangle$

$= \langle 3 + 3, -9 + 9, -24 + (-1)\rangle$ or $\langle 6, 0, -25\rangle$

8. $\vec{\mathbf{a}} = 2\vec{\mathbf{g}} - 5\vec{\mathbf{f}}$

$= 2\langle 3, 9, -1\rangle - 5\langle 1, -3, -8\rangle$

$= \langle 6, 18, -2\rangle - \langle 5, -15, -40\rangle$

$= \langle 6 - 5, 18 - (-15), -2 - (-40)\rangle$ or $\langle 1, 33, 38\rangle$

9. $\overrightarrow{EF} = \langle 6 - (-5), -6 - (-2), 6 - 4\rangle$

$= \langle 11, -4, 2\rangle$

$11\vec{\mathbf{i}} - 4\vec{\mathbf{j}} + 2\vec{\mathbf{k}}$

10. $\overrightarrow{EF} = \langle -12 - (-12), 17 - 15, -22 - (-9)\rangle$

$= \langle 0, 2, -13\rangle$

$2\vec{\mathbf{j}} - 13\vec{\mathbf{k}}$

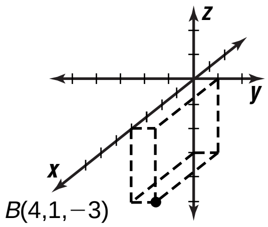
11. $|\langle 132, 3454, 0\rangle| = \sqrt{132^2 + 3454^2 + 0^2}$

$= \sqrt{11{,}947{,}540}$

≈ 3457 N

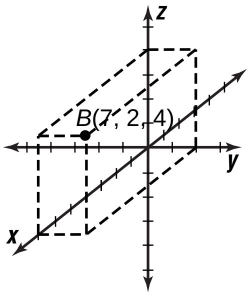
Pages 503–504 Exercises

12.



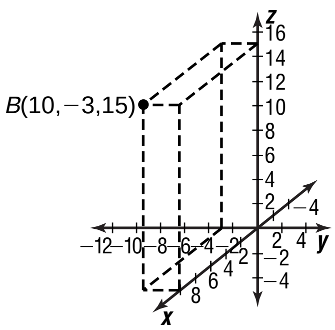
$$|\overrightarrow{OB}| = \sqrt{4^2 + 1^2 + (-3)^2} = \sqrt{26}$$

13.



$$|\overrightarrow{OB}| = \sqrt{7^2 + 2^2 + 4^2} = \sqrt{69}$$

14.



$$|\overrightarrow{OB}| = \sqrt{10^2 + (-3)^2 + 15^2} = \sqrt{334}$$

15. $\overrightarrow{TM} = \langle 3 - 2, 1 - 5, -4 - 4\rangle$ or $\langle 1, -4, -8\rangle$

$|\overrightarrow{TM}| = \sqrt{1^2 + (-4)^2 + (-8)^2} = \sqrt{81}$ or 9

16. $\overrightarrow{TM} = \langle -3 - (-2), 5 - 4, 2 - 7\rangle$ or $\langle -1, 1, -5\rangle$

$|\overrightarrow{TM}| = \sqrt{(-1)^2 + 1^2 + (-5)^2} = \sqrt{27}$ or $3\sqrt{3}$

17. $\overrightarrow{TM} = \langle 3 - 2, 1 - 5, 0 - 4\rangle$ or $\langle 1, -4, -4\rangle$

$|\overrightarrow{TM}| = \sqrt{1^2 + (-4)^2 + (-4)^2} = \sqrt{33}$

18. $\overrightarrow{TM} = \langle -1 - 3, 1 - (-5), 2 - 6\rangle$ or $\langle -4, 6, -4\rangle$

$|\overrightarrow{TM}| = \sqrt{(-4)^2 + 6^2 + (-4)^2} = \sqrt{68}$ or $2\sqrt{17}$

19. $\overrightarrow{TM} = \langle -2 - (-5), -1 - 8, -6 - 3\rangle$ or $\langle 3, -9, -9\rangle$

$|\overrightarrow{TM}| = \sqrt{3^2 + (-9)^2 + (-9)^2} = \sqrt{171}$ or $3\sqrt{19}$

20. $\overrightarrow{TM} = \langle 1-0, 4-6, -3-3\rangle$ or $\langle 1, -2, -6\rangle$
$|\overrightarrow{TM}| = \sqrt{1^2+(-2)^2+(-6)^2}$
$= \sqrt{41}$

21. $|\overrightarrow{CJ}| = \langle 3-(-1), -5-3, -4-10\rangle$
or $\langle 4, -8, -14\rangle$
$|\overrightarrow{CJ}| = \sqrt{4^2+(-8)^2+(-14)^2}$
$= \sqrt{276}$ or $2\sqrt{69}$

22. $\vec{\mathbf{u}} = 6\vec{\mathbf{w}} + 2\vec{\mathbf{z}}$
$= 6\langle 2, 6, -1\rangle + 2\langle 3, 0, 4\rangle$
$= \langle 12, 36, -6\rangle + \langle 6, 0, 8\rangle$
$= \langle 18, 36, 2\rangle$

23. $\vec{\mathbf{u}} = \frac{1}{2}\vec{\mathbf{v}} - \vec{\mathbf{w}} + 2\vec{\mathbf{z}}$
$\vec{\mathbf{u}} = \frac{1}{2}\langle 4, -3, 5\rangle - \langle 2, 6, -1\rangle + 2\langle 3, 0, 4\rangle$
$= \left\langle 2, -\frac{3}{2}, \frac{5}{2}\right\rangle - \langle 2, 6, -1\rangle + \langle 6, 0, 8\rangle$
$= \left\langle 6, -7\frac{1}{2}, 11\frac{1}{2}\right\rangle$

24. $\vec{\mathbf{u}} = \frac{3}{4}\vec{\mathbf{v}} - \vec{\mathbf{w}}$
$= \frac{3}{4}\langle 4, -3, 5\rangle - \langle 2, 6, -1\rangle$
$= \left\langle 3, -\frac{9}{4}, \frac{15}{4}\right\rangle - \langle 2, 6, -1\rangle$
$= \left\langle 1, -8\frac{1}{4}, 4\frac{3}{4}\right\rangle$

25. $\vec{\mathbf{u}} = 3\vec{\mathbf{v}} - \frac{2}{3}\vec{\mathbf{w}} + 2\vec{\mathbf{z}}$
$= 3\langle 4, -3, 5\rangle - \frac{2}{3}\langle 2, 6, -1\rangle + 2\langle 3, 0, 4\rangle$
$= \langle 12, -9, 15\rangle - \left\langle \frac{4}{3}, 4, -\frac{2}{3}\right\rangle + \langle 6, 0, 8\rangle$
$= \left\langle 16\frac{2}{3}, -13, 23\frac{2}{3}\right\rangle$

26. $\vec{\mathbf{u}} = 0.75\vec{\mathbf{v}} + 0.25\vec{\mathbf{w}}$
$= 0.75\langle 4, -3, 5\rangle + 0.25\langle 2, 6, -1\rangle$
$= \langle 3, -2.25, 3.75\rangle + \langle 0.5, 1.5, -0.25\rangle$
$= \langle 3.5, -0.75, 3.5\rangle$

27. $\vec{\mathbf{u}} = -4\vec{\mathbf{w}} + \vec{\mathbf{z}}$
$= -4\langle 2, 6, -1\rangle + \langle 3, 0, 4\rangle$
$= \langle -8, -24, 4\rangle + (3, 0, 4\rangle$
$= \langle -5, -24, 8\rangle$

28. $\frac{2}{3}\vec{\mathbf{f}} + 3\vec{\mathbf{g}} - \frac{2}{5}\vec{\mathbf{h}}$
$= \frac{2}{3}\langle -3, 4.5, -1\rangle + 3\langle -2, 1, 6\rangle - \frac{2}{5}\langle 6, -3, -3\rangle$
$= \left\langle -2, 3, -\frac{2}{3}\right\rangle + \langle -6, 3, 18\rangle - \left\langle \frac{12}{5}, -\frac{6}{5}, -\frac{6}{5}\right\rangle$
$= \left\langle -\frac{52}{5}, \frac{36}{5}, \frac{278}{15}\right\rangle$

29. $\overrightarrow{LB} = \langle 5-2, -6-2, 2-7\rangle$ or $\langle 3, -8, -5\rangle$
$3\vec{\mathbf{i}} - 8\vec{\mathbf{j}} - 5\vec{\mathbf{k}}$

30. $\overrightarrow{LB} = \langle -4-(-6), 5-1, -1-0\rangle$ or $\langle 2, 4, -1\rangle$
$2\vec{\mathbf{i}} + 4\vec{\mathbf{j}} - \vec{\mathbf{k}}$

31. $\overrightarrow{LB} = \langle 7-9, 3-7, -2-(-11)\rangle$ or $\langle -2, -4, 9\rangle$
$-2\vec{\mathbf{i}} - 4\vec{\mathbf{j}} + 9\vec{\mathbf{k}}$

32. $\overrightarrow{LB} = \langle -8-12, 7-2, -5-6\rangle$ or $\langle -20, 5, -11\rangle$
$-20\vec{\mathbf{i}} + 5\vec{\mathbf{j}} - 11\vec{\mathbf{k}}$

33. $\overrightarrow{LB} = \langle -8-(-1), 5-2, -10-(-4)\rangle$
or $\langle -7, 3, -6\rangle$
$-7\vec{\mathbf{i}} + 3\vec{\mathbf{j}} - 6\vec{\mathbf{k}}$

34. $\overrightarrow{LB} = \langle 6-(-9), 5-12, -5-(-5)\rangle$
or $\langle 15, -7, 0\rangle$
$15\vec{\mathbf{i}} - 7\vec{\mathbf{j}}$

35. $|\overrightarrow{G_1G_2}| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
$= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2} = |\overrightarrow{G_1G_2}|$
because $(x-y)^2 = (y-x)^2$ for all real numbers x and y.

36. If $\vec{\mathbf{m}} = \langle m_1, m_2, m_3\rangle$, then
$|\vec{\mathbf{m}}| = \sqrt{(m_1)^2+(m_2)^2+(m_3)^2}$. If $-\vec{\mathbf{m}} = \langle -m_1, -m_2, -m_3\rangle$, then $|-\vec{\mathbf{m}}| = \sqrt{(-m_1)^2+(-m_2)^2+(-m_3)^2}$.
Since $m_1{}^2 = (-m_1)^2$, $m_2{}^2 = (-m_2)^2$, and $m_3{}^2 = (-m_3)^2$, $|-\vec{\mathbf{m}}| = |\vec{\mathbf{m}}|$.

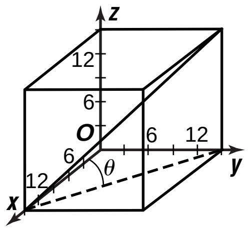
37. $\langle 3, -2, 4\rangle + \langle 6, 2, 5\rangle + \vec{\mathbf{F}} = \vec{\mathbf{O}}$
$\langle 9, 0, 9\rangle + \vec{\mathbf{F}} = \vec{\mathbf{O}}$
$\vec{\mathbf{F}} = -\langle 9, 0, 9\rangle$ or
$\langle -9, 0, -9\rangle$

38. $m = \frac{1}{2}(x_1+x_2, y_1+y_2, z_1+z_2)$
$= \frac{1}{2}(2+4, 3+5, 6+2)$
$= \frac{1}{2}(6, 8, 8)$
$= (3, 4, 4)$

39a. $\overrightarrow{OK} = \langle 1-0, 4-0, 0-0\rangle$ or $\langle 1, 4, 0\rangle$
$\vec{\mathbf{i}} + 4\vec{\mathbf{j}}$

39b. $\overrightarrow{TK} = \langle 1-2, 4-4, 0-0\rangle$ or $\langle -1, 0, 0\rangle$
$-\vec{\mathbf{i}}$

40. $\vec{\mathbf{c}} = \vec{\mathbf{b}} - \vec{\mathbf{a}}$
$\vec{\mathbf{c}} = \langle 3, 1, 5\rangle - \langle 1, 3, 1\rangle$
$\vec{\mathbf{c}} = \langle 2, -2, 4\rangle$

41a.


41b. Find distance between (0, 0, 0) and (15, 15, 15).
$d = \sqrt{(15-0)^2+(15-0)^2+(15-0)^2}$
$= \sqrt{675}$ or about 26 feet

41c. $\sin\theta = \frac{15}{26}$
$\theta = \sin^{-1}\left(\frac{15}{\sqrt{675}}\right)$
$\theta = 35.25°$

42. $|\overrightarrow{AB}| = \sqrt{(1-2)^2+(\sqrt{3}-0)^2+(0-0)^2}$
$= \sqrt{4}$ or 2
$|\overrightarrow{BC}| = \sqrt{(1-1)^2+\left(\frac{1}{3}-\sqrt{3}\right)^2+\left(\frac{2\sqrt{2}}{3}-0\right)^2}$
$= \frac{\sqrt{36-6\sqrt{3}}}{3}$ or ≈ 1.69
$|\overrightarrow{AC}| = \sqrt{(1-2)^2+\left(\frac{1}{3}-0\right)^2+\left(\frac{2\sqrt{2}}{3}-0\right)^2}$
$= \sqrt{2}$ or ≈ 1.41
No, the distances between the points are not equal. A and B are 2 units apart, B and C are 1.69 units apart, and A and C are 1.41 units apart.

43. $\langle 3, 5\rangle + \langle -1, 2\rangle = \langle 3+(-1), 5+2\rangle$
$= \langle 2, 7\rangle$

44. $\overrightarrow{AB} = \langle -3 - 5, 3 - 2\rangle$ or $\langle -8, 1\rangle$

$\overrightarrow{CD} = \langle d_1 - 0, d_2 - 0\rangle$ or $\langle d_1, d_2\rangle$

$\overrightarrow{AB} = \overrightarrow{CD}$

$\langle -8, 1\rangle = \langle d_1, d_2\rangle$

$D = (-8, 1)$

45. $\frac{\sin 2X}{1 - \cos 2X} = \cot X$

$\frac{2 \sin X \cos X}{1 - \cos^2 X + \sin^2 X} = \cot X$

$\frac{2 \sin X \cos X}{2 \sin^2 X} = \cot X$

$\frac{\cos X}{\sin X} = \cot X$

$\cot X = \cot X$

46. $\cos \theta = \frac{2}{3}$

$\sin^2 \theta = 1 - \cos^2 \theta$

$\sin^2 \theta = 1 - \left(\frac{4}{9}\right)$

$\sin^2 \theta = \frac{5}{9}$

$\sin \theta = \frac{\sqrt{5}}{3}$

47. $y = 6 \sin \frac{\theta}{2}$

amplitude $= |6|$ or 6

period $= \frac{2\pi}{k}$

$= \frac{2\pi}{\frac{1}{2}}$ or 4π

48. $16 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = \frac{8\pi}{15}$ radians per second

49. Yes, because substituting 7 for x and -2 for y results in the inequality $-2 < 180$ which is true.

$y < 4x^2 - 3x + 5$

$-2 < 4(7)^2 - 3(7) + 5$

$-2 < 180$

50. $\frac{3}{2} \colon \frac{3 + 1}{2 + 1} = \frac{4}{3}$ $\qquad \frac{3}{2} > \frac{4}{3}$

So, A, C, and D are not correct.

$\frac{2}{3} \colon \frac{2 + 1}{3 + 1} = \frac{3}{4}$ $\qquad \frac{2}{3} < \frac{3}{4}$

So, B is not correct.

The correct choice is E.

8-4 Perpendicular Vectors

Pages 508–509 Check for Understanding

1. Sample answer: Vector $\overline{\mathbf{v}} \times \overline{\mathbf{w}}$ is the negative of vector $\overline{\mathbf{w}} \times \overline{\mathbf{v}}$.

$$\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -1 & 0 & 3 \\ 1 & 2 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 3 \\ 2 & 4 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} -1 & 3 \\ 1 & 4 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} -1 & 0 \\ 0 & 2 \end{vmatrix} \vec{\mathbf{k}}$$

$$= -6\vec{\mathbf{i}} + 7\vec{\mathbf{j}} - 3\vec{\mathbf{k}}$$

$$\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 2 & 4 \\ -1 & 0 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 4 \\ 0 & 3 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} 1 & 4 \\ -1 & 3 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix} \vec{\mathbf{k}}$$

$$= 6\vec{\mathbf{i}} - 7\vec{\mathbf{j}} + 3\vec{\mathbf{k}}$$

2. $$\vec{\mathbf{a}} \times \vec{\mathbf{a}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ a_x & a_y & a_z \\ a_x & a_y & a_z \end{vmatrix}$$

$$= \begin{vmatrix} a_y & a_z \\ a_y & a_z \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} a_x & a_z \\ a_x & a_z \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} a_x & a_y \\ a_x & a_y \end{vmatrix} \vec{\mathbf{k}}$$

$$= (a_y a_z - a_y a_z)\vec{\mathbf{i}} - (a_x a_z - a_x a_z)\vec{\mathbf{j}} + (a_x a_y - a_x a_y)\vec{\mathbf{k}}$$

$$= 0\vec{\mathbf{i}} - 0\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$$

$$= \langle 0, 0, 0\rangle$$

$$= \vec{\mathbf{0}}$$

3. Sample answer: No, because a vector cannot be perpendicular to itself.

4. $\langle 5, 2\rangle \cdot \langle -3, 7\rangle = 5(-3) + 2(7)$

$= -15 + 14$

$= -1$, no

5. $\langle -8, 2\rangle \cdot \langle 4.5, 18\rangle = -8(4.5) + 2(18)$

$= -36 + 36$

$= 0$, yes

6. $\langle -4, 9, 8\rangle \cdot \langle 3, 2, -2\rangle = -4(3) + 9(2) + 8(-2)$

$= -12 + 18 - 16$

$= -10$, no

7. $$\langle 1, -3, 2\rangle \times \langle -2, 1, -5\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & -3 & 2 \\ -2 & 1 & -5 \end{vmatrix}$$

$$= \begin{vmatrix} -3 & 2 \\ 1 & -5 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} 1 & 2 \\ -2 & -5 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} 1 & -3 \\ -2 & 1 \end{vmatrix} \vec{\mathbf{k}}$$

$= 13\vec{\mathbf{i}} - \vec{\mathbf{j}} - 5\vec{\mathbf{k}}$ or $\langle 13, 1, -5\rangle$, yes

$\langle 13, 1, -5\rangle \cdot \langle 1, -3, 2\rangle$

$13(1) + 1(-3) + (-5)(2)$

$13 - 3 - 10 = 0$

$\langle 13, 1, -5\rangle \cdot \langle -2, 1, -5\rangle$

$13(-2) + 1(1) + (-5)(-5)$

$-26 + 1 + 25 = 0$

8. $$\langle 6, 2, 10\rangle \times \langle 4, 1, 9\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 6 & 2 & 10 \\ 4 & 1 & 9 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 10 \\ 1 & 9 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} 6 & 10 \\ 4 & 9 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} 6 & 2 \\ 4 & 1 \end{vmatrix} \vec{\mathbf{k}}$$

$= 8\vec{\mathbf{i}} - 14\vec{\mathbf{j}} - 2\vec{\mathbf{k}}$ or $\langle 8, -14, -2\rangle$, yes

$\langle 8, -14, -2\rangle \cdot \langle 6, 2, 10\rangle$

$8(6) + (-14)(2) + (-2)(10)$

$48 - 28 - 20 = 0$

$\langle 8, -14, -2\rangle \cdot \langle 4, 1, 9\rangle$

$8(4) + (-14)(1) + (-2)(9)$

$32 - 14 - 18 = 0$

9. Sample answer: Let $T(0, 1, 2)$, $U(-2, 2, 4)$, and $V(-1, -1, -1)$

$\overrightarrow{TU} = \langle -2, 1, 2\rangle$

$\overrightarrow{UV} = \langle 1, -3, -5\rangle$

$\overrightarrow{TU} \times \overrightarrow{UV} =$

$$= \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -2 & 1 & 2 \\ 1 & -3 & -5 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 \\ -3 & -5 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} -2 & 2 \\ 1 & -5 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} -2 & 1 \\ 1 & -3 \end{vmatrix} \vec{\mathbf{k}}$$

$= \vec{\mathbf{i}} - 8\vec{\mathbf{j}} + 5\vec{\mathbf{k}}$ or $\langle 1, -8, 5\rangle$

10. $\overrightarrow{AB} = (0.65, 0, 0.3) - (0, 0, 0)$
$= \langle 0.65, 0, 0.3 \rangle$
$\vec{\mathbf{F}} = \langle 0, 0, -32 \rangle$
$\vec{\mathbf{T}} = \overrightarrow{AB} \times \vec{\mathbf{F}} =$

$$= \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 0.65 & 0 & 0.3 \\ 0 & 0 & -32 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 0.3 \\ 0 & -32 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 0.65 & 0.3 \\ 0 & -32 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 0.65 & 0.3 \\ 0 & 0 \end{vmatrix}\vec{\mathbf{k}}$$

$= 0\vec{\mathbf{i}} - 20.8\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$
$|\vec{\mathrm{T}}| = \sqrt{0^2 + (-20.8)^2 + 0^2}$
$= 20.8$ foot-pounds

Pages 509–511 Exercises

11. $\langle 4.8 \rangle \cdot \langle 6, -3 \rangle = 4(6) + 8(-3)$
$= 24 - 24$
$= 0$, yes

12. $\langle 3, 5 \rangle \cdot \langle 4, -2 \rangle = 3(4) + 5(-2)$
$= 12 - 10$
$= 2$, no

13. $\langle 5, -1 \rangle \cdot \langle -3, 6 \rangle = 5(-3) + (-1)(6)$
$= -15 - 6$
$= -21$, no

14. $\langle 7, 2 \rangle \cdot \langle 0, -2 \rangle = 7(0) + 2(-2)$
$= 0 - 4$
$= -4$, no

15. $\langle 8, 4 \rangle \cdot (2, 4 \rangle = 8(2) + 4(4)$
$= 16 + 16$
$= 32$, no

16. $\langle 4, 9, -3 \rangle \cdot \langle -6, 7, 5 \rangle = 4(-6) + 9(7) + (-3)(5)$
$= -24 + 63 - 15$
$= 24$, no

17. $\langle 3, 1, 4 \rangle \cdot \langle 2, 8, -2 \rangle = 3(2) + 1(8) + 4(-2)$
$= 6 + 8 - 8$
$= 6$, no

18. $\langle -2, 4, 8 \rangle \cdot \langle 16, 4, 2 \rangle = -2(16) + 4(4) + 8(2)$
$= -32 + 16 + 16$
$= 0$, yes

19. $\langle 7, -2, 4 \rangle \cdot \langle 3, 8, 1 \rangle = 7(3) + (-2)(8) + 4(1)$
$= 21 - 16 + 4$
$= 9$, no

20. $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \langle 3, 12 \rangle \cdot \langle 8, -2 \rangle$
$= 24 - 24$
$= 0$, yes

$\vec{\mathbf{b}} \cdot \vec{\mathbf{c}} = \langle 8, -2 \rangle \cdot \langle 3, -2 \rangle$
$= 24 + 4$
$= 28$, no

$\vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = \langle 3, 12 \rangle \cdot \langle 3, -2 \rangle$
$= 9 - 24$
$= -15$, no

21. $\langle 0, 1, 2 \rangle \times \langle 1, 1, 4 \rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 0 & 1 & 2 \\ 1 & 1 & 4 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 0 & 2 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}\vec{\mathbf{k}}$$

$= 2\vec{\mathbf{i}} + 2\vec{\mathbf{j}} - \vec{\mathbf{k}}$ or $\langle 2, 2, -1 \rangle$, yes

$\langle 2, 2, -1 \rangle \cdot \langle 0, 1, 2 \rangle$
$2(0) + 2(1) + (-1)(2)$
$2 + 2 - 2 = 0$
$\langle 2, 2, -1 \rangle \cdot \langle 1, 1, 4 \rangle$
$2(1) + 2(1) + (-1)(4)$
$2 + 2 - 4 = 0$

22. $\langle 5, 2, 3 \rangle \times \langle -2, 5, 0 \rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 5 & 2 & 3 \\ -2 & 5 & 0 \end{vmatrix}$

$$= \begin{vmatrix} 2 & 3 \\ 5 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 5 & 3 \\ -2 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 5 & 2 \\ -2 & 5 \end{vmatrix}\vec{\mathbf{k}}$$

$= 15\vec{\mathbf{i}} - 6\vec{\mathbf{j}} + 29\vec{\mathbf{k}}$ or $\langle -15, -6, 29 \rangle$, yes

$\langle -15, -6, 29 \rangle \cdot \langle 5, 2, 3 \rangle$
$(-15)(5) + (-6)(2) + 29(3)$
$-75 - 12 + 87 = 0$
$\langle -15, -6, 29 \rangle \cdot \langle -2, 5, 0 \rangle$
$(-15)(-2) + (-6)(5) + 29(0)$
$30 - 30 + 0 = 0$

23. $\langle 3, 2, 0 \rangle \times \langle 1, 4, 0 \rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 3 & 2 & 0 \\ 1 & 4 & 0 \end{vmatrix}$

$$= \begin{vmatrix} 2 & 0 \\ 4 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{k}}$$

$= 0\vec{\mathbf{i}} - 0\vec{\mathbf{j}} + 10\vec{\mathbf{k}}$ or $\langle 0, 0, 10 \rangle$, yes

$\langle 0, 0, 10 \rangle \cdot \langle 3, 2, 0 \rangle$
$0(3) + 0(2) + 10(0)$
$0 + 0 + 0 = 0$
$\langle 0, 0, 10 \rangle \cdot \langle 1, 4, 0 \rangle$
$0(1) + 0(4) + 10(0)$
$0 + 0 + 0 = 0$

24. $\langle 1, -3, 2 \rangle \times \langle 5, 1, -2 \rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & -3 & 2 \\ 5 & 1 & -2 \end{vmatrix}$

$$= \begin{vmatrix} -3 & 2 \\ 1 & -2 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & 2 \\ 5 & -2 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & -3 \\ 5 & 1 \end{vmatrix}\vec{\mathbf{k}}$$

$= 4\vec{\mathbf{i}} + 12\vec{\mathbf{j}} + 16\vec{\mathbf{k}}$ or $\langle 4, 12, 16 \rangle$, yes

$\langle 4, 12, 16 \rangle \cdot \langle 1, -3, 2 \rangle$
$4(1) + 12(-3) + 16(2)$
$4 - 36 + 32 = 0$
$\langle 4, 12, 16 \rangle \cdot \langle 5, 1, -2 \rangle$
$4(5) + 12(1) + 16(-2)$
$20 + 12 - 32 = 0$

25. $\langle -3, -1, 2 \rangle \times \langle 4, -4, 0 \rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -3 & -1 & 2 \\ 4 & -4 & 0 \end{vmatrix}$

$$= \begin{vmatrix} -1 & 2 \\ -4 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -3 & 2 \\ 4 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -3 & -1 \\ 4 & -4 \end{vmatrix}\vec{\mathbf{k}}$$

$= 8\vec{\mathbf{i}} + 8\vec{\mathbf{j}} + 16\vec{\mathbf{k}}$ or $\langle 8, 8, 16 \rangle$, yes

$\langle 8, 8, 16 \rangle \cdot \langle -3, -1, 2 \rangle$
$8(-3) + 8(-1) + 16(2)$
$-24 - 8 + 32 = 0$
$\langle 8, 8, 16 \rangle \cdot \langle 4, -4, 0 \rangle$
$8(4) + 8(-4) + 16(0)$
$32 - 32 + 0 = 0$

26. $\langle 4, 0, -2\rangle \times \langle -7, 1, 0\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 4 & 0 & -2 \\ -7 & 1 & 0 \end{vmatrix}$

$= \begin{vmatrix} 0 & -2 \\ 1 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 4 & -2 \\ -7 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 4 & 0 \\ -7 & 1 \end{vmatrix}\vec{\mathbf{k}}$

$= 2\vec{\mathbf{i}} + 14\vec{\mathbf{j}} + 4\vec{\mathbf{k}}$ or $\langle 2, 14, 4\rangle$, yes

$\langle 2, 14, 4\rangle \cdot \langle 4, 0, -2\rangle$
$2(4) + 14(0) + 4(-2)$
$8 + 0 - 8 = 0$
$\langle 2, 14, 4\rangle \cdot \langle -7, 1, 0\rangle$
$2(-7) + 14(1) + 4(0)$
$-14 + 14 + 0 = 0$

27. Sample answer:
Let $\vec{\mathbf{v}} = \langle v_1, v_2, v_3\rangle$ and $-\vec{\mathbf{v}} = \langle -v_1, -v_2, -v_3\rangle$

$\vec{\mathbf{v}} \times (-\vec{\mathbf{v}}) = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ v_1 & v_2 & v_3 \\ -v_1 & -v_2 & -v_3 \end{vmatrix}$

$= \begin{vmatrix} v_2 & v_3 \\ -v_2 & -v_3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} v_1 & v_3 \\ -v_1 & -v_3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} v_1 & v_2 \\ -v_1 & -v_2 \end{vmatrix}\vec{\mathbf{k}}$

$= 0\vec{\mathbf{i}} - 0\vec{\mathbf{j}} + 0\vec{\mathbf{k}} = \mathbf{0}$

28. $\vec{\mathbf{a}} \times (\mathbf{b} + \vec{\mathbf{c}}) = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ (b_1 + c_1) & (b_2 + c_2) & (b_3 + c_3) \end{vmatrix}$

$= \begin{vmatrix} a_2 & a_3 \\ (b_2 + c_2) & (b_3 + c_3) \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ (b_1 + c_1) & (b_3 + c_3) \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ (b_1 + c_1) & (b_2 + c_2) \end{vmatrix}\vec{\mathbf{k}}$

$= [a_2 (b_3 + c_3) - a_3 (b_2 + c_2)]\vec{\mathbf{i}} -$
$[a_1 (b_3 + c_3) - a_3 (b_1 + c_1)]\vec{\mathbf{j}} +$
$[a_1 (b_2 + c_2) - a_2 (b_1 + c_1)]\vec{\mathbf{k}}$

$= [(a_2b_3 + a_2c_3) - (a_3b_2 + a_3c_2)]\vec{\mathbf{i}} -$
$[(a_1b_3 + a_1c_3) - (a_3b_1 + a_3c_1)]\vec{\mathbf{j}} +$
$[(a_1b_2 + a_1c_2) - (a_2b_1 + a_2c_1)]\vec{\mathbf{k}}$

$= [(a_2b_3 - a_3b_2) + (a_2c_3 - a_3c_2)]\vec{\mathbf{i}} -$
$[(a_1b_3 - a_3b_1) + (a_1c_3 - a_3c_1)]\vec{\mathbf{j}} +$
$[(a_1b_2 - a_2b_1) + (a_1c_2 - a_2c_1)]\vec{\mathbf{k}}$

$= (a_2b_3 - a_3b_2)\vec{\mathbf{i}} + (a_2c_3 - a_3c_2)\vec{\mathbf{i}} -$
$(a_1b_3 - a_3b_1)\vec{\mathbf{j}} - (a_1c_3 - a_3c_1)\vec{\mathbf{j}} +$
$(a_1b_2 - a_2b_1)\vec{\mathbf{k}} + (a_1c_2 - a_2c_1)\vec{\mathbf{k}}$

$= [(a_2b_3 - a_3b_2)\vec{\mathbf{i}} - (a_1b_3 - a_3b_1)\vec{\mathbf{j}} +$
$(a_1b_2 - a_2b_1)\vec{\mathbf{k}}] + [(a_2c_3 - a_3c_2)\vec{\mathbf{i}} -$
$(a_1c_3 - a_3c_1)\vec{\mathbf{j}} + (a_1c_2 - a_2c_1)\vec{\mathbf{k}}]$

$= \left[\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\right]\vec{\mathbf{k}} +$

$= \left[\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\right]\vec{\mathbf{k}} +$

$\left[\begin{vmatrix} a_2 & a_3 \\ c_2 & c_3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} a_1 & a_3 \\ c_1 & c_3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} a_1 & a_2 \\ c_1 & c_2 \end{vmatrix}\vec{\mathbf{k}}\right]$

$= (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) + (\vec{\mathbf{a}} \times \vec{\mathbf{c}})$

29. Sample answer:
Let $T(0, -2, 2)$, $U(1, 2, -3)$, and $V(4, 0, -1)$
$\overrightarrow{TU} = \langle 1, 4, -5\rangle$
$\overrightarrow{UV} = \langle 3, -2, 2\rangle$

$\overrightarrow{TU} \times \overrightarrow{UV} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 4 & -5 \\ 3 & -2 & 2 \end{vmatrix}$

$= \begin{vmatrix} 4 & -5 \\ -2 & 2 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & -5 \\ 3 & 2 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & 4 \\ 3 & -2 \end{vmatrix}\vec{\mathbf{k}}$

$= -2\vec{\mathbf{i}} - 17\vec{\mathbf{j}} - 14\vec{\mathbf{k}}$ or $\langle -2, -17, -14\rangle$

30. Sample answer:
Let $T(-2, 1, 0)$, $U(-3, 0, 0)$, and $V(5, 2, 0)$.
$\overrightarrow{TU} = \langle -1, -1, 0\rangle$
$\overrightarrow{UV} = \langle 8, 2, 0\rangle$

$\overrightarrow{TU} \times \overrightarrow{UV} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -1 & -1 & 0 \\ 8 & 2 & 0 \end{vmatrix}$

$= \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -1 & 0 \\ 8 & 0 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -1 & -1 \\ 8 & 2 \end{vmatrix}\vec{\mathbf{k}}$

$= 0\vec{\mathbf{i}} + 0\vec{\mathbf{j}} + 6\vec{\mathbf{k}}$ or $\langle 0, 0, 6\rangle$

31. Sample answer:
Let $T(0, 0, 1)$, $U(1, 0, 1)$, and $V(-1, -1, -1)$.
$\overrightarrow{TU} = \langle 1, 0, 0\rangle$
$\overrightarrow{UV} = \langle -2, -1, -2\rangle$

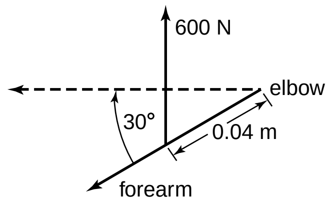
$\overrightarrow{TU} \times \overrightarrow{UV} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 0 & 0 \\ -2 & -1 & -2 \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 \\ -1 & -2 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & 0 \\ -2 & -2 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix}\vec{\mathbf{k}}$

$= 0\vec{\mathbf{i}} + 2\vec{\mathbf{j}} - \vec{\mathbf{k}}$ or $\langle 0, 2, -1\rangle$

32. The expression is false. $\vec{\mathbf{m}} \times \vec{\mathbf{n}}$ and $\vec{\mathbf{n}} \times \vec{\mathbf{m}}$ have the same magnitude but are opposite in direction.

33a.



33b. $\vec{\mathbf{T}} = \overrightarrow{AB} \times \vec{\mathbf{F}}$
$\overrightarrow{AB} = \langle 0.04 \cos(-30°), 0, 0.04 \sin(-30°)\rangle$
$= \langle 0.02(\sqrt{3}), 0, -0.02\rangle$
$\vec{\mathbf{F}} = \langle 0, 0, 600\rangle$

$\overrightarrow{AB} \times \vec{\mathbf{F}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 0.02\sqrt{3} & 0 & -0.02 \\ 0 & 0 & 600 \end{vmatrix}$

$= 0\vec{\mathbf{i}} - 12\sqrt{3}\,\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$

$\vec{\mathbf{T}} = |\overrightarrow{AB} \times \vec{\mathbf{F}}| = 12\sqrt{3}$ or about 21 N-m

34. $\vec{\mathbf{x}} \times \vec{\mathbf{y}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 2 & 3 & 0 \\ -1 & 1 & 4 \end{vmatrix}$

$= \begin{vmatrix} 3 & 0 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix}\vec{\mathbf{k}}$

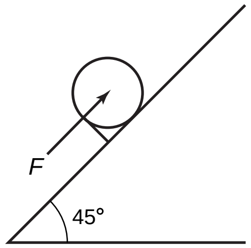
$= 12\vec{\mathbf{i}} - 8\vec{\mathbf{j}} + 5\vec{\mathbf{k}}$

$A = \frac{1}{2}|\vec{\mathbf{x}} \times \vec{\mathbf{y}}|$
$= \frac{1}{2}\sqrt{12^2 + (-8)^2 + (5)^2}$
$= \frac{1}{2}\sqrt{233}$

35a. $\vec{\mathbf{o}} = \langle 120, 310, 60\rangle$
$\vec{\mathbf{c}} = \langle 29, 18, 21\rangle$

35b. $\vec{\mathbf{o}} \cdot \vec{\mathbf{c}} = 120(29) + 310(18) + 60(21)$
$= \$10{,}320$

36a.



36b. $W = |\vec{\mathbf{F}}|\ |\vec{\mathbf{d}}| \cos\theta$
$W = 120 \cdot 4 \cdot \cos 45°$
$W \approx 339$ ft-lb

37a. $\vec{\mathbf{X}} = \langle 2-1, 5-0, 0-3\rangle$ or $\langle 1, 5, -3\rangle$
$\vec{\mathbf{Y}} = \langle 3-2, 1-5, 4-0\rangle$ or $\langle 1, -4, 4\rangle$

$$\vec{\mathbf{X}} \times \vec{\mathbf{Y}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 5 & -3 \\ 1 & -4 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} 5 & -3 \\ -4 & 4 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & -3 \\ 1 & 4 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & 5 \\ 1 & -4 \end{vmatrix}\vec{\mathbf{k}}$$

$$= 8\vec{\mathbf{i}} - 7\vec{\mathbf{j}} - 9\vec{\mathbf{k}} \text{ or } \langle 8, -7, -9\rangle$$

37b. The cross product of two vectors is always a vector perpendicular to the two vectors and the plane in which they lie.

38a. $v = \vec{\mathbf{p}} \cdot (\vec{\mathbf{q}} \times \vec{\mathbf{r}})$

$$\vec{\mathbf{q}} \times \vec{\mathbf{r}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 2 & 1 & -4 \\ -3 & 1 & -5 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -4 \\ 1 & -5 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 2 & -4 \\ -3 & -5 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix}\vec{\mathbf{k}}$$

$$= -\vec{\mathbf{i}} - 22\vec{\mathbf{j}} + 5\vec{\mathbf{k}} \text{ or } \langle -1, -22, 5\rangle$$

$\vec{\mathbf{p}} \cdot (\vec{\mathbf{q}} \times \vec{\mathbf{r}}) = \langle 0, 0, -1\rangle \cdot \langle -1, -22, 5\rangle$
$= 0(-1) + 0(-22) + (-1)(5)$
$= -5$ or 5 units^3

38b.
$$\begin{vmatrix} 0 & 0 & -1 \\ 2 & 1 & -4 \\ -3 & 1 & -5 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -4 \\ 1 & -5 \end{vmatrix}0 - \begin{vmatrix} 2 & -4 \\ -3 & -5 \end{vmatrix}0 + \begin{vmatrix} 2 & 1 \\ -3 & 1 \end{vmatrix}(-1)$$

$= -5$ or 5 units^3
They are the same.

39. Need $(k\vec{\mathbf{v}} + \vec{\mathbf{w}}) \cdot \vec{\mathbf{u}} = 0$.
$[k\langle 1, 2\rangle + \langle -1, 2\rangle] \cdot \langle 5, 12\rangle = 0$
$[\langle k, 2k\rangle + \langle -1, 2\rangle] \cdot \langle 5, 12\rangle = 0$
$\langle k-1, 2k+2\rangle \cdot \langle 5, 12\rangle = 0$
$(k-1)5 + (2k+2)12 = 0$
$5k - 5 + 24k + 24 = 0$
$29k + 19 = 0$
$k = -\frac{19}{29}$

40. $|\overrightarrow{BA}|^2 = |\vec{\mathbf{a}}|^2 + |\vec{\mathbf{b}}|^2 - 2|\vec{\mathbf{a}}|\ |\vec{\mathbf{b}}| \cos\theta$
$\left(\sqrt{(a_1-b_1)^2 + (a_2-b_2)^2}\right)^2$
$= \left(\sqrt{a_1^2 + a_2^2}\right)^2 + \left(\sqrt{b_1^2 + b_2^2}\right)^2$
$- 2\left(\sqrt{a_1^2 + a_2^2}\right)\left(\sqrt{b_1^2 + b_2^2}\right)\cos\theta$
$(a_1-b_1)^2 + (a_2-b_2)^2$
$= a_1^2 + a_2^2 + b_1^2 + b_2^2 - 2\left(\sqrt{a_1^2 + a_2^2}\right)$
$\left(\sqrt{b_1^2 + b_2^2}\right)\cos\theta\ a_1^2 - 2a_1b_1 + b_1^2$
$+ a_2^2 - 2a_2b_2 + b_2^2$
$= a_1^2 + a_2^2 + b_1^2 + b_2^2$
$- 2\left(\sqrt{a_1^2 + a_2^2}\right)\left(\sqrt{b_1^2 + b_2^2}\right)\cos\theta -$
$2a_1b_1 - 2a_2b_2$
$= -2\left(\sqrt{a_1^2 + a_2^2}\right)\left(\sqrt{b_1^2 + b_2^2}\right)\cos\theta$
$a_1b_1 + a_2b_2$
$= \sqrt{a_1^2 + a_2^2}\ \sqrt{b_1^2 + b_2^2}\cos\theta$
$a_1b_1 + a_2b_2$
$= |\vec{\mathbf{a}}|\ |\vec{\mathbf{b}}| \cos\theta\ \vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$
$= |\vec{\mathbf{a}}|\ |\vec{\mathbf{b}}| \cos\theta$

41. $\overrightarrow{AB} = \langle 5-3, 3-3, 2-(-1)\rangle$ or $\langle 2, 0, 3\rangle$

42. $D(8, 3)$
$E(0, -2)$
$\overrightarrow{DE} = \langle 0-8, -2-3\rangle$ or $\langle -8, -5\rangle$
$|\overrightarrow{DE}| = \sqrt{(-8)^2 + (-5)^2}$
$= \sqrt{89}$

43. $4x + y - 6 = 0$
$\sqrt{A^2 + B^2} = \sqrt{4^2 + 1^2}$ or $\sqrt{17}$
$\frac{4\sqrt{17}}{17}x + \frac{\sqrt{17}}{17}y - \frac{6\sqrt{17}}{17} = 0$
$p = \frac{6\sqrt{17}}{17} \approx 1.46$ units
$\sin\phi = \frac{\sqrt{17}}{17}$ $\quad\quad$ $\cos\phi = \frac{4\sqrt{17}}{17}$
$\tan\phi = \frac{1}{4}$
$\phi = 14°$

44. $A = 36°$, $b = 13$, and $c = 6$
$a^2 = b^2 + c^2 - 2\ bc \cos A$
$a^2 = 13^2 + 6^2 - 2(13)(6)\cos 36°$
$a \approx 8.9$
$\frac{\sin 36°}{8.9} \approx \frac{\sin B}{13}$
$B \approx \sin^{-1}\left(\frac{13 \sin 36°}{8.9}\right)$
$B \approx 59.41°$ or $59°25'$
$C \approx 180° - 36° - 59°25'$
$C \approx 84.59°$ or $84°35'$

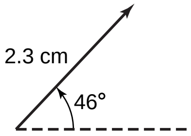
45. $\tan 73° = \frac{h}{4}$ $\quad\quad$ $\cos 73° = \frac{4}{\ell}$
$4 \tan 73° = h$ $\quad\quad$ $\ell = \frac{4}{\cos 73°}$
$13.1 = h$; 13.1 m $\quad\quad$ $\ell = 13.7$ m

46. $3 + \sqrt{3x-4} \geq 10$
$\sqrt{3x-4} \geq 7$
$3x - 4 \geq 49$
$x \geq 17.67$

47. $81 = 3^4$
$64 = 2^6 = (2^2)^3$ or $(2^3)^2$
$4 = 2^2$
$2 = 2^1$
$9 = 3^2$
So $64 = 4^3 = 8^2$
The correct choice is B.

Page 511 **Mid-Chapter Quiz**

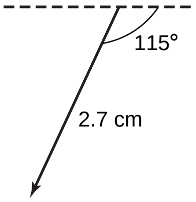
1.



$F_x = 2.3 \cos 46°$
$= 1.6$ cm

$F_y = 2.3 \sin 46°$
$= 1.7$ cm

2.



$F_x = 27 \cos 245°$
$= 11.4$ mm

$F_y = 27 \sin 245°$
$= 24.5$ mm

3. $\overrightarrow{CD} = \langle -4 - (-9), -3 - 2\rangle$ or $\langle 5, -5\rangle$
$|\overrightarrow{CD}| = \sqrt{5^2 + (-5)^2}$
$= 50$ or $5\sqrt{2}$

4. $\overrightarrow{CD} = \langle 5 - 3, 7 - 7, 2 - (-1)\rangle$ or $\langle 2, 0, 3\rangle$
$|\overrightarrow{CD}| = \sqrt{2^2 + 0^2 + 3^2}$
$= \sqrt{13}$

5. $\vec{\mathbf{r}} = \vec{\mathbf{t}} - 2\vec{\mathbf{s}}$
$= \langle -6, 2\rangle - 2\langle 4, -3\rangle$
$= \langle -6, 2\rangle - \langle 8, -6\rangle$
$= \langle -6 - 8, 2 + 6\rangle$ or $\langle -14, 8\rangle$

6. $\vec{\mathbf{r}} = 3\vec{\mathbf{u}} + \vec{\mathbf{v}}$
$= 3\langle 1, -3, -8\rangle + \langle 3, 9, -1\rangle$
$= \langle 3, -9, -24\rangle + \langle 3, 9, -1\rangle$
$= \langle 3 + 3, -9 + 9, -24 + (-1)\rangle$ or $\langle 6, 0, -25\rangle$

7. $\langle 3, 6\rangle \cdot \langle -4, 2\rangle = 3(-4) + 6(2)$
$= -12 + 12$
$= 0$; yes

8. $\langle 3, -2, 4\rangle \cdot \langle 1, -4, 0\rangle = 3(1) + (-2)(-4) + 4(0)$
$= 3 + 8$
$= 11$; no

9. $\langle 1, 3, 2\rangle \times \langle 2, -1, -1\rangle = \begin{vmatrix} \vec{\mathbf{u}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 1 & 3 & 2 \\ 2 & -1 & -1 \end{vmatrix}$

$= \begin{vmatrix} 3 & 2 \\ -1 & -1 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}\vec{\mathbf{k}}$

$= -\vec{\mathbf{i}} + 5\vec{\mathbf{j}} - 7\vec{\mathbf{k}}$ or $\langle -1, 5, -7\rangle$, yes
$\langle -1, 5, -7\rangle \cdot \langle 1, 3, 2\rangle$
$(-1)(1) + 5(3) + (-7)(2)$
$-1 + 15 - 14 = 0$
$\langle -1, 5, -7\rangle \cdot \langle 2, -1, -1\rangle$
$(-1)(2) + 5(-1) + (-7)(-1)$
$-2 - 5 + 7 = 0$

10. Let $X(2, 0, 4)$ and $Y(7, 4, 6)$.
$|XY| = \sqrt{(7 - 2)^2 + (4 - 0)^2 + (6 - 4)^2}$
$= \sqrt{45}$ or about 6.7 m

8-4B Graphing Calculator Exploration: Finding Cross Products

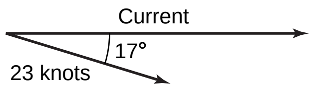
Page 512

1. $\langle -49, 32\ -55\rangle$
2. $\langle 168, -96, 76\rangle$
3. $\langle 0, 0, 0\rangle$
4. $\langle 11, 15, -3\rangle$
5. $\langle 0, 0, -7\rangle$
6. $\langle 0, 40, 0\rangle$
7. $\vec{\mathbf{u}} \times \vec{\mathbf{x}} = \langle 6, 6, -12\rangle$
$|\vec{\mathbf{u}} \times \vec{\mathbf{v}}| = \sqrt{6^2 + 6^2 + (-12)^2}$
$= \sqrt{216}$
8. $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = \langle 1, -13, -20\rangle$
$|\vec{\mathbf{u}} \times \vec{\mathbf{v}}| = \sqrt{1^2 + (-13)^2 + (-20)^2}$
$= \sqrt{570}$
9. Sample answer: Insert the following lines after the last line of the given program.
:Disp "LENGTH IS"
:Disp √ ((BZ − CY)^2 + (CX − AZ)^2 + (AY − BX)^2)

8-5 Applications with Vectors

Pages 516–517 **Check for Understanding**

1. Sample answer: Pushing an object up the slope requires less force because the component of the weight of the object in the direction of motion is $mg \sin \theta$. This is less than the weight mg of the object, which is the force that must be exerted to lift the object straight up.
2. The tension increases.
3. Sample answer: Forces are in equilibrium if the resultant force is $\vec{\mathbf{O}}$.
4.



5. $\vec{\mathbf{F}}_1 = 300\vec{\mathbf{i}}$
$\vec{\mathbf{F}}_2 = (170 \cos 55°)\vec{\mathbf{i}} + (170 \sin 55°)\vec{\mathbf{j}}$
$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{(300 + 170 \cos 55°)^2 + (170 \sin 55°)^2}$
≈ 421.19 N
$\tan \theta = \frac{170 \sin 55°}{300 + 170 \cos 55°}$
$\theta = \tan^{-1}\left(\frac{170 \sin 55°}{300 + 170 \cos 55°}\right)$
≈
6. $\vec{\mathbf{F}}_1 = 50\vec{\mathbf{i}}$
$\vec{\mathbf{F}}_2 = 100\vec{\mathbf{j}}$
$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{50^2 + 100^2}$
≈ 111.8 N
$\tan \theta = \frac{100}{50}$ or 2
$\theta = \tan^{-1} 2$
$\approx 63.43°$
7. horizontal $= 18 \cos 40°$
≈ 13.79 N
vertical $= 18 \sin 40°$
≈ 11.57 N

8. $\vec{\mathbf{F}}_1 = (33 \cos 90°)\vec{\mathbf{i}} + (33 \sin 90°)\vec{\mathbf{j}}$ or $33\vec{\mathbf{j}}$

$\vec{\mathbf{F}}_2 = (44 \cos 60°)\vec{\mathbf{i}} + (44 \sin 60°)\vec{\mathbf{j}}$
or $22\vec{\mathbf{i}} + 22\sqrt{3}\,\vec{\mathbf{j}}$

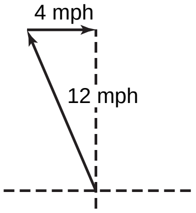
$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{22^2 + (33 + 22\sqrt{3})^2}$
≈ 74 N

$\tan\theta = \frac{33 + 22\sqrt{3}}{22}$ or $\frac{3 + 2\sqrt{3}}{2}$

$\theta = \tan^{-1}\left(\frac{3 + 2\sqrt{3}}{2}\right)$
$\approx 73°$

A force with magnitude 74 N and direction 73° + 180° or 253° will produce equilibrium.

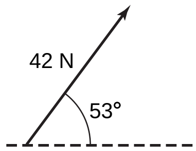
9a.



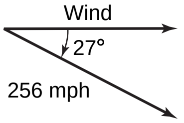
9b. If θ is the angle between the resultant path of the ferry and the line between the landings, then $\sin\theta = \frac{4}{12}$ or $\frac{1}{3}$. So $\theta = \sin^{-1}\frac{1}{3}$, or about 19.5°.

Pages 517–519 Exercises

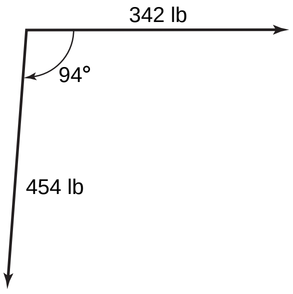
10.



11.



12.



13. $\vec{\mathbf{F}}_1 = 425\vec{\mathbf{i}}$

$\vec{\mathbf{F}}_2 = 390\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{425^2 + 390^2}$
≈ 576.82 N

$\tan\theta = \frac{390}{425}$ or $\frac{78}{85}$

$\theta = \tan^{-1}\frac{78}{85}$
$\approx 42.5°$

14. $\vec{\mathbf{v}}_1 = 65\vec{\mathbf{i}}$

$\vec{\mathbf{v}}_2 = (50 \cos 300°)\vec{\mathbf{i}} + (50 \sin 300°)\vec{\mathbf{j}}$ or $25\vec{\mathbf{i}} - 25\sqrt{3}\,\vec{\mathbf{j}}$

$|\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2| = \sqrt{90^2 + (-25\sqrt{3})^2}$
≈ 99.87 mph

$\tan\theta = -\frac{25\sqrt{3}}{90}$ or $-\frac{5\sqrt{3}}{18}$

$\theta = \tan^{-1}\left(-\frac{5\sqrt{3}}{18}\right)$

A positive value for θ is about 334.3°.

15. $\vec{\mathbf{v}}_1 = (115 \cos 60°)\vec{\mathbf{i}} + (115 \sin 60°)\vec{\mathbf{j}}$
or $57.5\vec{\mathbf{i}} + 57.5\sqrt{3}\,\vec{\mathbf{j}}$

$\vec{\mathbf{v}}_2 = (115 \cos 120°)\vec{\mathbf{i}} + (115 \sin 120°)\vec{\mathbf{j}}$
or $-57.5\vec{\mathbf{i}} + 57.5\sqrt{3}\,\vec{\mathbf{j}}$

$|\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2| = \sqrt{0^2 + (115\sqrt{3})^2}$
$= 115\sqrt{3}$
≈ 199.19 km/h

Since $\tan\theta$ is undefined and the vertical component is positive, $\theta = 90°$.

16. The force must be at least as great as the component of the weight of the object in the direction of the ramp. This is 100 sin 10°, or about 17.36 lb.

17. $\vec{\mathbf{F}}_1 = 105\vec{\mathbf{i}}$

$\vec{\mathbf{F}}_2 = (110 \cos 50°)\vec{\mathbf{i}} + (110 \sin 50°)\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{(105 + 110 \cos 50°)^2 + (110 \sin 50°)^2}$
≈ 194.87 N

$\tan\theta = \frac{110 \sin 50°}{105 + 110 \cos 50°}$

$\theta = \tan^{-1}\left(\frac{110 \sin 50°}{105 + 110 \cos 50°}\right)$
$\approx 25.62°$

18. $F = w \sin\theta$
$52.1 = 75 \sin\theta$
$\frac{52.1}{75} = \sin\theta$
$\sin^{-1}\left(\frac{52.1}{75}\right) = \theta$
$44° \approx \theta$

19. $\vec{\mathbf{F}}_1 = (250 \cos 25°)\vec{\mathbf{i}} + (250 \sin 25°)\vec{\mathbf{j}}$

$\vec{\mathbf{F}}_2 = (45 \cos 250°)\vec{\mathbf{i}} + (45 \sin 250°)\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| =$
$\sqrt{(250 \cos 25° + 45 \cos 250°)^2 + (250 \sin 25° + 45 \sin 250°)^2}$
≈ 220.5 lb

$\tan\theta = \frac{250 \sin 25° + 45 \sin 250°}{250 \cos 25° + 45 \cos 250°}$

$\theta = \tan^{-1}\left(\frac{250 \sin 25° + 45 \sin 250°}{250 \cos 25° + 45 \cos 250°}\right)$
$\approx 16.7°$

20. $\vec{\mathbf{F}}_1 = (70 \cos 330°)\vec{\mathbf{i}} + (70 \sin 330°)\vec{\mathbf{j}}$ or $35\sqrt{3}\,\vec{\mathbf{i}} - 35\vec{\mathbf{j}}$

$\vec{\mathbf{F}}_2 = (40 \cos 45°)\vec{\mathbf{i}} + (40 \sin 45°)\vec{\mathbf{j}}$ or $20\sqrt{2}\,\vec{\mathbf{i}} + 20\sqrt{2}\,\vec{\mathbf{j}}$

$\vec{\mathbf{F}}_3 = (60 \cos 135°)\vec{\mathbf{i}} + (60 \sin 135°)\vec{\mathbf{j}}$ or $-30\sqrt{2} + 30\sqrt{2}\,\vec{\mathbf{j}}$

$\tan\theta = \frac{-35 + 50\sqrt{2}}{35\sqrt{3} - 10\sqrt{2}}$

$\theta = \tan^{-1}\left(\frac{-35 + 50\sqrt{2}}{35\sqrt{3} - 10\sqrt{2}}\right)$
$\approx 37.5°$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3| = \sqrt{(35\sqrt{3} - 10\sqrt{2})^2 + (-35 + 50\sqrt{2})^2}$
≈ 58.6 lb

21. $\vec{\mathbf{F}}_1 = (23 \cos 60°)\vec{\mathbf{i}} + (23 \sin 60°)\vec{\mathbf{j}}$
or $11.5\vec{\mathbf{i}} + 11.5\sqrt{3}\vec{\mathbf{j}}$

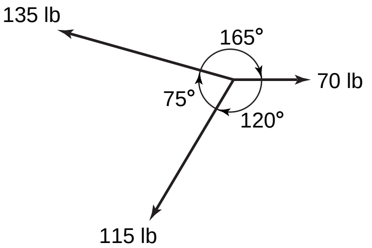
$\vec{\mathbf{F}}_2 = (23 \cos 120°)\vec{\mathbf{i}} + (23 \sin 120°)\vec{\mathbf{j}}$
or $-11.5\vec{\mathbf{i}} + 11.5\sqrt{3}\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{0^2 + (23\sqrt{3})^2}$
$= 23\sqrt{3}$
≈ 39.8 N

Since $\tan\theta$ is undefined and the vertical component is positive , $\theta = 90°$. A force with magnitude 39.8 N and direction 90° + 180° or 270° will produce equilibrium.

22. $a = g \sin 40°$
$= 32 \sin 40°$
$\approx 20.6 \text{ ft/s}^2$

23. $\vec{\mathbf{F_1}} = (36 \cos 20°)\vec{\mathbf{i}} + (36 \sin 20°)\vec{\mathbf{j}}$
$\vec{\mathbf{F_2}} = (48 \cos 222°\vec{\mathbf{i}} + (48 \sin 222°)\vec{\mathbf{j}}$
$|\vec{\mathbf{F_1}} + \vec{\mathbf{F_2}}| =$
$\sqrt{(36 \cos 20° + 48 \cos 222°)^2 + (36 \sin 20° + 48 \sin 222°)^2}$
≈ 19.9 N
$\tan \theta = \frac{36 \sin 20° + 48 \sin 222°}{36 \cos 20° + 48 \cos 222°}$
$\theta = \tan^{-1}\left(\frac{36 \sin 20° + 48 \sin 222°}{36 \cos 20° + 48 \cos 222°}\right)$
$\approx 264.7°$ or $5.3°$ west of south

24a.


24b. $\vec{\mathbf{F_1}} = 70\vec{\mathbf{i}}$
$\vec{\mathbf{F_2}} = (135 \cos 165°)\vec{\mathbf{i}} + (135 \sin 165°)\vec{\mathbf{j}}$
$|\vec{\mathbf{F_1}} + \vec{\mathbf{F_2}} + \vec{\mathbf{F_3}}| =$
$\sqrt{(70 + 135 \cos 165° + 115 \cos 240°)^2 + (135 \sin 165° + 115 \sin 240°)^2}$
≈ 134.5 lb
$\tan \theta = \frac{135 \sin 165° + 115 \sin 240°}{70 + 135 \cos 165° + 115 \cos 240°}$
$\theta = \tan^{-1}\left(\frac{135 \sin 165° + 115 \sin 240°}{70 + 135 \cos 165° + 115 \cos 240°}\right)$
$\approx 208.7°$ or $28.7°$ south of west
Since $\vec{\mathbf{F_1}} + \vec{\mathbf{F_2}} + \vec{\mathbf{F_3}} \neq 0$, the vectors are not in equilibrium.

25. $W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$
$= [(1600 \cos 50°)\vec{\mathbf{i}} + (1600 \sin 50°)\vec{\mathbf{j}}] \cdot 1500\vec{\mathbf{i}}$
$= (1600 \cos 50°)(1500) + (1600 \sin 50°)(0)$
$\approx 1{,}542{,}690$ N-m

26a. Sample answer: The horizontal forward force is $\vec{F} \cos \theta$. You can increase the horizontal forward force by decreasing the angle θ between the handle and the lawn.

26b. Sample answer: Pushing the lawnmower at a lower angle may cause back pain.

27a. $\tan \theta = \frac{3}{18}$ or $\frac{1}{6}$
$\theta = \tan^{-1} \frac{1}{6}$
$\approx 9.5°$ south of east

27b. $s = \sqrt{18^2 + 3^2}$
≈ 18.2 mph

28. $F \cos \theta = 100 \cos 25°$
≈ 90.63 N

29. $F_1 \cos 174.5° + F_2 \cos 6.2° = 0$
$F_1 \sin 174.5° + F_2 \sin 6.2° - 155 = 0$
The first equation gives $F_2 = -\frac{\cos 174.5°}{\cos 6.2°} F_1$.
Substitute into the second equation.
$F_1 \sin 174.5° - \frac{\cos 174.5° \sin 6.2°}{\cos 6.2°} F_1 - 155 = 0$

$F_1 (\sin 174.5° - \cos 174.5° \tan 6.2°) = 155$

$F_1 = \frac{155}{\sin 174.5° - \cos 174.5° \tan 6.2°}$
≈ 760 lb
$F_2 = -\frac{\cos 174.5°}{\cos 6.2°} F_1$
≈ 761 lb

30. Sample answer: Method b is better. Let F be the force exerted by the tractor, T be the tension in the two halves of the rope, and θ be the angle between the original line of the rope and half of the rope after it is pulled. At equilibrium, $2T \sin \theta - F = 0$, or $T = \frac{F}{2 \sin \theta}$. So, if $0° < \theta < 30°$, the force applied to the stump using method b is greater than the force exerted by the tractor.

31. Let T be the tension in each towline and suppose the axis of the ship is the vertical direction.
$2T \sin 70° - 6000 = 0$
$T = \frac{6000}{2 \sin 70°}$
≈ 3192.5 tons

32. Let T be the tension in each wire. The halves of the wire make angles of 30° and 150° with the horizontal.
$T \sin 30° + T \sin 150° - 25 = 0$
$\frac{1}{2}T + \frac{1}{2}T - 25 = 0$
$T = 25$ lb

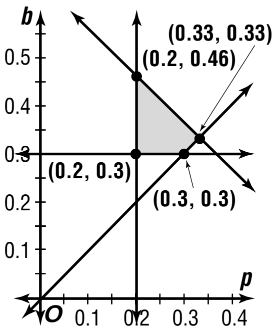
33. $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} = 9(-3) + 5(2) + 3(5)$
$= -2$
The vectors are not perpendicular since $\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \neq 0$.

34. $\vec{AB} = \langle 0 - 12, -11 - (-5), 21 - 18 \rangle$
$= \langle -12, -6, 3 \rangle$

35. $d = \frac{2{v_0}^2}{g} \sin \theta \cos \theta$
$= \frac{2 \cdot 100^2}{32} \sin 65° \cos 65°$
≈ 239.4 ft

36. Sample answer: A plot of the data suggests a quadratic function. Performing a quadratic regression and rounding the coefficients gives $y = 1.4x^2 - 2x + 3.9$.

37. $b \geq 0.3$
$p \geq 0.2$
$b + p \leq 0.66$
$b \geq p$



The vertices are at (0.2, 0.3), (0.3, 0.3), (0.33, 0.33) and (0.2, 0.46).
cost function $C(p, b) = 90p + 140b + 32(1 - p - b)$
$= 32 + 58p + 108b$
$C(0.2, 0.3) = 32 + 58(0.2) + 108(0.3)$ or 76
$C(0.3, 0.3) = 32 + 58(0.3) + 108(0.3)$ or 81.8
$C(0.33, 0.33) = 32 + 58(0.33) + 108(0.33)$ or 86.78
$C(0.2, 0.46) = 32 + 58(0.2) + 108(0.46)$ or 93.28
The minimum cost is \$76, using 30% beef and 20% pork.

38. $*4 - *(-3) = (4^3 - 4) - [(-3)^3 - (-3)]$
$= 60 - (-24)$
$= 84$
The correct choice is A.

8-6 Vectors and Parametric Equations

Pages 523–524 Check for Understanding

1. When $t = 0$, $x = 3$ and $y = -1$. When $t = 1$, $x = 7$ and $y = 1$. The graph is a line through $(3, -1)$ and $(7, 1)$.

2. Sample answer: For every single unit increment of t, x increases 1 unit and y increases 2 units. Then, the parametric equations of the line are $x = 3 + t$, $y = 6 + 2t$.

3. When $t = 0$, $x = 1$ and $y = 0$, so the line passes through $(1, 0)$. When $t = -1$, $x = 0$ and $y = 1$, so the line passes through $(0, 1)$, its y-intercept. The slope of the line is $\frac{1-0}{0-1}$ or -1.

4. $\langle x - (-4), y - 11\rangle = t\langle -3, 8\rangle$
$\langle x + 4, y - 11\rangle = t\langle -3, 8\rangle$
$x + 4 = -3t$ $\quad$ $y - 11 = 8t$
$x = -3t - 4$ $\quad$ $y = 8t + 11$

5. $\langle x - 1, y - 5\rangle = t\langle -7, 2\rangle$
$x - 1 = -7t$ $\quad$ $y - 5 = 2t$
$x = 1 - 7t$ $\quad$ $y = 5 + 2t$

6. $3x + 2y = 5$
$2y = -3x + 5$
$y = -\frac{3}{2}x + \frac{5}{2}$
$x = t$
$y = -\frac{3}{2}t + \frac{5}{2}$

7. $4x - 6y = -12$
$-6y = -4x - 12$
$y = \frac{2}{3}x + 2$
$x = t$
$y = \frac{2}{3}t + 2$

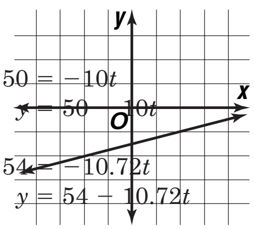
8. $x = -4t + 3$
$x - 3 = -4t$
$-\frac{1}{4}x + \frac{3}{4} = t$
$y = 5t - 3$
$y = 5\left(-\frac{1}{4}x + \frac{3}{4}\right) - 3$
$y = -\frac{5}{4}x + \frac{3}{4}$

9. $x = 9t$
$\frac{x}{9} = t$
$y = 4t + 2$
$y = 4\left(\frac{x}{9}\right) + 2$
$y = \frac{4}{9}x + 2$

10.

t	x	y
−1	−2	−2
0	2	−1
1	6	0
2	10	1

11a. receiver:
$x - 5 = 0t$ $\quad$ $y - 50 = -10t$
$x = 5$ $\quad$ $y = 50 - 10t$
defensive player:
$x - 10 = -0.9t$ $\quad$ $y - 54 = -10.72t$
$x = 10 - 0.9t$ $\quad$ $y = 54 - 10.72t$



11b. $50 - 10t = 0$
$50 = 10t$
$5 = t$
When $t = 5$, the coordinates of the defensive player are $(10 - 0.9(5), 54 - 10.72(5))$ or $(5.5, 0.4)$, so the defensive player has not yet caught the receiver.

524–525 Exercises

12. $\langle x - 5, y - 7\rangle = t\langle 2, 0\rangle$
$x - 5 = 2t$ $\quad$ $y - 7 = 0t$
$x = 5 + 2t$ $\quad$ $y = 7$

13. $\langle x - (-1), y - 4\rangle = t\langle 6, -10\rangle$
$\langle x + 1, y - 4\rangle = t\langle 6, -10\rangle$
$x + 1 = 6t$ $\quad$ $y - 4 = -10t$
$x = -1 + 6t$ $\quad$ $y = 4 - 10t$

14. $\langle x - (-6), y - 10\rangle = t\langle 3, 2\rangle$
$\langle x + 6, y - 10\rangle = t\langle 3, 2\rangle$
$x + 6 = 3t$ $\quad$ $y - 10 = 2t$
$x = -6 + 3t$ $\quad$ $y = 10 + 2t$

15. $\langle x - 1, y - 5\rangle = t\langle -7, 2\rangle$
$x - 1 = -7t$ $\quad$ $y - 5 = 2t$
$x = 1 - 7t$ $\quad$ $y = 5 + 2t$

16. $\langle x - 1, y - 0\rangle = t\langle -2, -4\rangle$
$\langle x - 1, y\rangle = t\langle -2, -4\rangle$
$x - 1 = -2t$ $\quad$ $y = -4t$
$x = 1 - 2t$

17. $\langle x - 3, y - (-5)\rangle = t\langle -2, 5\rangle$
$\langle x - 3, y + 5\rangle = t\langle -2, 5\rangle$
$x - 3 = -2t$ $\quad$ $y + 5 = 5t$
$x = 3 - 2t$ $\quad$ $y = -5 + 5t$

18. $x = t$
$y = 4t - 5$

19. $-3x + 4y = 7$
$4y = 3x + 7$
$y = \frac{3}{4}x + \frac{7}{4}$
$x = t$
$y = \frac{3}{4}t + \frac{7}{4}$

20. $2x - y = 3$
$-y = -2x + 3$
$y = 2x - 3$
$x = t$
$y = 2t - 3$

21. $9x + y = -1$
$y = -9x - 1$
$x = t$
$y = -9t - 1$

22. $2x + 3y = 11$
$3y = -2x + 11$
$y = -\frac{2}{3}x + \frac{11}{3}$
$x = t$
$y = -\frac{2}{3}t + \frac{11}{3}$

23. $-4x + y = -2$
$y = 4x - 2$
$x = t$
$y = 4t - 2$

24. $3x - 6y = -8$
$-6y = -3x - 8$
$y = \frac{1}{2}x + \frac{4}{3}$
The slope is $\frac{1}{2}$.
$y - 5 = \frac{1}{2}(x + 2)$
$y = \frac{1}{2}x + 6$
$x = t$
$y = \frac{1}{2}t + 6$

25. $x = 2t$

$\frac{x}{2} = t$

$y = 1 - t$

$y = 1 - \frac{x}{2}$

$y = -\frac{1}{2}x + 1$

26. $x = -7 + \frac{1}{2}t$

$x + 7 = \frac{1}{2}t$

$2x + 14 = t$

$y = 3t$

$y = 3(2x + 14)$

$y = 6x + 42$

27. $x = 4t - 11$

$x + 11 = 4t$

$\frac{1}{4}x + \frac{11}{4} = t$

$y = t + 3$

$y = \frac{1}{4}x + \frac{11}{4} + 3$

$y = \frac{1}{4}x + \frac{23}{4}$

28. $x = 4t - 8$

$x + 8 = 4t$

$\frac{1}{4}x + 2 = t$

$y = 3 + t$

$y = 3 + \frac{1}{4}x + 2$

$y = \frac{1}{4}x + 5$

29. $x = 3 + 2t$

$x - 3 = 2t$

$\frac{1}{2}x - \frac{3}{2} = t$

$y = -1 + 5t$

$y = -1 + 5\left(\frac{1}{2}x - \frac{3}{2}\right)$

$y = \frac{5}{2}x - \frac{17}{2}$

30. Regardless of the value of t, x is always 8, so the parametric equations represent the vertical line with equation $x = 8$.

31a. $\langle x - 11, y - (-4)\rangle = t\langle 3, 7\rangle$

$\langle x - 11, y + 4\rangle = t\langle 3, 7\rangle$

31b. $x - 11 = 3t$ $\quad$ $y + 4 = 7t$

$x = 3t + 11$ $\quad$ $y = 7t - 4$

31c. $x = 3t + 11$

$x - 11 = 3t$

$\frac{1}{3}x - \frac{11}{3} = t$

$y = 7t - 4$

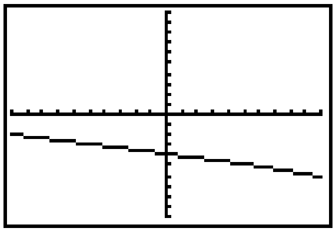
$y = 7\left(\frac{1}{3}x - \frac{11}{3}\right) - 4$

$y = \frac{7}{3}x - \frac{89}{3}$

32.

T	X1T	Y1T
1.0000	5.0000	-5.000
2.0000	10.000	-6.000
3.0000	15.000	-7.000
4.0000	20.000	-8.000
5.0000	25.000	-9.000
6.0000	30.000	-10.00
14.000	70.000	-18.00

T=1

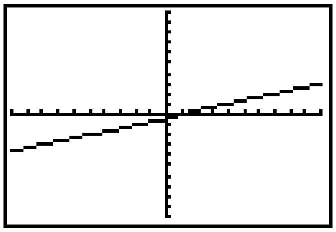


[−5, 5] Tstep: 1
[−10, 10] Xscl: 1
[−10, 10] Yscl: 1

33.

T	X1T	Y1T
1.0000	8.0000	2.0000
2.0000	11.000	3.0000
3.0000	14.000	4.0000
4.0000	17.000	5.0000
5.0000	20.000	6.0000
6.0000	23.000	7.0000
14.000	47.000	15.000

Y1T=2

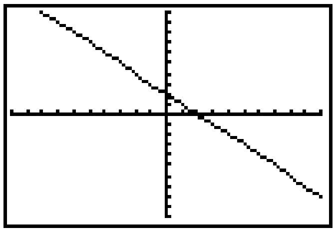


[−10, 10] Tstep: 1
[−20, 20] Xscl: 2
[−20, 20] Yscl: 2

34.

T	X1T	Y1T
1.0000	2.0000	0.0000
2.0000	3.0000	-1.000
3.0000	4.0000	-2.000
4.0000	5.0000	-3.000
5.0000	6.0000	-4.000
6.0000	7.0000	-5.000
14.000	15.000	-13.00

T=1



[−10, 10] Tstep: 1
[−10, 10] Xscl: 1
[−10, 10] Yscl: 1

35a. $x = 2 + 3t$ and $y = 4 + 7t$

If $t \geq 0$, then $x \geq 2$ and $y \geq 4$, so the part of the line to the right of point (2, 4) is obtained.

35b. $x < 0$

$2 + 3t < 0$

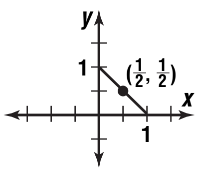
$3t < -2$

$t < -\frac{2}{3}$

36. $x + y = \cos^2 t + \sin^2 t$

$= 1$

$0 \leq \cos^2 t \leq 1$ and $0 \leq \sin^2 t \leq 1$, so the graph is the segment of the line with equation $x + y = 1$ from (1, 0) to (0, 1).



37a. target drone:

$x = 3 + (-1)t \qquad y = 4 + 0t$

$x = 3 - t \qquad y = 4$

missile:

$x = 2 + t \qquad y = 2 + 2t$

37b. $3 - t = 2 + t$

$1 = 2t$

$\frac{1}{2} = t$

When $t = \frac{1}{2}$, the missile has a y-coordinate of 3, not 4, so it does not intercept the drone.

38a. Ceres: $x = -1 + t,\ y = 4 - t,\ z = -1 + 2t$

Pallas: $x = -7 + 2t,\ y = -6 + 2t,\ z = -1 + t$

38b. Adding the equations for x and y for Ceres gives $x + y = 3$. Subtracting the equations for x and y for Pallas results in $x - y = -1$. The solution of this system is $x = 1$ and $y = 2$. Eliminating t from the equations for y and z results in the system $2y + z = 7,\ y - 2z = -4$ which has solution $y = 2$ and $z = 3$. Hence, the paths cross at (1, 2, 3).

38c. $-1 + t = 1 \Rightarrow t = 2$

$-7 + 2t = 1 \Rightarrow t = 4$

Ceres is at (1, 2, 3) when $t = 2$ but Pallas is at (1, 2, 3) when $t = 4$. The asteroids will not collide.

39. The line is parallel to the vector $\left\langle 0 - \left(-\frac{1}{3}\right), 5 - 1, -8 - 1\right\rangle$ or $\left\langle \frac{1}{3}, 4, -9\right\rangle$. The vector equation of the line is $\left\langle x - \left(-\frac{1}{3}\right), y = 1, z - 1\right\rangle = t\left\langle \frac{1}{3}, 4, -9\right\rangle$ or $\left\langle x + \frac{1}{3}, y - 1, z - 1\right\rangle = t\left\langle \frac{1}{3}, 4, -9\right\rangle$.

$x + \frac{1}{3} = \frac{1}{3}t \qquad y - 1 = 4t$

$x = -\frac{1}{3} + \frac{1}{3}t \qquad y = 1 + 4t$

$z - 1 = -9t$

$z = 1 - 9t$

40. $\vec{\mathbf{v}}_1 = (150 \cos 330°)\vec{\mathbf{i}} + (150 \sin 330°)\vec{\mathbf{j}}$

$\vec{\mathbf{v}}_2 = (50 \cos 245°)\vec{\mathbf{i}} + (50 \sin 245°)\vec{\mathbf{j}}$

$|\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2| =$

$\sqrt{(150 \cos 330° + 50 \cos 245°)^2 + (150 \sin 330° + 50 \sin 245°)^2}$

≈ 162.2 km/h

$\tan \theta = \frac{150 \sin 330° + 50 \sin 245°}{150 \cos 330° + 50 \cos 245°}$

$\theta = \tan^{-1}\left(\frac{150 \sin 330° + 50 \sin 245°}{150 \cos 330° + 50 \cos 245°}\right)$

$\approx -47°53'4''$ or $47°53'4''$ south of east

41. $\langle 1, 3\rangle \cdot \langle 3, -2\rangle = 1(3) + 3(-2)$

$= -3$

Since the inner product is not 0, the vectors are not perpendicular.

42. Since $A < 90°$, $a < b$, and $a < b \sin A$, no solution exists.

43. A graphing calculator indicates that there is one real zero and that it is close to 1. $f(1) = 0$, so the zero is exactly 1.

44. $x = \frac{3}{2}y - 2$

$x + 2 = \frac{3}{2}y$

$\frac{2}{3}x + \frac{4}{3} = y$

$y = \frac{2}{3}x + \frac{4}{3}$

45. The slope is 1.

$y - 1 = 1[x - (-3)]$

$y - 1 = x + 3$

$x - y + 4 = 0$

46. The linear velocity of the belt around the larger pulley is $(120 \text{ rpm})\left(2\pi \cdot \frac{9}{2} \text{ in./rev}\right) = 1080\pi$ in./min. The linear velocity around the smaller pulley must be the same, so its angular velocity is $(1080\pi \text{ in./min})\left(\frac{1 \text{ rev}}{2\pi \cdot 3 \text{ in.}}\right) = 180$ rpm. The correct choice is D.

8-6B Graphing Calculator Exploration: Modeling with Parametric Equations

Page 526

1. $408.7t = 418.3(t - 0.0083)$

$408.7t = 418.3t - 3.47189$

$-9.6t = -3.47189$

$t = \frac{3.47189}{9.6}$

≈ 0.362 hr or 21.7 min

2. $d = rt$

$= 408.7\left(\frac{3.47189}{9.6}\right)$

≈ 147.8 mi

3. The time for plane 1 to fly 500 miles is $\frac{500}{408.7}$. The time for plane 2 is $\frac{500}{418.3} + 0.0083$. Suppose the speed of plane 1 is increased by a mph.

$\frac{500}{408.7 + a} = \frac{500}{418.3} + 0.0083$

$\frac{408.7 + a}{500} = \frac{1}{\frac{500}{418.3} + 0.0083}$

$a = \frac{500}{\frac{500}{418.3} + 0.0083} - 408.7$

≈ 6.7 mph

8-7 Modeling Motion Using Parametric Equations

Page 531 Check for Understanding

1. Sample answer: a rocket launched at 90° to the horizontal; tip-off in basketball

2. Equal magnitude with opposite direction.

3. The greater the angle of the head of the golf club, the greater the angle of initial velocity of the ball.

4. $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta$

$= 50 \sin 40°$

≈ 32.14 ft/s

5. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta$

$= 20 \cos 50°$

≈ 12.86 m/s

6. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta$

$= 45 \cos 32°$

≈ 38.16 ft/s

$|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta$

$= 45 \sin 32°$

≈ 23.85 ft/s

7. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}| \cos \theta$

$= 7.5 \cos 20°$

≈ 7.05 m/s

$|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}| \sin \theta$

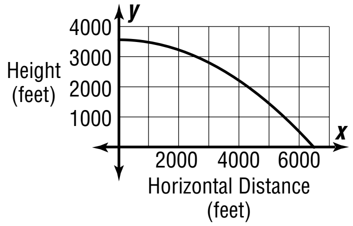
$= 7.5 \sin 20°$

≈ 2.57 m/s

8a. $300 \text{ mph} \left(\frac{5280 \text{ ft}}{\text{mile}}\right)\left(\frac{\text{h}}{3600 \text{ s}}\right) = 440 \text{ ft/s}$

$x = t|\vec{v}| \cos \theta$
$x = t(440) \cos 0^\circ$
$x = 440t$
$y = t|\vec{v}| \sin \theta - \frac{1}{2}gt^2 + h$
$y = t(440) \sin 0^\circ - \frac{1}{2}(32)t^2 + 3500$
$y = -16t^2 + 3500$

8b. Sample graph:



8c. $-16t^2 + 3500 = 0$
$-16t^2 = -3500$
$t^2 = \frac{-3500}{-16}$
$t = \sqrt{\frac{3500}{16}}$
$t \approx 14.8 \text{ s}$

8d. $x = 440t$
$= 440(14.8)$
$= 6512 \text{ ft}$

Pages 531–533 Exercises

9. $|\vec{v}_x| = |\vec{v}| \cos \theta$ $= 65 \cos 60^\circ$ $= 32.5 \text{ ft/s}$
$|\vec{v}_y| = |\vec{v}| \sin \theta$ $= 65 \sin 60^\circ$ $= 56.29 \text{ ft/s}$

10. $|\vec{v}_x| = |\vec{v}| \cos \theta$ $= 47 \cos 10.7^\circ$ $\approx 46.18 \text{ m/s}$
$|\vec{v}_y| = |\vec{v}| \sin \theta$ $= 47 \sin 10.7^\circ$ $\approx 8.73 \text{ m/s}$

11. $|\vec{v}_x| = |\vec{v}| \cos \theta$ $= 1200 \cos 42^\circ$ $\approx 891.77 \text{ ft/s}$
$|\vec{v}_y| = |\vec{v}| \sin \theta$ $= 1200 \sin 42^\circ$ $\approx 802.96 \text{ ft/s}$

12. $|\vec{v}_x| = |\vec{v}| \cos \theta$ $= 17 \cos 28^\circ$ $\approx 15.01 \text{ ft/s}$
$|\vec{v}_y| = |\vec{v}| \sin \theta$ $= 17 \sin 28^\circ$ $\approx 7.98 \text{ ft/s}$

13. $|\vec{v}_x| = |\vec{v}| \cos \theta$ $= 69 \cos 37^\circ$ $\approx 55.11 \text{ yd/s}$
$|\vec{v}_y| = |\vec{v}| \sin \theta$ $= 69 \sin 37^\circ$ $\approx 41.53 \text{ yd/s}$

14. $|\vec{v}_x| = |\vec{v}| \cos \theta$ $= 46 \cos 19^\circ$ $\approx 43.49 \text{ km/h}$
$|\vec{v}_y| = |\vec{v}| \sin \theta$ $= 46 \sin 19^\circ$ $\approx 14.98 \text{ km/h}$

15a. $x = t|\vec{v}| \cos \theta$
$x = 175t \cos 35^\circ$
$y = t|\vec{v}| \sin \theta - \frac{1}{2}gt^2$
$y = 175t \sin 35^\circ - 16t^2$

15b. $y = 0$
$175t \sin 35^\circ - 16t^2 = 0$
$t(175 \sin 35^\circ - 16t) = 0$
$175 \sin 35^\circ - 16t = 0$
$175 \sin 35^\circ = 16t$
$\frac{175 \sin 35^\circ}{16} = t$
$x = 175t \cos 35^\circ$
$= 175 \left(\frac{175 \sin 35^\circ}{16}\right) \cos 35^\circ$
$\approx 899.32 \text{ ft or } 299.77 \text{ yd}$

16. To find the time the projectile stays in the air, set $y = 0$ and solve for t.
$t|\vec{v}| \sin \theta - \frac{1}{2}gt^2 = 0$
$t(|\vec{v}| \sin \theta - \frac{1}{2}gt) = 0$
$|\vec{v}| \sin \theta - \frac{1}{2}gt = 0$
$|\vec{v}| \sin \theta = \frac{1}{2}gt$
$\frac{2|\vec{v}| \sin \theta}{g} = t$

The greater the angle, the greater the time the projectile stays in the air. To find the horizontal distance covered, substitute the expression for t in the equation for x.
$x = t|\vec{v}| \cos \theta$
$= \frac{2|\vec{v}| \sin \theta}{g} |\vec{v}| \cos \theta$
$= \frac{|\vec{v}|^{2g} \sin 2\theta}{g}$

As the angle increases from 0° to 45°, the horizontal distance increases. As the angle increases from 45° to 90°, the horizontal distance decreases.

17a. $y = 300$ when $t = 7$
$7|\vec{v}| \sin 78^\circ - \frac{1}{2}(32)7^2 = 300$
$7|\vec{v}| \sin 78^\circ - 784 = 300$
$7|\vec{v}| \sin 78^\circ = 1084$
$|\vec{v}| = \frac{1084}{7 \sin 78^\circ}$
$|\vec{v}| \approx 158.32 \text{ ft/s}$

17b. $x = \frac{1}{3}t|\vec{v}| \cos \theta + 50 \text{ yd}$
$= \frac{1}{3}(7)(158.32) \cos 78^\circ + 50$
$\approx 127 \text{ yd}$

18. $x = t|\vec{v}| \cos \theta$
$\frac{x}{|\vec{v}| \cos \theta} = t$
$y = t|\vec{v}| \sin \theta - \frac{1}{2}gt^2$
$y = \frac{x}{|\vec{v}| \cos \theta} |\vec{v}| \sin \theta - \frac{1}{2}g\left(\frac{x}{|\vec{v}| \cos \theta}\right)^2$
$y = x \tan \theta - \frac{g}{2|\vec{v}|^2 \cos^2 \theta}$

The presence of the x^2-term (due to the force of gravity) means that y is a quadratic function of x. Therefore, the path of a projectile is a parabolic arc.

19. To find the time the projectile stays in the air if the initial velocity is $\vec{v}$, set $y = 0$ and solve for t.
$t|\vec{v}| \sin \theta - \frac{1}{2}gt^2 = 0$
$t\left(|\vec{v}| \sin \theta - \frac{1}{2}gt\right) = 0$
$|\vec{v}| \sin \theta - \frac{1}{2}gt = 0$
$|\vec{v}| \sin \theta = \frac{1}{2}gt$
$\frac{2|\vec{v}| \sin \theta}{g = t} = t$

To find the range, substitute this expression for t in the equation for x.
$x = t|\vec{v}| \cos \theta$
$= \frac{2|\vec{v}| \sin \theta}{g} |\vec{v}| \cos \theta$
$= \frac{|\vec{v}|^2 \sin 2\theta}{g}$

If the magnitude of the initial velocity is doubled to $2|\vec{\mathbf{v}}|$, the range becomes $\frac{(2|\vec{\mathbf{v}}|)^2 \sin 2\theta}{g}$ or $4\frac{|\vec{\mathbf{v}}|^2 \sin 2\theta}{g}$. The projectile will travel four times as far.

20a. $800 \text{ km/h}\left(\frac{1000 \text{ m}}{\text{km}}\right)\left(\frac{\text{h}}{3600 \text{ s}}\right) \approx 222.2 \text{ m/s}$

$x = t|\vec{\mathbf{v}}| \cos\theta$

$x = -222.2\, t \cos 45°$

$y = t|\vec{\mathbf{v}}| \sin\theta - \frac{1}{2}gt^2$

$y = -222.2t \sin 45° - \frac{1}{2}(9.8)t^2$

$y = -222.2t \sin 45° - 4.9t^2$

The negative coefficient in the t-term in the equation for y indicates that the aircraft is descending. The negative coefficient in the equation for x is arbitrary.

20b. $y = -222.2t \sin 45° - 4.9t^2$

$= -222.2(2.5) \sin 45° - 4.9\,(2.5)^2$

≈ -423.4

The aircraft has descended about 423.4 m.

20c. $\frac{423.4 \text{ m}}{2.5 \text{ s}} \approx 169 \text{ m/s}$

or

$169 \text{ m/s}\left(\frac{\text{km}}{1000 \text{ m}}\right)\left(\frac{3600 \text{ s}}{\text{h}}\right) = 608.4 \text{ km/h}$

21a. $70 \text{ mph}\left(\frac{5280 \text{ ft}}{\text{mi}}\right)\left(\frac{\text{h}}{3600 \text{ s}}\right) = \frac{308}{3} \text{ ft/s}$

$y = 0$

$t\left(\frac{308}{3}\right) \sin 35° - 16t^2 + 10 = 0$

$t = \frac{\frac{-308}{3}\sin 35° - \sqrt{\left(\frac{308}{3}\sin 35°\right)^2 - 4(-16)10}}{}$

$t \approx$

3.84 s

$x = t|\vec{\mathbf{v}}| \cos\theta$

≈ 323.2 ft

21b. $y = 8$

$t\left(\frac{308}{3}\right) \sin 35° - 16t^2 + 10 = 8$

$-16t^2 + t\left(\frac{308}{3}\right) \sin 35° + 2 = 0$

$t = \frac{-\frac{308}{3}\sin 35° - \sqrt{\left(\frac{308}{3}\sin 35°\right)^2 - 4(-16)2}}{}$

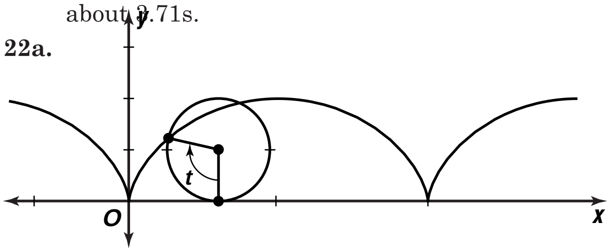
$t \approx 3.71$ s

$x = t|\vec{\mathbf{v}}| \cos\theta$

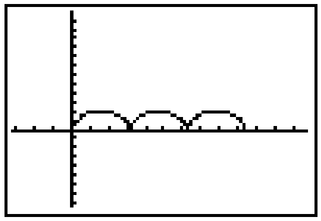
≈ 312.4 ft

21c. From the calculations in part b, the time is about 3.71s.

22a.



22b.



23a. $y = 300$ when $t = 4.8$

$4.8|\vec{\mathbf{v}}| \sin 82° - \frac{1}{2}(32)(4.8)^2 = 300$

$4.8|\vec{\mathbf{v}}| \sin 82° - 368.64 = 668.64$

$|\vec{\mathbf{v}}| = \frac{668.64}{4.8 \sin 82°}$

$|\vec{\mathbf{v}}| \approx 140.7$ ft/s

23b. $x = \frac{1}{3}t|\vec{\mathbf{v}}| \cos\theta + 100$

≈ 131.3 yd

24a. $x = t|\vec{\mathbf{v}}| \cos\theta$ $\quad y = t|\vec{\mathbf{v}}| \sin\theta - \frac{1}{2}gt^2 + h$

$x = 155t \cos 22°$ $\quad y = 155t \sin 22° - 16t^2 + 3$

24b. $x = 420$

$155t \cos 22° = 420$

$t = \frac{420}{155 \cos 22°}$

$y = 155t \sin 22° - 16t^2 + 3$

$= 155\left(\frac{420}{155 \cos 22°}\right) \sin 22° - 16\left(\frac{420}{155 \cos 22°}\right)^2 + 3$

≈ 36.04 ft

Since $36.04 > 15$, the ball will clear the fence.

24c. $y = 0$

$155t \sin 22° - 16t^2 + 3 = 0$

$t = \frac{-155 \sin 22° - \sqrt{(155 \sin 22°)^2 - 4(-16)3}}{}$

$t \approx$

3.68 s

$x = t|\vec{\mathbf{v}}| \cos\theta$

≈ 528.86 ft

25. $x = 11 - t$

$x - 11 = -t$

$-x + 11 = t$

$y = 8 - 6t$

$y = 8 - 6(-x + 11)$

$y = 6x - 58$

26a. $mg \sin\theta = 300(9.8) \sin 22°$

≈ 1101.3 N

26b. $mg \cos\theta = 300(9.8) \cos 22°$

≈ 2725.9 N

27. $\cos A = \frac{17.4}{21.9}$

$A = \cos^{-1}\frac{17.4}{21.9}$

$A \approx 37°$

28. $2(2x - y + z) = 2(2)$

$$\begin{array}{r} x + 3y - 2z = -3.25 \\ \hline 5x + y \qquad = 0.75 \end{array}$$

$$-1(2x - y + z) = -1(2)$$

$$\begin{array}{r} -4x - 5y + z = 2.5 \\ \hline -6x - 4y \qquad = 0.5 \end{array}$$

$$4(5x + y) = 4(0.75)$$

$$\begin{array}{r} -6x - 4y = 0.5 \\ \hline 14x = 3.5 \end{array}$$

$$x = \frac{3.5}{14}$$

$$x = 0.25$$

$$5x + y = 0.75 \qquad 2x - y + z = 2$$

$$5(0.25) + y = 0.75 \qquad 2(0.25) + 0.5 + z = 2$$

$$y = -0.5 \qquad z = 1$$

29. $\pi \cdot 5^2 - \pi \cdot 3^2 = 25\pi - 9\pi$

$$= 16\pi$$

The correct choice is B.

Page 534 History of Mathematics

1. $\frac{7°12'}{360°} = \frac{7.2°}{360°}$

$$= \frac{1}{50}$$

$$\frac{1}{50} = \frac{5000 \text{ stadia}}{x}$$

$$x = 50(5000)$$

$$x = 250{,}000 \text{ stadia}$$

250,000(500) = 125,000,000 ft

125,000,000 ÷ 5280 ≈ 23,674 mi

The actual circumference of Earth is about 24,901.55 miles.

2. See students' work. No solution exists.

3. See students' work.

8-8 Transformation Matrices in Three-Dimensional Space

Pages 539–540 Check for Understanding

1. Matrix T multiplies x-coordinates by -2 and y- and z-coordinates by 2, so it produces a reflection over the yz-plane and increases the dimensions two-fold.

2. $\overrightarrow{CC'} = \langle 8 - 6, 8 - 7, 2 - 3\rangle$ or $\langle 2, 1, -1\rangle$

The matrix is $\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$.

3. $VU = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = T$, so the transformations are the same.

4a-c.

Transformation	Orientation	Site	Shape
Reflection	yes	no	no
Translation	no	no	no
Dilation	no	yes	no

5a. $\overrightarrow{BE} = \langle 0 - 5, 2 - 5, 4 - 0\rangle$ or $\langle -5, -3, 4\rangle$

$A(5, 5 + (-3), 0) = A(5, 2, 0)$

$C(5 + (-5), 5, 0) = C(0, 5, 0)$

$D(5 + (-5), 5 + (-3), 0) = D(0, 2, 0)$

$F(5, 5 + (-3), 0 + 4) = F(5, 2, 4)$

$G(5, 5, 0 + 4) = G(5, 5, 4)$

$H(5 + (-5), 5, 0 + 4) = H(0, 5, 4)$

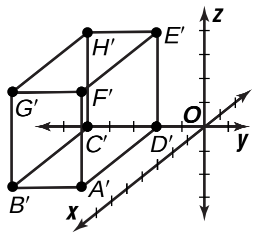
The matrix is $\begin{bmatrix} 5 & 5 & 0 & 0 & 0 & 5 & 5 & 0 \\ 2 & 5 & 5 & 2 & 2 & 2 & 5 & 5 \\ 0 & 0 & 0 & 0 & 4 & 4 & 4 & 4 \end{bmatrix}$.

5b. $\begin{bmatrix} 5+4 & 5+4 & 0+4 & 0+4 & 0+4 & 5+4 & 5+4 & 0+4 \\ 2+(-1) & 5+(-1) & 5+(-1) & 2+(-1) & 2+(-1) & 2+(-1) & 5+(-1) & 5+(-1) \\ 0+2 & 0+2 & 0+2 & 0+2 & 4+2 & 4+2 & 4+2 & 4+2 \end{bmatrix}$

$= \begin{bmatrix} 9 & 9 & 4 & 4 & 4 & 9 & 9 & 4 \\ 1 & 4 & 4 & 1 & 1 & 1 & 4 & 4 \\ 2 & 2 & 2 & 2 & 6 & 6 & 6 & 6 \end{bmatrix}$

5c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 5 & 0 & 0 & 0 & 5 & 5 & 0 \\ 2 & 5 & 5 & 2 & 2 & 2 & 5 & 5 \\ 0 & 0 & 0 & 0 & 4 & 4 & 4 & 4 \end{bmatrix}$

$= \begin{bmatrix} 5 & 5 & 0 & 0 & 0 & 5 & 5 & 0 \\ -2 & -5 & -5 & -2 & -2 & -2 & -5 & -5 \\ 0 & 0 & 0 & 0 & 4 & 4 & 4 & 4 \end{bmatrix}$



The image is the reflection over the xz-plane.

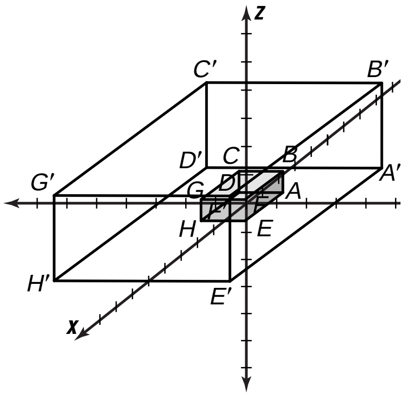
5d. The dimensions of the resulting figure are half the original.

6a. The scale factor of the dilation is 4. The translation increases x-coordinates by 2. The matrices are

$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ and

$T = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

6b. Sample answer: If the original prism has vertices $A(-3, 3, 0)$ $B(-3, 3, 3)$, $C(-3, -3, 3)$, $D(-3, -3, 0)$, $E(5, 3, 0)$, $F(5, 3, 3)$, $G(5, -3, 3)$, and $H(5, -3, 0)$, then the image has vertices $A'(-10, 12, 0)$, $B'(-10, 12, 12)$, $C'(-10, -12, 12)$, $D'(-10, -12, 0)$, $E'(22, 12, 0)$, $F'(22, 12, 12)$, $G'(22, -12, 12)$, and $H'(22, -12, 0)$.



Pages 540–542 Exercises

7. $\overrightarrow{FB} = \langle 3 - 3, 1 - 7, 4 - 4\rangle$ or $\langle 0, -6, 0\rangle$

$A(2, 3, + (-6), 2) = A(2, -3, 2)$

$C(4, 7 + (-6), -1) = C(4, 1, -1)$

The matrix is $\begin{bmatrix} 2 & 3 & 4 & 4 & 2 & 3 \\ -3 & 1 & 1 & 7 & 3 & 7 \\ 2 & 4 & -1 & -1 & 2 & 4 \end{bmatrix}$.

8. $\overrightarrow{AH} = \langle 4 - (-3), 1 - (-2), -2 - 2\rangle$ or $\langle 7, 3, -4\rangle$

$B(-3, -2 + 3, 2) = B(-3, 1, 2)$

$C(-3, -2 + 3, 2 + (-4)) = C(-3, 1, -2)$

$D(-3, -2, 2 + (-4)) = D(-3, -2, -2)$

$E(-3 + 7, -2, 2 + (-4)) = E(4, -2, -2)$

$F(-3 + 7, -2, 2) = F(4, -2, 2)$

$G(-3 + 7, -2 + 3, 2) = G(4, 1, 2)$

The matrix is $\begin{bmatrix} -3 & -3 & -3 & -3 & 4 & 4 & 4 & 4 \\ -2 & 1 & 1 & -2 & -2 & -2 & 1 & 1 \\ 2 & 2 & -2 & -2 & -2 & 2 & 2 & -2 \end{bmatrix}$

9. $\overrightarrow{CF} = \langle 6 - 4, 0 - (-1), 0 - 2\rangle$ or $\langle 2, 1, -2\rangle$

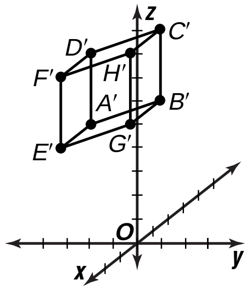
$D(2 + 2, -2 + 1, 3 + (-2)) = D(4, -1, 1)$

$E(1 + 2, 0 + 1, 4 + (-2)) = E(3, 1, 2)$

The matrix is $\begin{bmatrix} 2 & 1 & 4 & 4 & 3 & 6 \\ -2 & 0 & -1 & -1 & 1 & 0 \\ 3 & 4 & 2 & 1 & 2 & 0 \end{bmatrix}$.

10. $$\begin{bmatrix} 0+0 & 0+0 & 0+0 & 0+0 & 2+0 & 2+0 & 2+0 & 2+0 \\ 0+(-2) & 3+(-2) & 3+(-2) & 0+(-2) & 0+(-2) & 0+(-2) & 3+(-2) & 3+(-2) \\ 1+4 & 2+4 & 5+4 & 4+4 & 1+4 & 4+4 & 5+4 & 2+4 \end{bmatrix}$$

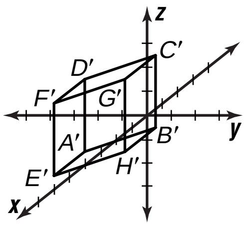
$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ -2 & 1 & 1 & -2 & -2 & -2 & 1 & 1 \\ 5 & 6 & 9 & 8 & 5 & 8 & 9 & 6 \end{bmatrix}$$



The result is a translation of −2 units along the *y*-axis and 4 units along the *z*-axis.

11. $$\begin{bmatrix} 0+1 & 0+1 & 0+1 & 0+1 & 2+1 & 2+1 & 2+1 & 2+1 \\ 0+(-2) & 3+(-2) & 3+(-2) & 0+(-2) & 0+(-2) & 0+(-2) & 3+(-2) & 3+(-2) \\ 1+(-2) & 2+(-2) & 5+(-2) & 4+(-2) & 1+(-2) & 4+(-2) & 5+(-2) & 2+(-2) \end{bmatrix}$$

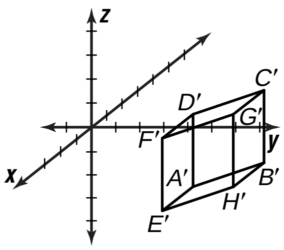
$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 3 & 3 & 3 & 3 \\ -2 & 1 & 1 & -2 & -2 & -2 & 1 & 1 \\ -1 & 0 & 3 & 2 & -1 & 2 & 3 & 0 \end{bmatrix}$$



The result is a translation of 1 unit along the *x*-axis, −2 units along the *y*-axis, and −2 units along the *z*-axis.

12. $$\begin{bmatrix} 0+1 & 0+1 & 0+1 & 0+1 & 2+1 & 2+1 & 2+1 & 2+1 \\ 0+5 & 3+5 & 3+5 & 0+5 & 0+5 & 0+5 & 3+5 & 3+5 \\ 1+(-3) & 2+(-3) & 5+(-3) & 4+(-3) & 1+(-3) & 4+(-3) & 5+(-3) & 2+(-3) \end{bmatrix}$$

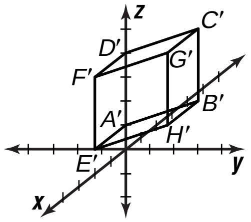
$$= \begin{bmatrix} 1 & 1 & 1 & 1 & 3 & 3 & 3 & 3 \\ 5 & 8 & 8 & 5 & 5 & 5 & 8 & 8 \\ -2 & -1 & 2 & 1 & -2 & 1 & 2 & -1 \end{bmatrix}$$



The results is a translation of 1 unit along the *x*-axis, 5 units along the *y*-axis, and −3 units along the *z*-axis.

13. $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&0&0&2&2&2&2\\0&3&3&0&0&0&3&3\\1&2&5&4&1&4&5&2\end{bmatrix}$

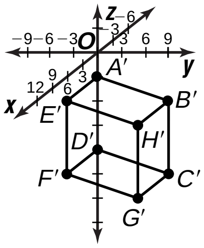
$=\begin{bmatrix}0&0&0&0&2&2&2&2\\0&3&3&0&0&0&3&3\\1&2&5&4&1&4&5&2\end{bmatrix}$



The transformation does not change the figure.

14. $\begin{bmatrix}1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}0&0&0&0&2&2&2&2\\0&3&3&0&0&0&3&3\\1&2&5&4&1&4&5&2\end{bmatrix}$

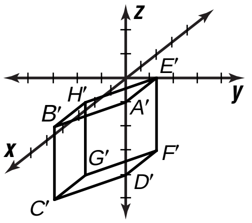
$=\begin{bmatrix}0&0&0&0&2&2&2&2\\0&-3&-3&0&0&0&-3&-3\\-1&-2&-5&-4&-1&-4&-5&-2\end{bmatrix}$



The transformation results in reflections over the xy − and xz-planes.

15. $\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}0&0&0&0&2&2&2&2\\0&3&3&0&0&0&3&3\\1&2&5&4&1&4&5&2\end{bmatrix}$

$=\begin{bmatrix}0&0&0&0&-2&-2&-2&-2\\0&-3&-3&0&0&0&-3&-3\\-1&-2&-5&-4&-1&-4&-5&-2\end{bmatrix}$



The transformation results in reflections over all three coordinate planes.

16. The matrix results in a dilation of scale factor 2, so the figure is twice the original size.

17. $\begin{bmatrix}3&0&0\\0&3&0\\0&0&-3\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&-1\end{bmatrix}\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$, so the figure is three times the original size and reflected over the xy-plane.

18. $\begin{bmatrix}-0.75&0&0\\0&-0.75&0\\0&0&-0.75\end{bmatrix}=\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&-1\end{bmatrix}$
$\begin{bmatrix}0.75&0&0\\0&0.75&0\\0&0&0.75\end{bmatrix}$, so the figure is three-fourths the original site and reflected over all three coordinate planes.

19a. $\begin{bmatrix}2x\\2y\\5z\end{bmatrix}=\begin{bmatrix}2&0&0\\0&2&0\\0&0&5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}$, so the transformation can be represented by the matrix $\begin{bmatrix}2&0&0\\0&2&0\\0&0&5\end{bmatrix}$.

19b. The transformation will magnify the x- and y-dimensions two-fold, and the z-dimension 5-fold.

20a. $\begin{bmatrix}23.6&23.6&23.6&23.6&23.6\\72&72&72&72&72\\0&0&0&0&0\end{bmatrix}$

20b. $\begin{bmatrix}20+23.6&136+23.6&247+23.6&302+23.6&351+23.6\\-58+72&-71+72&-74+72&-83+72&-62+72\\27+0&53+0&59+0&37+0&52+0\end{bmatrix}$

$=\begin{bmatrix}43.6&159.6&270.6&325.6&374.6\\14&1&-2&-11&10\\27&53&59&37&52\end{bmatrix}$

20c. The result is a translation 23.6 units along the x-axis and 72 units along the y-axis.

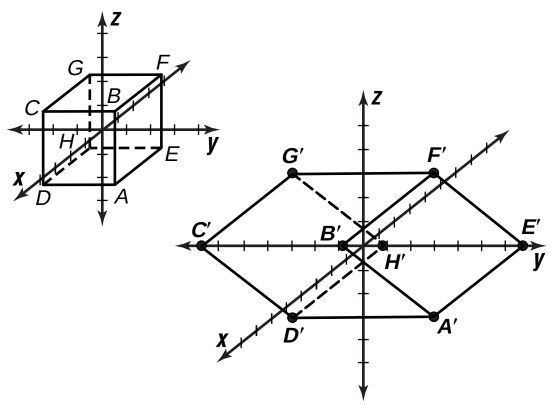
21. The matrix $\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}$ would reflect the prism over the yz-plane. The matrix $\begin{bmatrix}0.5&0&0\\0&0.5&0\\0&0&0.5\end{bmatrix}$ would reduce its dimensions by half.

$\begin{bmatrix}0.5&0&0\\0&0.5&0\\0&0&0.5\end{bmatrix}\begin{bmatrix}-1&0&0\\0&1&0\\0&0&1\end{bmatrix}=\begin{bmatrix}-0.5&0&0\\0&0.5&0\\0&0&0.5\end{bmatrix}$

22a. Placing a non zero element in the first row and third column will skew the cube so that the top is no longer directly above the bottom.

Sample answer: $\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix}$

22b. Sample graphs:



23. The first transformation reflects the figure over all three coordinate planes. The second transformation stretches the dimensions along the y- and z-axes and skews it along the xy-plane. (The first row of T changes the x-coordinate of (x, y, z) to $x + 2z$.)

24. To multiply the x-coordinate by 3, the first row of the matrix must be 3 0 0. Since the y-coordinate is multiplied by 2, the second row is 0 2 0. To convert a z-coordinate to $x - 4z$, use a third row of 1 0 −4.

The matrix is $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & -4 \end{bmatrix}$.

25a. The x-coordinates are unchanged, the y-coordinates increase, and the z-coordinates decrease, so the movement is dip-slip.

25b. $$\begin{bmatrix} 123.9 & -41.3 & 201.7 & 73.8 & -129.4 & 36.4 \\ 88.0 & 145.8 & -28.3 & -82.6 & 97.1 & -123.9 \\ 205.3 & 246.6 & 261.5 & 212.0 & -166.4 & -85.3 \end{bmatrix}$$
$$- \begin{bmatrix} 123.9 & -41.3 & 201.7 & 73.8 & -129.4 & 36.4 \\ 86.4 & 144.2 & -29.9 & -84.2 & 95.5 & -125.5 \\ 206.5 & 247.8 & 262.7 & 213.2 & -165.2 & -84.1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1.6 & 1.6 & 1.6 & 1.6 & 1.6 & 1.6 \\ -1.2 & -1.2 & -1.2 & -1.2 & -1.2 & -1.2 \end{bmatrix}$$

26a.

La Shawna	Jaimie
$x = 0$	$x = 35t$
$y = -16t^2 + 150$	$y = -16t^2 + 150$

$-16t^2 + 150 = 0$

$150 = 16t^2$

$\frac{150}{16} = t^2$

$\sqrt{\frac{150}{16}} = t$

$3.06 \approx t$

$x = 35t$

$= 35\sqrt{\frac{150}{16}}$

≈ 107 ft

26b. Since the stones have the same parametric equations for y, they land at the same time. In part a, it was calculated that the elapsed time is about 3.06 seconds.

27. $x = -5t - 1$

$x + 1 = -5t$

$\frac{x+1}{-5} = t$

$y = 2t + 10$

$y = 2\left(\frac{x+1}{-5}\right) + 10$

$y = -\frac{2}{5}x + \frac{48}{5}$

28. $\sec\left(\cos^{-1}\frac{2}{5}\right) = \frac{1}{\cos\left(\cos^{-1}\frac{2}{5}\right)}$

$= \frac{1}{\frac{2}{5}}$

$= \frac{5}{2}$

29. $80x^3 + 80x^2 + 80x = 24.2$

$80x^3 + 80x^2 + 80x - 24.2 = 0$

A graphing calculator indicates that there is a solution between 0 and 1. By Descartes' Rule of Signs, it is the only solution. When $x = 0.2$, $80x^3 + 80x^2 + 80x - 24.2 = -4.36$ and when $x = 0.3$, $80x^3 + 80x^2 + 80x - 24.2 = 9.16$. So the solution to the nearest tenth is 0.2.

30. Divide each side of the equations by 2, 3, 4, and 6, respectively, so that the left side is $x + 2y$.

I. $x + 2y = 4$ II. $y = 4$

III. $x + 2y = 2$ IV. $x + 2y = \frac{8}{3}$

Only I and II are equivalent, so the correct choice is A.

Chapter 8 Study Guide and Assessment

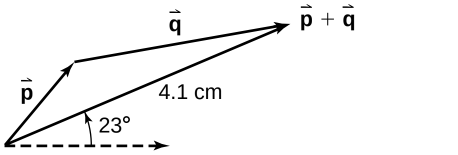
Page 543 Understanding and Using the Vocabulary

1. resultant **2.** unit
3. magnitude **4.** cross
5. inner **6.** vector
7. parallel **8.** standard
9. direction **10.** components

544–546 Skills and Concepts

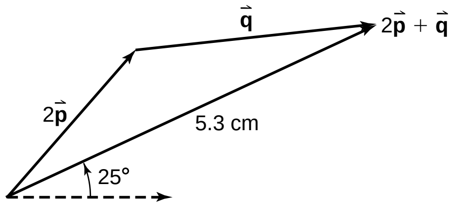
11. 1.3 cm, 50° **12.** 2.9 cm, 10°

13.



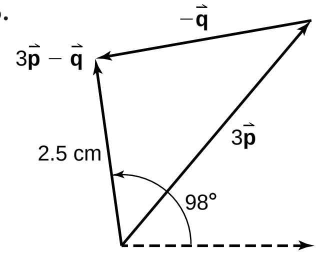
4.1 cm, 23°

14.



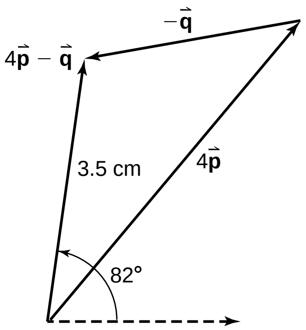
5.3 cm, 25°

15.



2.5 cm, 98°

16.



3.5 cm, 82°

17. $h = 1.3 \cos 50°$ $\quad v = 1.3 \sin 50°$
$h = 0.8$ cm $\quad v = 1$ cm

18. $h = 2.9 \cos 10°$ $\quad v = 2.9 \sin 10°$
$h = 2.9$ cm $\quad v = 0.5$ cm

19. $\overrightarrow{CD} = \langle 7 - 2, 15 - 3\rangle$ or $\langle 5, 12\rangle$
$|\overrightarrow{CD}| = \sqrt{5^2 + 12^2}$
$= \sqrt{169}$ or 13

20. $\overrightarrow{CD} = \langle 4 - (-2), 12 - 8\rangle$ or $\langle 6, 4\rangle$
$|\overrightarrow{CD}| = \sqrt{6^2 + 4^2}$
$= \sqrt{52}$ or $2\sqrt{13}$

21. $\overrightarrow{CD} = \langle 0 - 2, 9 - (-3)\rangle$ or $\langle -2, 12\rangle$
$|\overrightarrow{CD}| = \sqrt{(-2)^2 + 12^2}$
$= \sqrt{148}$ or $2\sqrt{37}$

22. $\overrightarrow{CD} = \langle -5 - (-6), -4 - 4\rangle$ or $\langle 1, -8\rangle$
$|\overrightarrow{CD}| = \sqrt{1^2 + (-8)^2}$
$= \sqrt{65}$

23. $\vec{\mathbf{u}} = \vec{\mathbf{v}} + \vec{\mathbf{w}}$
$\vec{\mathbf{u}} = \langle 2, -5\rangle + \langle 3, -1\rangle$
$\vec{\mathbf{u}} = \langle 2 + 3, -5 + (-1)\rangle$ or $\langle 5, -6\rangle$

24. $\vec{\mathbf{u}} = \vec{\mathbf{v}} - \vec{\mathbf{w}}$
$\vec{\mathbf{u}} = \langle 2, -5\rangle - \langle 3, -1\rangle$
$\vec{\mathbf{u}} = \langle 2 - 3, -5 - (-1)\rangle$ or $\langle -1, -4\rangle$

25. $\vec{\mathbf{u}} = 3\vec{\mathbf{v}} + 2\vec{\mathbf{w}}$
$\vec{\mathbf{u}} = 3\langle 2, -5\rangle + 2\langle 3, -1\rangle$
$\vec{\mathbf{u}} = \langle 6, -15\rangle + \langle 6, -2\rangle$
$\vec{\mathbf{u}} = \langle 6 + 6, -15 + (-2)\rangle$ or $\langle 12, -17\rangle$

26. $\vec{\mathbf{u}} = 3\vec{\mathbf{v}} - 2\vec{\mathbf{w}}$
$\vec{\mathbf{u}} = 3\langle 2, -5\rangle - 2\langle 3, -1\rangle$
$\vec{\mathbf{u}} = \langle 6, -15\rangle - \langle 6, -2\rangle$
$\vec{\mathbf{u}} = \langle 6 - 6, -15 - (-2)\rangle$ or $\langle 0, -13\rangle$

27. $\overrightarrow{EF} = \langle 6 - 2, -2 - (-1), 1 - 4\rangle$ or $\langle 4, -1, -3\rangle$
$|\overrightarrow{EF}| = \sqrt{4^2 + (-1)^2 + (-3)^2}$
$= \sqrt{26}$

28. $\overrightarrow{EF} = \langle -1 - 9, 5 - 8, 11 - 5\rangle$ or $\langle -10, -3, 6\rangle$
$|\overrightarrow{EF}| = \sqrt{(-10)^2 + (-3)^2 + 6^2}$
$= \sqrt{145}$

29. $\overrightarrow{EF} = \langle 2 - (-4), -1 - (-3), 7 - 0)$ or $\langle 6, 2, 7\rangle$
$|\overrightarrow{EF}| = \sqrt{6^2 + 2^2 + 7^2}$
$= \sqrt{89}$

30. $\overrightarrow{EF} = \langle -4 - 3, 0 - 7, 5 - (-8)\rangle$ or $\langle -7, -7, 13\rangle$
$|\overrightarrow{EF}| = \sqrt{(-7)^2 + (-7)^2 + 13^2}$
$= \sqrt{267}$

31. $\vec{\mathbf{u}} = 2\vec{\mathbf{w}} - 5\vec{\mathbf{v}}$
$\vec{\mathbf{u}} = 2\langle 4, -1, 5\rangle - 5\langle -1, 7, -4\rangle$
$\vec{\mathbf{u}} = \langle 8, -2, 10\rangle - \langle -5, 35, -20\rangle$
$\vec{\mathbf{u}} = \langle 8 - (-5), -2 - 35, 10 - (-20)\rangle$
$\vec{\mathbf{u}} = \langle 13, -37, 30\rangle$

32. $\vec{\mathbf{u}} = 0.25\vec{\mathbf{v}} + 0.4\vec{\mathbf{w}}$
$\vec{\mathbf{u}} = 0.25\langle -1, 7, -4\rangle + 0.4\langle 4, -1, 5\rangle$
$\vec{\mathbf{u}} = \langle -0.25, 1.75, -1\rangle + \langle 1.6, -0.4, 2\rangle$
$\vec{\mathbf{u}} = \langle -0.25 + 1.6, 1.75 + (-0.4), -1 + 2\rangle$
$\vec{\mathbf{u}} = \langle 1.35, 1.35, 1\rangle$

33. $\langle 5, -1\rangle \cdot \langle -2, 6\rangle = 5(-2) + (-1)6$
$= -10 - 6$
$= -16$; no

34. $\langle 2, 6\rangle \cdot \langle 3, -4\rangle = 2(3) + 6(-4)$
$= 6 - 24$
$= -18$; no

35. $\langle 4, 1, -2\rangle \cdot \langle 3, -4, 4\rangle = 4(3) + 1(-4) + (-2)4$
$= 12 - 4 - 8$
$= 0$; yes

36. $\langle 2, -1, 4\rangle \cdot \langle 6, -2, 1\rangle = 2(6) + (-1)(-2) + 4(1)$
$= 12 + 2 + 4$
$= 18$; no

37. $\langle 5, 2, -10\rangle \cdot \langle 2, -4, -4\rangle$
$= 5(2) + 2(-4) + (-10)(-4)$
$= 10 - 8 + 40$
$= 42$; no

38. $\langle 5, -2, 5\rangle \times \langle -1, 0, -3\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 5 & -2 & 5 \\ -1 & 0 & -3 \end{vmatrix}$
$= \begin{vmatrix} -2 & 5 \\ 0 & -3 \end{vmatrix}\vec{\mathbf{u}} - \begin{vmatrix} 5 & 5 \\ -1 & -3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 5 & -2 \\ -1 & 0 \end{vmatrix}\vec{\mathbf{k}}$
$= 6\vec{\mathbf{i}} + 10\vec{\mathbf{j}} - 2\vec{\mathbf{k}}$ or $\langle 6, 10, -2\rangle$
$\langle 6, 10, -2\rangle \cdot \langle 5, -2, 5\rangle$
$6(5) + 10(-2) + (-2)(5)$
$30 - 20 - 10 = 0$
$\langle 6, 10, -2\rangle \cdot \langle -1, 0, -3\rangle$
$6(-1) + 10(0) + (-2)(-3)$
$-6 + 0 + 6 = 0$

39. $\langle -2, -3, 1\rangle \times \langle 2, 3, -4\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -2 & -3 & 1 \\ 2 & 3 & -4 \end{vmatrix}$
$= \begin{vmatrix} -3 & 1 \\ 3 & -4 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -2 & 1 \\ 2 & -4 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -2 & -3 \\ 2 & 3 \end{vmatrix}\vec{\mathbf{k}}$
$= 9\vec{\mathbf{i}} - 6\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$ or $\langle 9, -6, 0\rangle$
$\langle 9, -6, 0\rangle \cdot \langle -2, -3, 1\rangle$
$9(-2) + (-6)(-3) + 0(1)$
$-18 + 18 + 0 = 0$
$\langle 9, -6, 0\rangle \cdot \langle 2, 3, -4\rangle$
$9(2) + (-6)(3) + 0(-4)$
$18 - 18 + 0 = 0$

40. $\langle -1, 0, 4\rangle \times \langle 5, 2, -1\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -1 & 0 & 4 \\ 5 & 2 & -1 \end{vmatrix}$

$= \begin{vmatrix} 0 & 4 \\ 2 & -1 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -1 & 4 \\ 5 & -1 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -1 & 0 \\ 5 & 2 \end{vmatrix}\vec{\mathbf{k}}$

$= -8\vec{\mathbf{i}} + 19\vec{\mathbf{j}} - 2\vec{\mathbf{k}}$ or $\langle -8, 19, -2\rangle$

$\langle -8, 19, -2\rangle \cdot \langle -1, 0, 4\rangle$

$(-8)(-1) + 19(0)(-2)(4)$

$8 + 0 - 8 = 0$

$\langle -8, 19, -2\rangle \cdot \langle 5, 2, -1\rangle$

$(-8)(5) + 19(2) + (-2)(-1)$

$-40 + 38 + 2 = 0$

41. $\langle 7, 2, 1\rangle \times \langle 2, 5, 3\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 7 & 2 & 1 \\ 2 & 5 & 3 \end{vmatrix}$

$= \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 7 & 1 \\ 2 & 3 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 7 & 2 \\ 2 & 5 \end{vmatrix}\vec{\mathbf{k}}$

$= \vec{\mathbf{i}} - 19\vec{\mathbf{j}} + 31\vec{\mathbf{k}}$ or $\langle 1, -19, 31\rangle$

$\langle 1, -19, 31\rangle \cdot \langle 7, 2, 1\rangle$

$1(7) + (-19)2 + 31(1)$

$7 + (-38) + 31 = 0$

$\langle 1, -19, 31\rangle \cdot \langle 2, 5, 3\rangle$

$1(2) + (-19)5 + 31(3)$

$2 + (-95) + 93 = 0$

42. Sample answer:

Let $x(1, 2, 3)$, $y(-4, 2, -1)$ and $z(5, -3, 0)$

$\overrightarrow{xy} = \langle -4 - 1, 2 - 2, -1 - 3\rangle$ or $\langle -5, 0, -4\rangle$

$\overrightarrow{yz} = \langle 5 - (-4), -3 - 2, 0 - (-1)\rangle$ or $\langle 9, -5, 1\rangle$

$\langle -5, 0, -4\rangle \times \langle 9, -5, 1\rangle = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -5 & 0 & -4 \\ 9 & -5 & 1 \end{vmatrix}$

$= \begin{vmatrix} 0 & -4 \\ -5 & 1 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} -5 & -4 \\ 9 & 1 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} -5 & 0 \\ 9 & -5 \end{vmatrix}\vec{\mathbf{k}}$

$= -20\vec{\mathbf{i}} - 31\vec{\mathbf{j}} + 25\vec{\mathbf{k}}$ or $\langle -20, -31, 25\rangle$

43. $\vec{\mathbf{F}}_1 = 320\vec{\mathbf{i}}$

$\vec{\mathbf{F}}_2 = 260\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{320^2 + 260^2}$

≈ 412.31 N

$\tan\theta = \frac{260}{320}$ or $\frac{13}{16}$

$\theta = \tan^{-1}\frac{13}{16}$

$\approx 39.09°$

44. $\vec{\mathbf{v}}_1 = 12\vec{\mathbf{j}}$

$\vec{\mathbf{v}}_2 = (30\cos 116°)\vec{\mathbf{i}} + (30\sin 116°)\vec{\mathbf{j}}$

$|\vec{\mathbf{v}}_1 + \vec{\mathbf{v}}_2| = \sqrt{(30\cos 116°)^2 + (12 + 30\sin 116°)^2}$

≈ 41 m/s

$\tan\theta = \frac{12 + 30\sin 116°}{30\cos 116°}$

$\theta = \tan^{-1}\left(\frac{12 + 30\sin 116°}{30\cos 116°}\right)$

$\approx 108.65°$

45. $\langle x - 3, y - (-5)\rangle = t\langle 4, 2\rangle$

$\langle x - 3, y + 5\rangle = t\langle 4, 2\rangle$

$x - 3 = 4t$ $y + 5 = 2t$

$x = 3 + 4t$ $y = -5 + 2t$

46. $\langle x - (-1), y - 9\rangle = t\langle -7, -5\rangle$

$\langle x + 1, y - 9\rangle = t\langle -7, -5\rangle$

$x + 1 = -7t$ $y - 9 = -5t$

$x = -1 - 7t$ $y = 9 - 5t$

47. $\langle x - 4, y - 0\rangle = t\langle 3, -6\rangle$

$\langle x - 4, y\rangle = t\langle 3, -6\rangle$

$x - 4 = 3t$ $y = -6t$

$x = 4 + 3t$

48. $x = t$

$y = -8t - 7$

$= -7 - 8t$

49. $x = t$

$y = -\frac{1}{2}t + \frac{5}{2}$

50. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}|\cos\theta$ $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}|\sin\theta$

$= 15\cos 55°$ $= 15\sin 55°$

≈ 8.60 ft/s ≈ 12.29 ft/s

51. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}|\cos\theta$ $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}|\sin\theta$

$= 13.2\cos 66°$ $= 13.2\sin 66°$

≈ 5.37 ft/s ≈ 12.06 ft/s

52. $|\vec{\mathbf{v}}_x| = |\vec{\mathbf{v}}|\cos\theta$ $|\vec{\mathbf{v}}_y| = |\vec{\mathbf{v}}|\sin\theta$

$= 18\cos 28°$ $= 18\sin 28°$

≈ 15.89 m/s ≈ 8.45 m/s

53. $\overrightarrow{CH} = \langle -4 - 3, -2 - 4, 2 - (-1)\rangle$ or $\langle -7, -6, 3\rangle$

$A(3, 4 + (-6), -1 + 3) = A(3, -2, 2)$

$B(3, 4 + (-6), -1) = B(3, -2, -1)$

$D(3, 4, -1 + 3) = D(3, 4, 2)$

$E(3 + (-7), 4, -1 + 3) = E(-4, 4, 2)$

$F(3 + (-7), 4, -1) = F(-4, 4, -1)$

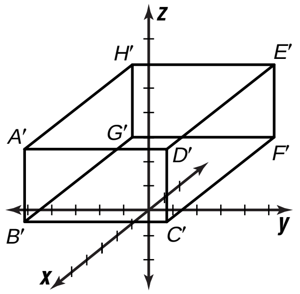
$G(3 + (-7), 4 + (-6), -1) = G(-4, -2, -1)$

The matrix for the figure is

$$\begin{bmatrix} 3 & 3 & 3 & 3 & -4 & -4 & -4 & -4 \\ -2 & -2 & 4 & 4 & 4 & 4 & -2 & -2 \\ 2 & -1 & -1 & 2 & 2 & -1 & -1 & 2 \end{bmatrix}.$$

The matrix for the translated figure is

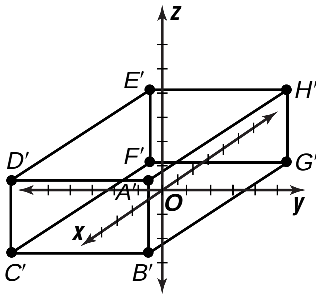
$$\begin{bmatrix} 5 & 5 & 5 & 5 & -2 & -2 & -2 & -2 \\ -2 & -2 & 4 & 4 & 4 & 4 & -2 & -2 \\ 5 & 2 & 2 & 5 & 5 & 2 & 2 & 5 \end{bmatrix}.$$



The figure moves 2 units along the x-axis and 3 units along the z-axis.

54. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 3 & 3 & -4 & -4 & -4 & -4 \\ -2 & -2 & 4 & 4 & 4 & 4 & -2 & -2 \\ 2 & -1 & -1 & 2 & 2 & -1 & -1 & 2 \end{bmatrix}$

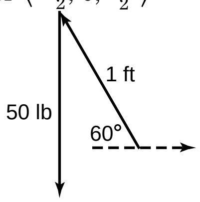
$= \begin{bmatrix} 3 & 3 & 3 & 3 & -4 & -4 & -4 & -4 \\ 2 & 2 & -4 & -4 & -4 & -4 & 2 & 2 \\ 2 & -1 & -1 & 2 & 2 & -1 & -1 & 2 \end{bmatrix}$



The figure is reflected over the *xz*-plane.

Page 547 Applications and Problem Solving

55. $\overrightarrow{AB} = \langle 1 \cos 120°, 0, 1 \sin 120° \rangle$ or $\left\langle -\frac{1}{2}, 0, \frac{\sqrt{3}}{2} \right\rangle$



$\vec{\mathbf{F}} = \langle 0, 0, -50 \rangle$

$\vec{\mathbf{T}} = \overrightarrow{AB} \times \vec{\mathbf{F}}$

$= \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ -\frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 0 & -50 \end{vmatrix}$

$= \begin{vmatrix} 0 & \frac{\sqrt{3}}{2} \\ 0 & -50 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -50 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} -\frac{1}{2} & 0 \\ 0 & 0 \end{vmatrix} \vec{\mathbf{k}}$

$= 0\vec{\mathbf{i}} - 25\vec{\mathbf{j}} + 0\vec{\mathbf{k}}$ or $\langle 0, -25, 0 \rangle$

$|\vec{\mathbf{T}}| = \sqrt{0^2 + (-25)^2 + 0^2}$

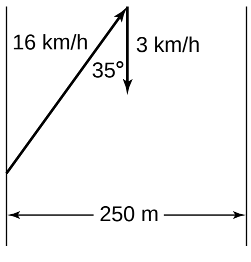
$= 25$ lb-ft

56. $y = t|\vec{\mathbf{v}}| \sin \theta - \frac{1}{2}gt^2 + h$

$= 0.5(38) \sin 40° - \frac{1}{2}(32)(0.5)^2 + 2$

≈ 10.2 ft

57a.



$\vec{\mathbf{b}} = (16 \cos 55°)\vec{\mathbf{c}} + (16 \sin 55°)\vec{\mathbf{j}}$

$\vec{\mathbf{c}} = -3\vec{\mathbf{j}}$

$|\vec{\mathbf{b}} + \vec{\mathbf{c}}| = \sqrt{(16 \cos 55°)^2 + (16 \sin 55° - 3)^2}$

≈ 13.7 km/h

57b. $\frac{u}{250} = \frac{16 \sin 55° - 3}{16 \cos 55°}$

$u = 250\left(\frac{16 \sin 55° - 3}{16 \cos 55°}\right)$

$u \approx 275.3$ m

58. $\vec{\mathbf{F}}_1 = 90\mathbf{i}$

$\vec{\mathbf{F}}_2 = (70 \cos 30°)\vec{\mathbf{i}} + (70 \sin 30°)\vec{\mathbf{j}}$ or $35\sqrt{3}\,\vec{\mathbf{i}} + 35\vec{\mathbf{j}}$

$|\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2| = \sqrt{(90 + 35\sqrt{3})^2 + 35^2}$

≈ 154.6 N

$\tan \theta = \frac{35}{90 + 35\sqrt{3}}$ or $\frac{7}{18 + 7\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{7}{18 + 7\sqrt{3}}\right)$

$\approx 13.1°$

Page 547 Open-Ended Assessment

1a. Sample answer: $X(4, -1)$, $Y(1, 1)$

$\overrightarrow{XY} = \langle 1 - 4, 1 - (-1) \rangle$ or $\langle -3, 2 \rangle$

1b. $|\overrightarrow{XY}| = \sqrt{(-3)^2 + 2^2}$ or $\sqrt{13}$

The magnitude of $\overrightarrow{XY}$ only depends on the differences of the coordinates of *X* and *Y*, not the actual coordinates.

2a. Sample answer: $P(1, 1)$, $Q(3, 3)$, $R(3, 1)$, $S(5, 3)$

$\overrightarrow{PQ} = \langle 3 - 1, 3 - 1 \rangle$ or $\langle 2, 2 \rangle$

$\overrightarrow{RS} = \langle 5 - 3, 3 - 1 \rangle$ or $\langle 2, 2 \rangle$

$\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are parallel because they have the same direction. In fact, they are the same vector.

2b. Sample answer: $\vec{\mathbf{a}} = \langle 8, -4 \rangle$, $\vec{\mathbf{b}} = \langle 3, 6 \rangle$

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 8(3) + (-4)6$ or 0

$\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are perpendicular because their inner product is 0.

Chapter 8 SAT & ACT Preparation

Page 549 SAT and ACT Practice

1. Recall that the formula for the area of a parallelogram is base times height. You know the base is 5, but you don't know the height. Don't be fooled by the segment *BD*; it is not the height of the parallelogram. Try another method to find the area. The parallelogram is made up of two triangles. Find the area of each triangle. Since *ABCD* is a parallelogram, *AB* = *DC* and *AD* = *BC*. The two triangles are both right triangles, and they share a common side, *BD*. By SAS, the two triangles are congruent. So you can find the area of one triangle and multiply by 2. The hypotenuse of the triangle is 5 and one side is 3. Use the Pythagorean Theorem to find the other side.

$5^2 = 3^2 + b^2$

$25 = 9 + b^2$

$16 = b^2$

$4 = b$

The height is 4.

Use the formula for the area of a triangle.

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(4)(3)$ or 6

Since the parallelogram consists of two triangles, the area of the parallelogram is 2 × 6 or 12. The correct choice is A.

2. In order to write the equation of a circle, you need to know the coordinates of the center and the length of the radius. The general equation for a circle is $(x - h)^2 + (y - k)^2 = r^2$, where the center is (h, k) and the radius is r. From the coordinates of points A and B, you know the length of the side is 4. So the center Q, has coordinates (0, 4).
To calculate the length of the radius, draw the radius OB. This creates a 45°-45°-90° right triangle. The two legs each have length 2. The hypotenuse has length $2\sqrt{2}$.

$$(x - 4)^2 + (y - 0)^2 = (2\sqrt{2})^2$$
$$(x - 4)^2 + y^2 = 4(2)$$
$$(x - 4)^2 + y^2 = 8$$

The correct choice is B.

3. Write the equation for the perimeter of a rectangle. then replace x with its value in terms of y. Solve the equation for y.

$$p = 2x + 2y$$
$$p = 2\left(\frac{2}{3}y\right) + 2y$$
$$p = \frac{4}{3}y + 2y$$
$$p = \frac{10}{3}y$$
$$\frac{3p}{10} = y$$

The correct choice is B.

4. Recall the triangle Inequality Theorem: the sum of the lengths of any two sides of a triangle is greater than the length of the third side. Let x represent the length of the third side.

$$40 + 80 > x$$
$$120 > x$$
$$40 + x > 80$$
$$x > 40$$

Since x must be greater than 40, x cannot be equal to 40. The correct choice is A. To check your answer, notice that the other answer choices are greater than 40 and less than 120, so they are all possible values for x.

5. Since the answer choices have fractional exponents of x, start by rewriting the expression with fractional exponents. Simplify the fractions and use the rules for exponents to combine terms.

$$\sqrt[3]{x^2} \cdot \sqrt[9]{x^3} = x^{\frac{2}{3}} \cdot x^{\frac{3}{9}}$$
$$= x^{\frac{2}{3}} \cdot x^{\frac{1}{3}}$$
$$= x^{\left(\frac{2}{3} + \frac{1}{3}\right)}$$
$$= x^1 \text{ or } x$$

The correct choice is E.

6. This figure looks more complex than it is. A *semi-circle* is just one half of a circle. Notice that the answer choices include π, so don't convert to decimals. Find the radius of each semi-circle. Calculate the area of each semi-circle.
The area of the shaded region is the area of the large semi-circle minus the area of the medium semi-circle plus the area of the small semi-circle.

Large semi-circle area $= \frac{1}{2}\pi\, 3^2 = \frac{9\pi}{2}$

Medium semi-circle area $= \frac{1}{2}\pi\, 2^2 = \frac{4\pi}{2}$

Small semi-circle area $= \frac{1}{2}\pi\, 1^2 = \frac{1\pi}{2}$

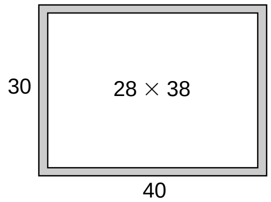
Shaded area $= \frac{9\pi}{2} - \frac{4\pi}{2} + \frac{1\pi}{2} = \frac{6\pi}{2} = 3\pi$

The correct choice is A.

7. The only values for which a rational function is undefined are values which make the denominator 0. Since $f(x) = \frac{x^2 - 3x + 2}{x - 1}$,
the denominator is only 0 when $x - 1 = 0$ or $x = 1$.

The correct choice is D.

8. Start by sketching a diagram of the counter



Use your calculator to find the area of the whole counter and then subtract the area of the white tiles in the center. The white tiles cover an area of $(30 - 2)(40 - 2)$ or (28)(38).

$(30)(40) = 1200$
$(28)(38) = 1064$
Red tiles $= 1200 - 1064 = 136$

The correct choice is B.

9. First, find the slope of the line containing the points (–2, 6) and (4, –3).

$$m = \frac{-3 - 6}{4 - -2}$$
$$m = \frac{-9}{6} \text{ or } -\frac{3}{2}$$

The point-slope form of the line is

$$y - 6 = -\frac{3}{2}(x - -2).$$
$$y - 6 = -\frac{3}{2}x - 3$$
$$y = -\frac{3}{2}x + 3$$

So the y-intercept of the line is 3.

The correct choice is B.

10. Write an expression for the sum of the areas of the two triangles. Recall the area of a triangle is one half the base times the height.

$\frac{1}{2}(AC)(AB) + \frac{1}{2}(CE)(ED)$

From the figure, you know that $\triangle ABC$ and $\triangle CDE$ are both isosceles, because of the angles marked $x°$ and because $\overline{BCD}$ is a line segment. These two triangles have equal corresponding angles.

Since they are isosceles triangles, $AC = AB$ and $CE = ED$. Use these equivalent lengths in the expressions for the area sum.

$$\frac{1}{2}(AC)(AB) + \frac{1}{2}(CE)(ED) = \frac{1}{2}(AC)^2 + \frac{1}{2}(CE)^2$$
$$= \frac{1}{2}[(AC)^2 + (CE)^2]$$

Using the Pythagorean Theorem for $\triangle ACE$, you know that $(AC)^2 + (CE)^2 = (AE)^2$ or 1.

So the sum of the two areas is $\frac{1}{2}(1) = \frac{1}{2}$. You can grid the answer either as .5 or as 1/2.