65. **(a)** $N(t) = 4 \cdot 2^t$

(b) 4 days: $4 \cdot 2^4 = 64$ guppies

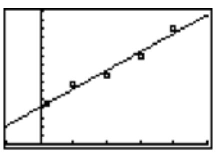
1 week: $4 \cdot 2^7 = 512$ guppies

(c)
$$N(t) = 2000$$
$$4 \cdot 2^t = 2000$$
$$2^t = 500$$
$$\ln 2^t = \ln 500$$
$$t \ln 2 = \ln 500$$

$t = \frac{\ln 500}{\ln 2} \approx 8.9658$ There will be 2000 guppies after 8.9658 days, or after nearly 9 days.

(d) Because it suggests the number of guppies will continue to double indefinitely and become arbitrarily large, which is impossible due to the finite size of the tank and the oxygen supply in the water.

66. **(a)** $y = 41.770x + 414.342$



[–5, 25] by [0, 1500]

(b) $y = 41.770(22) + 414.342 = 1333$
$1333 - 1432 = -99$

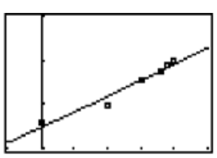
The estimate is 99 less than the actual number.

(c) $y = mx + b$

$m = 41.770$

The slope represents the approximate annual increase in the number of doctorates earned by Hispanic Americans per year.

67. **(a)** $y = (17467.361)(1.00398)^x =$



[–5, 25] by [17000, 20000]

(b) $(17467.361)\ (1.00398)^{23} = 19{,}138$ thousand or 19,138,000

$19{,}138{,}000 - 19{,}190{,}000 = -52{,}000$

The prediction is less than the actual by 52,000.

(c) $\frac{17{,}558}{(19{,}138)(23)} = 0.0398$ or 4%

68. **(a)** $m = 1$

(b) $y = -x - 1$

(c) $y = x + 3$

(d) 2

69. **(a)** $(2, \infty)$ $x - 2 > 0$

(b) $(-\infty, \infty)$ all real numbers

(c)
$$f(x) = 1 - ln(x-2)$$
$$0 = 1 - ln(x-2)$$
$$1 = ln(x-2)$$
$$e^1 = x - 2$$
$$x = e^1 + 2 \approx 4.718$$

(d)
$$y = 1 - ln(x-2)$$
$$x = 1 - ln(y-2)$$
$$x - 1 = -ln(y-2)$$
$$e^{1-x} = y - 2$$
$$y = e^{1-x} + 2$$

(e)
$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f(2 + e^{1-x})$$
$$= 1 - ln(2 + e^{1-x} - 2) = 1 - ln(e^{1-x})$$
$$= 1 - (1 - x) = x$$
$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(1 - ln(x-2))$$
$$= 2 + e^{1-(1-ln(x-2))} = 2 + e^{ln(x-2)}$$
$$= 2 + (x - 2) = x$$

70. **(a)** $(-\infty, \infty)$ all real numbers

(b) $[-2, 4]$ $1 - 3\cos(2x)$ oscillates between -2 and 4

(c) π

(d) Even. $\cos(-\theta) = \cos(\theta)$

(e) $x \approx 2.526$

Chapter 2
Limits and Continuity

Section 2.1 Rates of Change and Limits (pp. 59–69)

Quick Review 2.1

1. $f(2) = 2(2^3) - 5(2)^2 + 4 = 0$

2. $f(2) = \frac{4(2)^2 - 5}{2^3 + 4} = \frac{11}{12}$

3. $f(2) = \sin\left(\pi \cdot \frac{2}{2}\right) = \sin \pi = 0$

4. $f(2) = \frac{1}{2^2 - 1} = \frac{1}{3}$

5. $|x| < 4$
$-4 < x < 4$

6. $|x| < c^2$
$-c^2 < x < c^2$

7. $|x-2| < 3$
$-3 < x-2 < 3$
$-1 < x < 5$

8. $|x-c| < d^2$
$-d^2 < x-c < d^2$
$-d^2 + c < x < d^2 + c$

9. $\frac{x^2-3x-18}{x+3} = \frac{(x+3)(x-6)}{x+3} = x-6,\ x \neq -3$

10. $\frac{2x^2-x}{2x^2+x-1} = \frac{x(2x-1)}{(2x-1)(x+1)} = \frac{x}{x+1},\ x \neq \frac{1}{2}$

Section 2.1 Exercises

1. $\frac{\Delta y}{\Delta t} = \frac{16(3)^2-16(0)^2}{3-0} = 48 \text{ ft/sec}$

2. $\frac{\Delta y}{\Delta t} = \frac{16(4)^2-16(0)^2}{4-0} = 64 \text{ ft/sec}$

3. $\frac{\Delta y}{\Delta t} = \frac{16(3+h)^2-16(3)^2}{h}$, say $h = 0.01$

$$= \frac{16(3+0.01)^2-16(9)}{0.01} = \frac{16(9.0601)-16(9)}{0.01}$$
$$= \frac{144.9616-144}{0.01} = \frac{0.9616}{0.01} = 96.16 \text{ ft/sec}$$

Confirm Algebraically

$$\frac{\Delta y}{\Delta t} = \frac{16(3+h)^2-16(3)^2}{h}$$
$$= \frac{16(9+6h+h^2)-144}{h} = \frac{96h+16h^2}{h} = (96+16h) \text{ ft/sec}$$

if $h = 0$, then $\frac{\Delta y}{\Delta t} = 96 \frac{\text{ft}}{\text{sec}}$

4. $\frac{\Delta y}{\Delta t} = \frac{16(4+h)^2-16(4)^2}{h}$
say $h = 0.01$

$$\frac{16(4+0.01)^2-16(4)^2}{0.01}$$
$$= \frac{16(16.0801)-16(16)}{0.01}$$
$$= \frac{257.2816-256}{0.01}$$
$$= \frac{1.2816}{0.01} = 128.16 \frac{\text{ft}}{\text{sec}}$$

Confirm Algebraically

$$\frac{\Delta y}{\Delta t} = \frac{16(4+h)^2-16(4)^2}{h}$$
$$= \frac{16(16+8h+h^2)-256}{h}$$
$$= \frac{128h+16h^2}{h}$$
$$= (128+16h)\frac{\text{ft}}{\text{sec}}$$

if $h = 0$, then $\frac{\Delta y}{\Delta t} = 128 \frac{\text{ft}}{\text{sec}}$

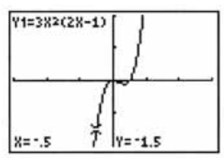
5. $\lim_{x \to c} (2x^3 - 3x^2 + x - 1)$
$= 2c^3 - 3c^2 + c - 1$

6. $\lim_{x \to c} \frac{x^4 - x^3 + 1}{x^2 + 9}$

$$= \frac{c^4 - c^3 + 1}{c^2 + 9}$$

7. $\lim_{x \to -1/2} 3x^2(2x-1) = 3\left(-\frac{1}{2}\right)^2\left[2\left(-\frac{1}{2}\right)-1\right] = 3\left(\frac{1}{4}\right)(-2)$
$= -\frac{3}{2}$

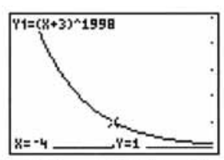
Graphical support:



[−3, 3] by [−2, 2]

8. $\lim_{x \to -4} (x+3)^{1998} = (-4+3)^{1998} = (-1)^{1998} = 1$

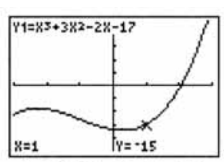
Graphical support:



[−4.001, −3.999] by [0, 5]

9. $\lim_{x \to 1} (x^3 + 3x^2 - 2x - 17) = (1)^3 + 3(1)^2 - 2(1) - 17$
$= 1 + 3 - 2 - 17 = -15$

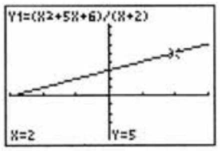
Graphical support:



[−3, 3] by [−25, 25]

10. $\lim_{y\to 2} \frac{y^2+5y+6}{y+2} = \frac{2^2+5(2)+6}{2+2} = \frac{20}{4} = 5$

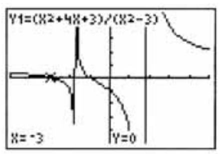
Graphical support:



[−3, 3] by [−5, 10]

11. $\lim_{y\to -3} \frac{y^2+4y+3}{y^2-3} = \frac{(-3)^2+4(-3)+3}{(-3)^2-3} = \frac{0}{6} = 0$

Graphical support:

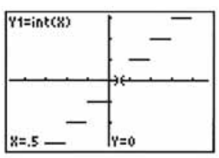


[−5, 5] by [−5, 5]

12. $\lim_{x\to 1/2} \text{int}\, x = \text{int}\,\frac{1}{2} = 0$

Note that substitution cannot always be used to find limits of the int function. Its use here can be justified by the Sandwich Theorem, using $g(x) = h(x) = 0$ on the interval (0, 1).

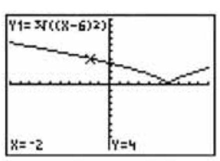
Graphical support:



[−4.7, 4.7] by [−3.1, 3.1]

13. $\lim_{x\to -2} (x-6)^{2/3} = (-2-6)^{2/3} = \sqrt[3]{(-8)^2} = \sqrt[3]{64} = 4$

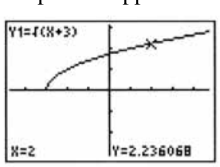
Graphical support:



[−10, 10] by [−10, 10]

14. $\lim_{x\to 2} \sqrt{x+3} = \sqrt{2+3} = \sqrt{5}$

Graphical support:



[−4.7, 4.7] by [−3.1, 3.1]

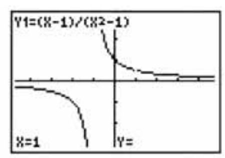
15. You cannot use substitution because the expression $\sqrt{x-2}$ is not defined at $x = -2$. Since the expression is not defined at points near $x = -2$, the limit does not exist.

16. You cannot use substitution because the expression $\frac{1}{x^2}$ is not defined at $x = 0$. Since $\frac{1}{x^2}$ becomes arbitrarily large as x approaches 0 from either side, there is no (finite) limit. (As we shall see in Section 2.2, we may write $\lim_{x\to 0} \frac{1}{x^2} = \infty$.)

17. You cannot use substitution because the expression $\frac{|x|}{x}$ is not defined at $x = 0$. Since $\lim_{x\to 0^-} \frac{|x|}{x} = -1$ and $\lim_{x\to 0^+} \frac{|x|}{x} = 1$, the left- and right-hand limits are not equal and so the limit does not exist.

18. You cannot use substitution because the expression $\frac{(4+x)^2-16}{x}$ is not defined at $x = 0$. Since $\frac{(4+x)^2-16}{x} = \frac{8x+x^2}{x} = 8+x$ for all $x \neq 0$, the limit exists and is equal to $\lim_{x\to 0} (8+x) = 8+0 = 8$.

19.



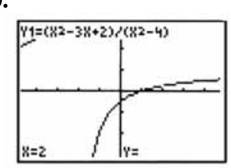
[−4.7, 4.7] by [−3.1, 3.1]

$\lim_{x\to 1} \frac{x-1}{x^2-1} = \frac{1}{2}$

Algebraic confirmation:

$$\lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{x-1}{(x+1)\,(x-1)} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{1+1} = \frac{1}{2}$$

20.



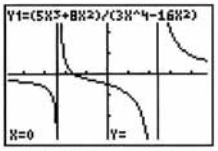
[−4.7, 4.7] by [−3.1, 3.1]

$\lim_{t\to 2} \frac{t^2-3t+2}{t^2-4} = \frac{1}{4}$

Algebraic confirmation:

$$\lim_{t\to 2} \frac{t^2-3t+2}{r^2-4} = \lim_{t\to 2} \frac{(t-1)(t-2)}{} = \lim_{t\to 2} \frac{t-1}{t+2} = \frac{2-1}{2+2} = \frac{1}{4}$$

21.



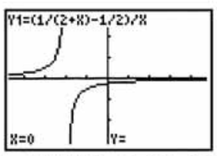
[−4.7, 4.7] by [−3.1, 3.1]

$$\lim_{x\to 0}\frac{5x^3+8x^2}{3x^4-16x^2}=-\frac{1}{2}$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{5x^3+8x^2}{3x^4-16x^2}=\lim_{x\to 0}\frac{x^2(5x+8)}{x^2(3x^2-16)}$$
$$=\lim_{x\to 0}\frac{5x+8}{3x^2-16}$$
$$=\frac{5(0)+8}{3(0)^2-16}$$
$$=\frac{8}{-16}=-\frac{1}{2}$$

22.



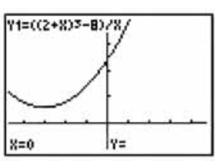
[−4.7, 4.7] by [−3.1, 3.1]

$$\lim_{x\to 0}\frac{\frac{1}{2+x}-\frac{1}{2}}{x}=-\frac{1}{4}$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{\frac{1}{2+x}-\frac{1}{2}}{x}=\lim_{x\to 0}\frac{2-(2+x)}{}$$
$$=\lim_{x\to 0}\frac{-x}{x(2+x)\,(2)}$$
$$=\lim_{x\to 0}\frac{-1}{2(2+x)}$$
$$=\frac{-1}{2(2+0)}=-\frac{1}{4}$$

23.



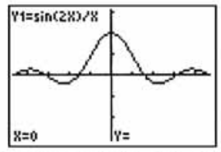
[−4.7, 4.7] by [−5, 20]

$$\lim_{x\to 0}\frac{(2+x)^3-8}{x}=12$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{(2+x)^3-8}{x}=\lim_{x\to 0}\frac{12x+6x^2+x^3}{x}$$
$$=\lim_{x\to 0}(12+6x+x^2)$$
$$=12+6(0)+(0)^2=12$$

24.



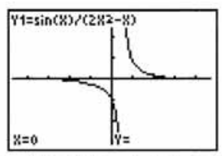
[−4.7, 4.7] by [−3.1, 3.1]

$$\lim_{x\to 0}\frac{\sin 2x}{x}=2$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{\sin 2x}{x}=2\lim_{x\to 0}\frac{\sin 2x}{2x}=2(1)=2$$

25.



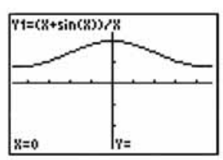
[−4.7, 4.7] by [−3.1, 3.1]

$$\lim_{x\to 0}\frac{\sin x}{2x^2-x}=-1$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{\sin x}{2x^2-x}=\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{2x-1}\right)$$
$$=\left(\lim_{x\to 0}\frac{\sin x}{x}\right)\left(\lim_{x\to 0}\frac{1}{2x-1}\right)=(1)\left(\frac{1}{2(0)-1}\right)=-1$$

26.



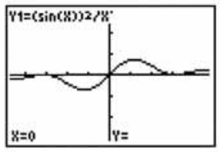
[−4.7, 4.7] by [−3.1, 3.1]

$$\lim_{x\to 0}\frac{x+\sin x}{x}=2$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{x+\sin x}{x}=\lim_{x\to 0}\left(1+\frac{\sin x}{x}\right)$$
$$=\left(\lim_{x\to 0}1\right)+\left(\lim_{x\to 0}\frac{\sin x}{x}\right)$$
$$=1+1=2$$

27.



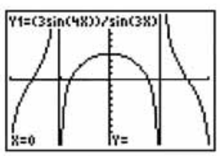
[−4.7, 4.7] by [−3.1, 3.1]

$$\lim_{x\to 0}\frac{\sin^2 x}{x}=0$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{\sin^2 x}{x}=\lim_{x\to 0}\left(\sin x\cdot\frac{\sin x}{x}\right)$$
$$=\left(\lim_{x\to 0}\sin x\right)\cdot\left(\lim_{x\to 0}\frac{\sin x}{x}\right)$$
$$=(\sin 0)(1)=0$$

28.



[−2, 2] by [−10, 10]

$$\lim_{x\to 0}\frac{3\sin 4x}{\sin 3x}=4$$

Algebraic confirmation:

$$\lim_{x\to 0}\frac{3\sin 4x}{\sin 3x}=4\lim_{x\to 0}\left(\frac{\sin 4x}{4x}\cdot\frac{3x}{\sin 3x}\right)$$
$$=4\left(\lim_{x\to 0}\frac{\sin 4x}{4x}\right)\div\left(\lim_{x\to 0}\frac{\sin 3x}{3x}\right)$$
$$=4(1)\div(1)=4$$

29. Answers will vary. One possible graph is given by the window

[–4.7, 4.7] by [–15, 15] with *Xscl* = 1 and *Yscl* = 5.

30. Answers will vary. One possible graph is given by the window

[–4.7, 4.7] by [–15, 15] with *Xscl* = 1 and *Yscl* = 5.

31. Since int $x = 0$ for x in (0, 1), $\lim_{x\to 0^+}\text{int}\,x=0$.

32. Since int $x = -1$ for x in (–1, 0), $\lim_{x\to 0^-}\text{int}\,x=-1$.

33. Since int $x = 0$ for x in (0, 1), $\lim_{x\to 0.01}\text{int}\,x=0$.

34. Since int $x = 1$ for x in (1, 2), $\lim_{x\to 2^-}\text{int}\,x=1$.

35. Since $\frac{x}{|x|}=1$ for $x>0$, $\lim_{x\to 0^+}\frac{x}{|x|}=1$.

36. Since $\frac{x}{|x|}=-1$ for $x<0$, $\lim_{x\to 0^-}\frac{x}{|x|}=-1$.

37. **(a)** True

(b) True

(c) False, since $\lim_{x\to 0^-}f(x)=0$.

(d) True, since both are equal to 0.

(e) True, since (d) is true.

(f) True

(g) False, since $\lim_{x\to 0}f(x)=0$.

(h) False, $\lim_{x\to 1^-}f(x)=1$, but $\lim_{x\to 1}f(x)$ is undefined.

(i) False, $\lim_{x\to 1^+}f(x)=0$, but $\lim_{x\to 1}f(x)$ is undefined.

(j) False, since $\lim_{x\to 2^-}f(x)=0$.

38. **(a)** True

(b) False, since $\lim_{x\to 2}f(x)=1$.

(c) False, since $\lim_{x\to 2}f(x)=1$.

(d) True

(e) True

(f) True, since $\lim_{x\to 1^-}f(x)\neq\lim_{x\to 1^+}f(x)$.

(g) True, since both are equal to 0.

(h) True

(i) True, since $\lim_{x\to c}f(x)=1$ for all c in (1, 3).

39. **(a)** $\lim_{x\to 3^-}f(x)=3$

(b) $\lim_{x\to 3^+}f(x)=-2$

(c) $\lim_{x\to 3}f(x)$ does not exist, because the left- and right-hand limits are not equal.

(d) $f(3)=1$

40. **(a)** $\lim_{t\to -4^-}g(t)=5$

(b) $\lim_{t\to -4^+}g(t)=2$

(c) $\lim_{t\to -4}g(t)$ does not exist, because the left- and right-hand limits are not equal.

(d) $g(-4)=2$

41. (a) $\lim_{h\to 0^-} f(h) = -4$

(b) $\lim_{h\to 0^+} f(h) = -4$

(c) $\lim_{h\to 0} f(h) = -4$

(d) $f(0) = -4$

42. (a) $\lim_{s\to -2^-} p(s) = 3$

(b) $\lim_{s\to -2^+} p(s) = 3$

(c) $\lim_{s\to -2} p(s) = 3$

(d) $p(-2) = 3$

43. (a) $\lim_{x\to 0^-} F(x) = 4$

(b) $\lim_{x\to 0^+} F(x) = -3$

(c) $\lim_{x\to 0} F(x)$ does not exist, because the left- and right-hand limits are not equal.

(d) $F(0) = 4$

44. (a) $\lim_{x\to 2^-} G(x) = 1$

(b) $\lim_{x\to 2^+} G(x) = 1$

(c) $\lim_{x\to 2} G(x) = 1$

(d) $G(2) = 3$

45. $y_1 = \dfrac{x^2+x-2}{x-1} = \dfrac{(x-1)(x+2)}{x-1} = x+2,\ x \neq 1$

(c)

46. $y_1 = \dfrac{x^2-x-2}{x-1} = \dfrac{(x+1)(x-2)}{x-1}$

(b)

47. $y_1 = \dfrac{x^2-2x+1}{x-1} = \dfrac{(x-1)^2}{x-1} = x-1,\ x \neq 1$

(d)

48. $y_1 = \dfrac{x^2+x-2}{x+1} = \dfrac{(x-1)(x+2)}{x+1}$

(a)

49. (a) $\lim_{x\to 4} (g(x)+3) = \left(\lim_{x\to 4} g(x)\right) + \left(\lim_{x\to 4} 3\right) = 3+3 = 6$

(b) $\lim_{x\to 4} x f(x) = \left(\lim_{x\to 4} x\right)\left(\lim_{x\to 4} f(x)\right) = 4\cdot 0 = 0$

(c) $\lim_{x\to 4} g^2(x) = \left(\lim_{x\to 4} g(x)\right)^2 = 3^2 = 9$

(d) $\lim_{x\to 4} \dfrac{g(x)}{f(x)-1} = \dfrac{\lim_{x\to 4} g(x)}{\left(\lim_{x\to 4} f(x)\right) - \left(\lim_{x\to 4} 1\right)} = \dfrac{3}{0-1} = -3$

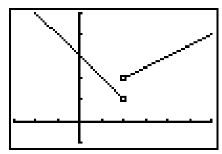
50. (a) $\lim_{x\to b} (f(x)+g(x)) = \left(\lim_{x\to b} f(x)\right) + \left(\lim_{x\to b} g(x)\right)$
$= 7 + (-3) = 4$

(b) $\lim_{x\to b} (f(x)\cdot g(x)) = \left(\lim_{x\to b} f(x)\right)\left(\lim_{x\to b} g(x)\right)$
$= (7)\,(-3) = -21$

(c) $\lim_{x\to b} 4g(x) = 4\lim_{x\to b} g(x) = 4(-3) = -12$

(d) $\lim_{x\to b} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to b} f(x)}{\lim_{x\to b} g(x)} = \dfrac{7}{-3} = -\dfrac{7}{3}$

51. (a)

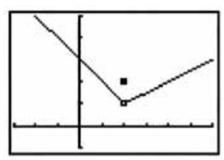


[–3, 6] by [–1, 5]

(b) $\lim_{x\to 2^+} f(x) = 2;\ \lim_{x\to 2^-} f(x) = 1$

(c) No, because the two one-sided limits are different.

52. (a)

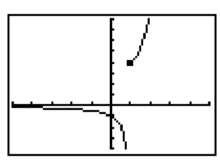


[–3, 6] by [–1, 5]

(b) $\lim_{x\to 2^+} f(x) = 1;\ \lim_{x\to 2^-} f(x) = 1$

(c) Yes. The limit is 1.

53. (a)

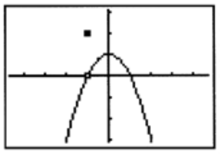


[–5, 5] by [–4, 8]

(b) $\lim_{x\to 1^+} f(x) = 4;\ \lim_{x\to 1^-} f(x)$ does not exist.

(c) No, because the left-hand limits does not exist.

54. (a)

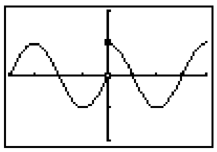


[–4.7, 4.7] by [–3.1, 3.1]

(b) $\lim_{x \to -1^+} f(x) = 0; \lim_{x \to -1^-} f(x) = 0$

(c) Yes. The limit is 0.

55. (a)



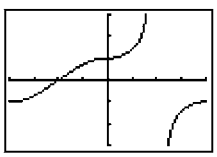
[–2π, 2π] by [–2, 2]

(b) $(-2\pi,\ 0) \cup (0,\ 2\pi)$

(c) $c = 2\pi$

(d) $c = -2\pi$

56. (a)

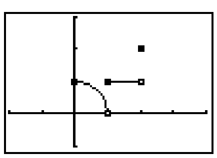


[–π, π] by [–3, 3]

(b) $\left(-\pi, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

(c) $c = \pi$ **(d)** $c = -\pi$

57. (a)

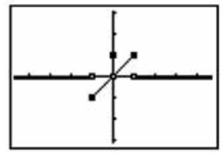


[–2, 4] by [–1, 3]

(b) (0, 1) ∪ (1, 2)

(c) $c = 2$ **(d)** $c = 0$

58. (a)

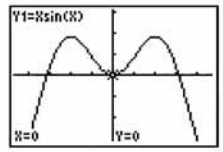


[–4.7, 4.7] by [–3.1, 3.1]

(b) $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(c) None **(d)** None

59.



[−4.7, 4.7] by [−3.1, 3.1]

$\lim_{x \to 0} (x \sin x) = 0$

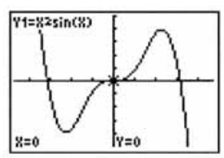
Confirm using the Sandwich Theorem, with $g(x) = -|x|$ and $h(x) = |x|$.

$|x \sin x| = |x| \cdot |\sin x| \le |x| \cdot 1 = |x|$

$-|x| \le x \sin x \le |x|$

Because $\lim_{x \to 0} (-|x|) = \lim_{x \to 0} |x| = 0,$ the Sandwich Theorem gives $\lim_{x \to 0} (x \sin x) = 0.$

60.



[−4.7, 4.7] by [−5, 5]

$\lim_{x \to 0} (x^2 \sin x) = 0$

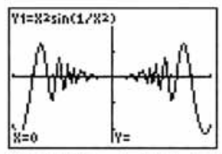
Confirm using the Sandwich Theorem, with $g(x) = -x^2$ and $h(x) = x^2$.

$|x^2 \sin x| = |x^2| \cdot |\sin x| \le |x^2| \cdot 1 = x^2.$

$-x^2 \le x^2 \sin x \le x^2$

Because $\lim_{x \to 0} (-x^2) = \lim_{x \to 0} x^2 = 0,$ the Sandwich Theorem gives $\lim_{x \to 0} (x^2 \sin x) = 0$

61.



[−0.5, 0.5] by [−0.25, 0.25]

$$\lim_{x\to 0}\left(x^2 \sin\frac{1}{x^2}\right)=0$$

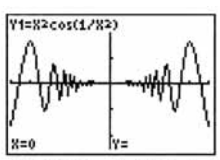
Confirm using the Sandwich Theorem, with $g(x) = -x^2$ and $h(x) = x^2$.

$$\left|x^2 \sin\frac{1}{x^2}\right| = \left|x^2\right|\cdot\left|\sin\frac{1}{x^2}\right| \le \left|x^2\right|\cdot 1 = x^2.$$

$$-x^2 \le x^2 \sin\frac{1}{x^2} \le x^2$$

Because $\lim_{x\to 0}(-x^2)=\lim_{x\to 0}x^2=0$, the Sandwich Theorem give $\lim_{x\to 0}\left(x^2\sin\frac{1}{x^2}\right)=0.$

62.



[−0.5, 0.5] by [−0.25, 0.25]

$$\lim_{x\to 0}\left(x^2 \cos\frac{1}{x^2}\right)=0$$

Confirm using the Sandwich Theorem, with $g(x) = -x^2$ and $h(x) = x^2$.

$$\left|x^2 \cos\frac{1}{x^2}\right| = \left|x^2\right|\cdot\left|\cos\frac{1}{x^2}\right| \le \left|x^2\right|\cdot 1 = x^2.$$

$$-x^2 \le x^2 \cos\frac{1}{x^2} \le x^2$$

Because $\lim_{x\to 0}(-x^2)=\lim_{x\to 0}x^2=0$, the Sandwich Theorem give $\lim_{x\to 0}\left(x^2\cos\frac{1}{x^2}\right)=0.$

63. (a) In three seconds, the ball falls $4.9\,(3)^2 = 44.1$ m, so its average speed is $\frac{44.1}{3}=14.7$ m/sec.

(b) The average speed over the interval from time $t = 3$ to time $3 + h$ is

$$\frac{\Delta y}{\Delta t}=\frac{4.9(3+h)^2-4.9(3)^2}{(3+h)-3}=\frac{4.9(6h+h^2)}{h}$$
$$=29.4+4.9h$$

Since $\lim_{h\to 0}(29.4+4.9h)=29.4,$ the instantaneous speed is 29.4 m/sec.

64. (a) $y = gt^2$

$20 = g(4^2)$

$g=\frac{20}{16}=\frac{5}{4}$ or 1.25

(b) Average speed $=\frac{20}{4}=5$ m/sec

(c) If the rock had not been stopped, its average speed over the interval from time $t = 4$ to time $t = 4 + h$ is

$$\frac{\Delta y}{\Delta t}=\frac{1.25(4+h)^2-1.25(4)^2}{(4+h)-4}=\frac{1.25(8h+h^2)}{h}$$
$$=10+1.25h$$

Since $\lim_{h\to 0}(10+1.25h)=10,$ the instantaneous speed is 10 m/sec.

65. True. The definition of a limit.

66. True

$$\lim_{x\to 0}\left(\frac{x+\sin x}{x}\right)=\lim_{x\to 0}\left(1+\frac{\sin x}{x}\right)=1+\lim_{x\to 0}\frac{\sin x}{x}=2$$

$\sin x \approx x$ as $x\to 0$.

67. C.

68. B.

69. E.

70. C.

71. (a)

x	–0.1	–0.01	–0.001	–0.0001
$f(x)$	–0.054402	–0.005064	–0.000827	–0.000031

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	–0.054402	–0.005064	–0.000827	–0.000031

The limit appears to be 0.

72. (a)

x	–0.1	–0.01	–0.001	–0.0001
$f(x)$	0.5440	0.5064	–0.8269	0.3056

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	–0.5440	–0.5064	0.8269	–0.3056

There is no clear indication of a limit.

73. (a)

x	–0.1	–0.01	–0.001	–0.0001
$f(x)$	2.0567	2.2763	2.2999	2.3023

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	2.5893	2.3293	2.3052	2.3029

The limit appears to be approximately 2.3.

74. (a)

x	–0.1	–0.01	–0.001	–0.0001
$f(x)$	0.074398	–0.009943	0.000585	0.000021

(b)

x	0.1	0.01	0.001	0.0001
$f(x)$	–0.074398	0.009943	–0.000585	–0.000021

The limit appears to be 0.

75. (a) Because the right-hand limit at zero depends only on the values of the function for positive x-values near zero.

(b) Area of $\Delta OAP = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(1)(\sin\theta) = \frac{\sin\theta}{2}$

Area of sector $OAP = \frac{(\text{angle})(\text{radius})^2}{2} = \frac{\theta(1)^2}{2} = \frac{\theta}{2}$

Area of $\Delta OAT = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(1)(\tan\theta) = \frac{\tan\theta}{2}$

(c) This is how the areas of the three regions compare.

(d) Multiply by 2 and divide by sin θ.

(e) Take reciprocals, remembering that all of the values involved are positive.

(f) The limits for cos θ and 1 are both equal to 1. Since $\frac{\sin\theta}{\theta}$ is between them, it must also have a limit of 1.

(g) $\frac{\sin(-\theta)}{-\theta} = \frac{-\sin\theta}{-\theta} = \frac{\sin\theta}{\theta}$

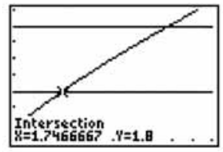
(h) If the function is symmetric about the y-axis, and the right-hand limit at zero is 1, then the left-hand limit at zero must also be 1.

(i) The two one-sided limits both exist and are equal to 1.

76. (a) The limit can be found by substitution.

$$\lim_{x\to 2} f(x) = f(2) = \sqrt{3(2)-2} = \sqrt{4} = 2$$

(b) The graphs of $y_1 = f(x)$, $y_2 = 1.8$, and $y_3 = 2.2$ are shown.

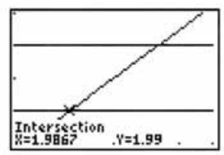


[1.5, 2.5] by [1.5, 2.3]

The intersections of y_1 with y_2 and y_3 are at $x \approx 1.7467$ and $x = 2.28$, respectively, so we may choose any value of a in [1.7467, 2) (approximately) and any value of b in (2, 2.28].

One possible answer: $a = 1.75$, $b = 2.28$.

(c) The graphs of $y_1 = f(x)$, $y_2 = 1.99$, and $y_3 = 2.01$ are shown.



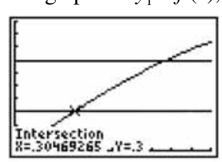
[1.97, 2.03] by [1.98, 2.02]

The intersections of y_1 with y_2 and y_3 are at $x = 1.9867$ and $x \approx 2.0134$, respectively, so we may choose any value of a in [1.9867, 2), and any value of b in (2, 2.0134] (approximately).

One possible answer: $a = 1.99$, $b = 2.01$.

77. (a) $f\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

(b) The graphs of $y_1 = f(x)$, $y_2 = 0.3$, and $y_3 = 0.7$ are shown.



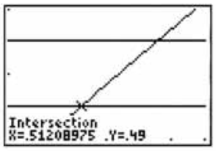
[0, 1] by [0, 1]

The intersections of y_1 with y_2 and y_3 are at $x \approx 0.3047$ and $x \approx 0.7754$, respectively, so we may choose any value of a in $\left[0.3047, \frac{\pi}{6}\right)$, and any value of b in $\left(\frac{\pi}{6}, 0.7754\right]$, where the interval endpoints are approximate.

One possible answer: $a = 0.305$, $b = 0.775$.

77. Continued

(c) The graphs of $y_1 = f(x)$, $y_2 = 0.49$, and $y_3 = 0.51$ are shown.



[0.49, 0.55] by [0.48, 0.52]

The intersections of y_1 with y_2 and y_3 are at $x \approx 0.5121$ and $x \approx 0.5352$, respectively, so we may choose any value of a in $\left[0.5121, \frac{\pi}{6}\right)$, and any value of b in $\left(\frac{\pi}{6}, 0.5352\right]$, where the interval endpoints are approximate.

One possible answer: $a = 0.513$, $b = 0.535$.

78. Line segment OP has endpoints (0, 0) and $(a,\ a^2)$, so its midpoint is $\left(\frac{0+a}{2}, \frac{0+a^2}{2}\right) = \left(\frac{a}{2}, \frac{a^2}{2}\right)$ and its slope is $\frac{a^2-0}{a-0} = a$. The perpendicular bisector is the line through $\left(\frac{a}{2}, \frac{a^2}{2}\right)$ with slope $-\frac{1}{a}$, so its equation is $y = -\frac{1}{a}\left(x - \frac{a}{2}\right) + \frac{a^2}{2}$, which is equivalent to $y = -\frac{1}{a}x + \frac{1+a^2}{2}$. Thus the y-intercept is $b = \frac{1+a^2}{2}$. As the point P approaches the origin along the parabola, the value of a approaches zero. Therefore,

$$\lim_{P\to 0} b = \lim_{a\to 0} \frac{1+a^2}{2} = \frac{1+0^2}{2} = \frac{1}{2}.$$

Section 2.2 Limits Involving Infinity (pp. 70–77)

Exploration 1 Exploring Theorem 5

1. Neither $\lim_{x\to\infty} f(x)$ or $\lim_{x\to\infty} g(x)$ exist. In this case, we can describe the behavior of f and g as $x \to \infty$ by writing $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} g(x) = \infty$. We cannot apply the quotient rule because both limits must exist. However, from Example 5,

$$\lim_{x\to\infty} \frac{5x + \sin x}{x} = \lim_{x\to\infty}\left(5 + \frac{\sin x}{x}\right) = 5 + 0 = 5,$$

so the limit of the quotient exists.

2. Both f and g oscillate between 0 and 1 as $x \to \infty$, taking on each value infinitely often. We cannot apply the sum rule because neither limit exists. However,

$$\lim_{x\to\infty} (\sin^2 x + \cos^2 x) = \lim_{x\to\infty} (1) = 1,$$

so the limit of the sum exists.

3. The limit of f and g as $x \to \infty$ do not exist, so we cannot apply the difference rule to $f - g$. We can say that $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} g(x) = \infty$. We can write the difference as $f(x) - g(x) = \ln(2x) - \ln(x+1) = \ln\frac{2x}{x+1}$. We can use graphs or tables to convince ourselves that this limit is equal to ln 2.

4. The fact that the limits of f and g as $x \to \infty$ do not exist does not necessarily mean that the limits of $f + g$, $f - g$ or $\frac{f}{g}$ do not exist, just that Theorem 5 cannot be applied.

Quick Review 2.2

1. $y = 2x - 3$

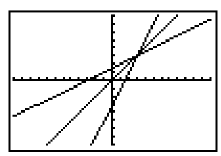
$y + 3 = 2x$

$\frac{y+3}{2} = x$

Interchange x and y.

$\frac{x+3}{2} = y$

$f^{-1}(x) = \frac{x+3}{2}$



[–12, 12] by [–8, 8]

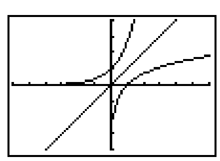
2. $y = e^x$

$\ln y = x$

Interchange x and y.

$\ln x = y$

$f^{-1}(x) = \ln x$



[–6, 6] by [–4, 4]

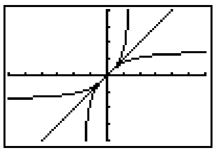
3. $y = \tan^{-1} x$

$\tan y = x, \; -\frac{\pi}{2} < y < \frac{\pi}{2}$

Interchange x and y.

$\tan x = y, -\frac{\pi}{2} < x < \frac{\pi}{2}$

$f^{-1}(x) = \tan x, -\frac{\pi}{2} < x < \frac{\pi}{2}$



[–6, 6] by [–4, 4]

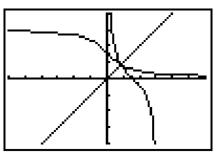
4. $y = \cot^{-1} x$

$\cot y = x, 0 < x < \pi$

Interchange x and y.

$\cot x = y, 0 < y < \pi$

$f^{-1}(x) = \cot x, 0 < x < \pi$



[–6, 6] by [–4, 4]

5.

$$\begin{array}{r l}
 & \frac{2}{3} \\
3x^3+4x-5 \,\big)\! & 2x^3-3x^2+x\;\;-1 \\
 & 2x^3+0x^2+\frac{8}{3}x-\frac{10}{3} \\
\hline
 & -3x^2-\frac{5}{3}x+\frac{7}{3}
\end{array}$$

$q(x) = \frac{2}{3}$

$r(x) = -3x^2 - \frac{5}{3}x + \frac{7}{3}$

6.

$$\begin{array}{r l}
 & 2x^2+2x+1 \\
x^3-x^2+1 \,\big)\! & 2x^5+\;0x^4-\;\;x^3+0x^2+\;x-1 \\
 & 2x^5-\;2x^4+\;0x^3+2x^2 \\
\hline
 & \qquad 2x^4-\;x^3-2x^2+\;\;x-1 \\
 & \qquad 2x^4-2x^3+0x^2+2x \\
\hline
 & \qquad\qquad x^3-2x^2-\;\;x-1 \\
 & \qquad\qquad x^3-\;x^2+0x+1 \\
\hline
 & \qquad\qquad\quad -x^2-\;\;x-2
\end{array}$$

$q(x) = 2x^2 + 2x + 1$

$r(x) = -x^2 - x - 2$

7. **(a)** $f(-x) = \cos(-x) = \cos x$

(b) $f\left(\frac{1}{x}\right) = \cos\left(\frac{1}{x}\right)$

8. **(a)** $f(-x) = e^{-(-x)} = e^{x}$

(b) $f\left(\frac{1}{x}\right) = e^{-1/x}$

9. **(a)** $f(-x) = \frac{\ln(-x)}{-x} = -\frac{\ln(-x)}{x}$

(b) $f\left(\frac{1}{x}\right) = \frac{\ln 1/x}{1/x} = x\ln x^{-1} = -x\ln x$

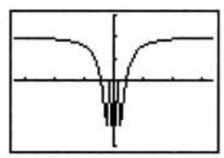
10. **(a)** $f(-x) = \left(-x + \frac{1}{-x}\right)\sin(-x) = -\left(x + \frac{1}{x}\right)(-\sin x)$

$= \left(x + \frac{1}{x}\right)\sin x$

(b) $f\left(\frac{1}{x}\right) = \left(\frac{1}{x} + \frac{1}{1/x}\right)\sin\left(\frac{1}{x}\right) = \left(\frac{1}{x} + x\right)\sin\left(\frac{1}{x}\right)$

Section 2.2 Exercises

1.



[−5, 5] by [−1.5, 1.5]

X	Y1
100	.99995
200	.99999
300	.99999
400	1
-100	.99995
-200	.99999
-300	.99999

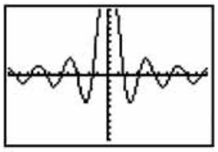
Y1 = cos(1/X)

(a) $\lim_{x\to\infty} f(x) = 1$

(b) $\lim_{x\to-\infty} f(x) = 1$

(c) $y = 1$

2.



[−10, 10] by [−1, 1]

X	Y1
100	-.0087
200	-.0043
500	.00165
1000	9.3E-4
-100	-.0087
-200	-.0043
-500	.00165

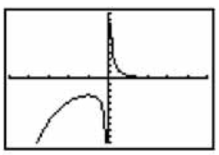
Y1=sin(2X)/X

(a) $\lim_{x\to\infty} f(x) = 0$

(b) $\lim_{x\to-\infty} f(x) = 0$

(c) $y = 0$

3.



[−5, 5] by [−10, 10]

X	Y1
100	4E-46
200	7E-90
300	0
400	0
-100	-3E41
-200	-4E84
-300	ERROR

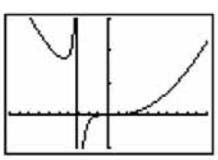
Y1=e^(-X)/X

(a) $\lim_{x\to\infty} f(x) = 0$

(b) $\lim_{x\to-\infty} f(x) = -\infty$

(c) $y = 0$

4.



[−10, 10] by [−100, 300]

X	Y1
100	29125
200	118226
300	267326
400	476426
-100	30927
-200	121826
-300	272726

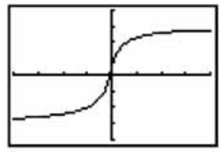
Y1=(3X³-X+1)/(X...

(a) $\lim_{x\to\infty} f(x) = \infty$

(b) $\lim_{x\to-\infty} f(x) = \infty$

(c) No horizontal asymptotes.

5.



[−20, 20] by [−4, 4]

X	Y1
100	2.951
200	2.9752
300	2.9834
400	2.9876
-100	-2.931
-200	-2.965
-300	-2.977

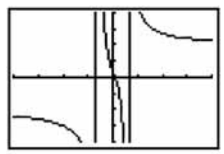
Y1=(3X+1)/(abs(...

(a) $\lim_{x\to\infty} f(x) = 3$

(b) $\lim_{x\to-\infty} f(x) = -3$

(c) $y = 3$, $y = -3$

6.



[−20, 20] by [−4, 4]

X	Y1
100	2.0515
200	2.0254
500	2.0101
1000	2.005
-100	-2.072
-200	-2.036
-500	-2.014

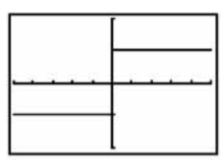
Y1=(2X-1)/(abs(...

(a) $\lim_{x\to\infty} f(x) = 2$

(b) $\lim_{x\to-\infty} f(x) = -2$

(c) $y = 2$, $y = -2$

7.



[−5, 5] by [−2, 2]

X	Y1
100	1
200	1
500	1
1000	1
-100	-1
-200	-1
-500	-1

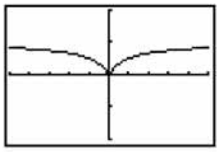
Y1=X/abs(X)

(a) $\lim_{x\to\infty} f(x) = 1$

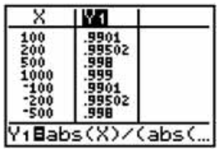
(b) $\lim_{x\to-\infty} f(x) = -1$

(c) $y = 1$, $y = -1$

8.



[−5, 5] by [−2, 2]



X	Y1
100	.9901
200	.99502
500	.998
1000	.999
-100	.9901
-200	.99502
-500	.998

Y1=abs(X)/(abs(...

(a) $\lim_{x\to\infty} f(x) = 1$

(b) $\lim_{x\to-\infty} f(x) = 1$

(c) $y = 1$

9. $0 \le 1 - \cos x \le 2$. So, for $x > 0$ we have $0 \le 1 - \cos x \le \frac{1}{x^2}$.

By the Sandwich Theorem,

$$0 = \lim_{x\to\infty} (0) = \lim_{x\to\infty} \frac{1-\cos x}{x^2} = \lim_{x\to\infty} \frac{1}{x^2} = 0.$$

10. $0 \le 1 - \cos x \le 2$. So, for $x > 0$ we have $-\frac{1}{x^2} \le 1 - \cos x \le 0$.

By the Sandwich Theorem,

$$0 = \lim_{x\to-\infty} (0) = \lim_{x\to-\infty} \frac{1-\cos x}{x^2} = \lim_{x\to-\infty} \frac{1}{x^2} = 0.$$

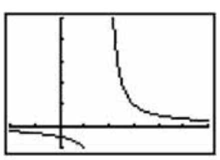
11. $-1 \le \sin x \le 1$

$$\lim_{x\to-\infty} \frac{-1}{x} = \lim_{x\to-\infty} \frac{1}{x} = 0.$$

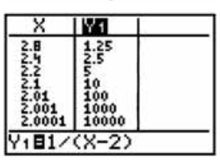
12. $-1 \le \sin(x^2) \le 1$

$$\lim_{x\to\infty} \frac{-1}{x} = \lim_{x\to\infty} \frac{1}{x} = 0.$$

13.



[−2, 6] by [−1, 5]

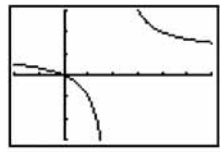


X	Y1
2.8	1.25
2.4	2.5
2.2	5
2.1	10
2.01	100
2.001	1000
2.0001	10000

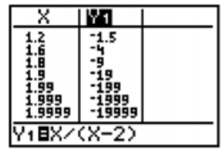
Y1=1/(X-2)

$$\lim_{x\to 2^+} \frac{1}{x-2} = \infty$$

14.



[−2, 6] by [−3, 3]

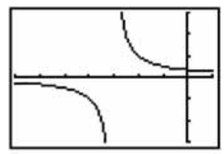


X	Y1
1.2	-1.5
1.6	-4
1.8	-9
1.9	-19
1.99	-199
1.999	-1999
1.9999	-19999

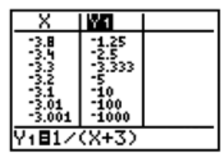
Y1=X/(X-2)

$$\lim_{x\to 2^-} \frac{x}{x-2} = -\infty$$

15.



[−7, 1] by [−3, 3]

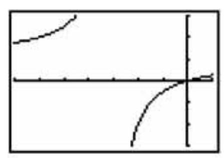


X	Y1
-3.8	-1.25
-3.4	-2.5
-3.3	-3.333
-3.2	-5
-3.1	-10
-3.01	-100
-3.001	-1000

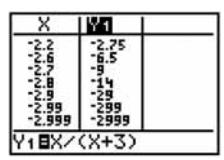
Y1=1/(X+3)

$$\lim_{x\to -3^-} \frac{x}{x+3} = -\infty$$

16.



[−7, 1] by [−3, 3]

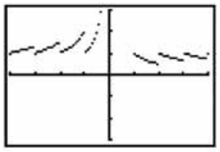


X	Y1
-2.2	-2.75
-2.6	-6.5
-2.7	-9
-2.8	-14
-2.9	-29
-2.99	-299
-2.999	-2999

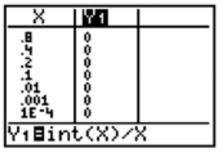
Y1=X/(X+3)

$$\lim_{x\to -3^+} \frac{1}{x+3} = -\infty$$

17.



[−4, 4] by [−3, 3]

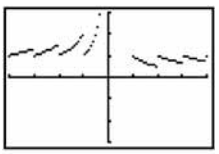


X	Y1
.8	0
.4	0
.2	0
.1	0
.01	0
.001	0
1E-4	0

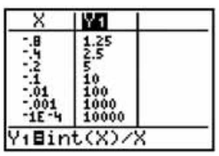
Y1=int(X)/X

$$\lim_{x\to 0^+} \frac{\text{int } x}{x} = 0$$

18.



[−4, 4] by [−3, 3]

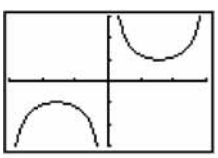


X	Y1
-.8	1.25
-.4	2.5
-.2	5
-.1	10
-.01	100
-.001	1000
-1E-4	10000

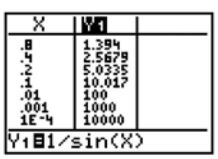
Y1=int(X)/X

$$\lim_{x\to 0^-} \frac{\text{int } x}{x} = \infty$$

19.



[−3, 3] by [−3, 3]

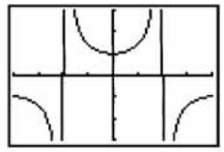


X	Y1
.8	1.394
.4	2.5679
.2	5.0335
.1	10.017
.01	100
.001	1000
1E-4	10000

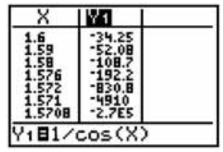
Y1=1/sin(X)

$$\lim_{x\to 0^+} \csc x = \infty$$

20.



[−π, π] by [−3, 3]



X	Y1
1.6	-34.25
1.59	-52.08
1.58	-108.7
1.576	-192.2
1.572	-830.8
1.571	-4910
1.5708	-2.7E5

Y1=1/cos(X)

$$\lim_{x\to(\pi/2)^+} \sec x = -\infty$$

21. $y = \left(2 - \frac{x}{x+1}\right)\left(\frac{x^2}{5+x^2}\right) = \left(\frac{2(x+1)-x}{x+1}\right)\left(\frac{x^2}{5+x^2}\right)$

$$= \left(\frac{x+2}{x+1}\right)\left(\frac{x^2}{5+x^2}\right) = \frac{x^3+2x^2}{x^3+x^2+5x+5}$$

An end behavior model for y is $\frac{x^3}{x^3} = 1.$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} 1 = 1$$
$$\lim_{x\to-\infty} y = \lim_{x\to-\infty} 1 = 1$$

22. $y = \left(\frac{2}{x} + 1\right)\left(\frac{5x^2-1}{x^2}\right) = \left(\frac{2+x}{x}\right)\left(\frac{5x^2-1}{x^2}\right)$

$$= \frac{5x^3+10x^2-x-2}{x^3}$$

An end behavior model for y is $\frac{5x^3}{x^3} = 5.$

$$\lim_{x\to\infty} y = \lim_{x\to\infty} 5 = 5$$
$$\lim_{x\to-\infty} y = \lim_{x\to-\infty} 5 = 5$$

23. Use the method of Example 10 in the text.

$$\lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)}{1+\frac{1}{x}} = \lim_{x\to 0^+} \frac{\cos x}{1+x} = \frac{\cos(0)}{1+0} = \frac{1}{1} = 1$$

$$\lim_{x\to-\infty} \frac{\cos\left(\frac{1}{x}\right)}{1+\frac{1}{x}} = \lim_{x\to 0^-} \frac{\cos x}{1+x} = \frac{\cos(0)}{1+0} = \frac{1}{1} = 1$$

24. Note that $y = \frac{2x + \sin x}{x} = 2 + \frac{\sin x}{x}$.

So, $\lim_{x\to\infty} y = \lim_{x\to\infty} 2 + \lim_{x\to\infty} \frac{\sin x}{x} = 2 + 0 = 2.$

Similarly, $\lim_{x\to-\infty} y = 2.$

25. Use $y = \frac{\sin x}{2x^2 + x} = \frac{\sin x}{x} \bullet \frac{1}{2x+1}$

$\lim_{x\to\pm\infty} \frac{\sin x}{x} = 0$

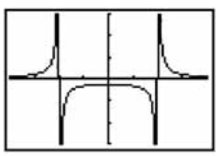
$\lim_{x\to\pm\infty} \frac{1}{2x+1} = 0$

So, $\lim_{x\to\infty} y = 0$ and $\lim_{x\to-\infty} y = 0.$

26. $y = \frac{1}{2}\frac{\sin x}{x} + \frac{1}{x}\frac{\sin x}{x}$

So, $\lim_{x\to\infty} y = 0$ and $\lim_{x\to-\infty} y = 0.$

27.

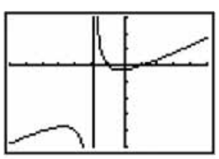


[−4, 4] by [−3, 3]

(a) $x = -2$, $x = 2$

(b) Left-hand limit at -2 is ∞.
Right-hand limit at -2 is $-\infty$.
Left-hand limit at 2 is $-\infty$.
Right-hand limit at 2 is ∞.

28.

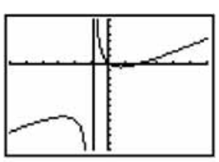


[−7, 5] by [−5, 3]

(a) $x = -2$

(b) Left-hand limit at -2 is $-\infty$.
Right-hand limit at -2 is ∞.

29.



[−6, 6] by [−12, 6]

(a) $x = -1$

(b) Left-hand limit at -1 is $-\infty$.
Right-hand limit at -1 is ∞.

30.

[−2, 4] by [−2, 2]

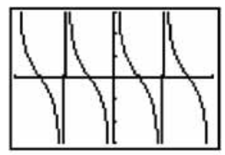
(a) $x = -\frac{1}{2}$, $x = 3$

(b) Left-hand limit at $-\frac{1}{2}$ is ∞.

Right-hand limit at $-\frac{1}{2}$ is $-\infty$.

Left-hand limit at 3 is ∞.
Right-hand limit at 3 is $-\infty$.

31.

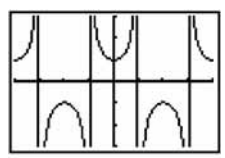


[-2π, 2π] by [−3, 3]

(a) $x = k\pi$, k any integer

(b) at each vertical asymptote:
Left-hand limit is $-\infty$.
Right-hand limit is ∞.

32.



[-2π, 2π] by [−3, 3]

(a) $x = \frac{\pi}{2} + n\pi$, n any integer

(b) If n is even:

Left-hand limit is ∞.
Right-hand limit is $-\infty$.
If n is odd:
Left-hand limit is $-\infty$.
Right-hand limit is ∞.

33. $f(x)=\dfrac{\tan x}{\sin x}=\dfrac{1}{\sin x}\dfrac{\sin x}{\cos x}=\dfrac{1}{\cos x}$

$\cos x=0$ at: $a=(4k+1)\dfrac{\pi}{2}$ and $b=(4k+3)\dfrac{\pi}{2}$ where k is any real integer.

$\lim\limits_{x\to a^-} f(x)=\infty$, $\lim\limits_{x\to a^+} f(x)=-\infty$, $\lim\limits_{x\to b^-} f(x)=-\infty$, $\lim\limits_{x\to b^+} f(x)=\infty$.

34. $f(x)=\dfrac{\cot x}{\cos x}=\dfrac{\cos x}{\sin x}\dfrac{1}{\cos x}=\dfrac{1}{\sin x}$

$\sin x=0$ at $a=2k\pi$ and $b=(2k+1)\pi$ where k is any real integer.

$\lim\limits_{x\to a^-} f(x)=-\infty$, $\lim\limits_{x\to a^+} f(x)=\infty$, $\lim\limits_{x\to b^-} f(x)=\infty$, $\lim\limits_{x\to b^+} f(x)=-\infty$.

35. An end behavior model is $\dfrac{2x^3}{x}=2x^2$. (a)

36. An end behavior model is $\dfrac{x^5}{2x^2}=0.5x^3$. (c)

37. An end behavior model is $\dfrac{2x^4}{-x}=-2x^3$. (d)

38. An end behavior model is $\dfrac{x^4}{-x^2}=-x^2$. (b)

39. (a) $3x^2$

(b) None

40. (a) $-4x^3$

(b) None

41. (a) $\dfrac{x}{2x^2}=\dfrac{1}{2x}$

(b) $y=0$

42. (a) $\dfrac{3x^2}{x^2}=3$

(b) $y=3$

43. (a) $\dfrac{4x^3}{x}=4x^2$

(b) None

44. (a) $\dfrac{-x^4}{x^2}=-x^2$

(b) None

45. (a) The function $y=e^x$ is a right end behavior model because $\lim\limits_{x\to\infty}\dfrac{e^x-2x}{e^x}=\lim\limits_{x\to\infty}\left(1-\dfrac{2x}{e^x}\right)=1-0=1.$

(b) The function $y=-2x$ is a left end behavior model because $\lim\limits_{x\to-\infty}\dfrac{e^x-2x}{-2x}=\lim\limits_{x\to-\infty}\left(-\dfrac{e^x}{2x}+1\right)=0+1=1.$

46. (a) The function $y=x^2$ is a right end behavior model because $\lim\limits_{x\to\infty}\dfrac{x^2+e^{-x}}{x^2}=\lim\limits_{x\to\infty}\left(1+\dfrac{e^{-x}}{x^2}\right)=1+0=1.$

(b) The function $y=e^{-x}$ is a left end behavior model because $\lim\limits_{x\to-\infty}\dfrac{x^2+e^{-x}}{e^{-x}}=\lim\limits_{x\to-\infty}\left(\dfrac{x^2}{e^{-x}}+1\right)$ $=\lim\limits_{x\to-\infty}(x^2e^x+1)=0+1=1.$

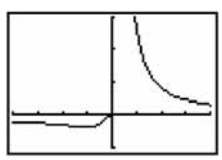
47. (a, b) The function $y=x$ is both a right end behavior model and a left end behavior model because

$$\lim_{x\to\pm\infty}\left(\frac{x+\ln|x|}{x}\right)=\lim_{x\to\pm\infty}\left(1+\frac{\ln|x|}{x}\right)=1+0=1.$$

48. (a, b) The function $y=x^2$ is both a right end behavior model and a left end behavior model because

$$\lim_{x\to\pm\infty}\left(\frac{x^2+\sin x}{x^2}\right)=\lim_{x\to\pm\infty}\left(1+\frac{\sin x}{x^2}\right)=1.$$

49.



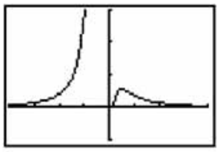
[−4, 4] by [−1, 3]

The graph of $y=f\left(\dfrac{1}{x}\right)=\dfrac{1}{x}e^{1/x}$ is shown.

$$\lim_{x\to\infty} f(x)=\lim_{x\to0^+} f\left(\frac{1}{x}\right)=\infty$$

$$\lim_{x\to\infty^-} f(x)=\lim_{x\to0^-} f\left(\frac{1}{x}\right)=0$$

50.



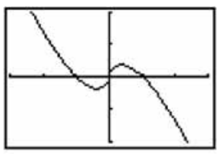
[−4, 4] by [−1, 3]

The graph of $y = f\left(\frac{1}{x}\right) = \frac{1}{x^2}e^{-1/x}$ is shown.

$\lim_{x\to\infty} f(x) = \lim_{x\to 0^+} f\left(\frac{1}{x}\right) = 0$

$\lim_{x\to -\infty} f(x) = \lim_{x\to 0^-} f\left(\frac{1}{x}\right) = \infty$

51.



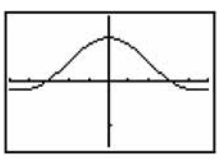
[−3, 3] by [−2, 2]

The graph of $y = f\left(\frac{1}{x}\right) = x\ln\left|\frac{1}{x}\right|$ is shown.

$\lim_{x\to\infty} f(x) = \lim_{x\to 0^+} f\left(\frac{1}{x}\right) = 0$

$\lim_{x\to -\infty} f(x) = \lim_{x\to 0^-} f\left(\frac{1}{x}\right) = 0$

52.



[−5, 5] by [−1.5, 1.5]

The graph of $y = f\left(\frac{1}{x}\right) = \frac{\sin x}{x}$ is shown.

$\lim_{x\to\infty} f(x) = \lim_{x\to 0^+} f\left(\frac{1}{x}\right) = 1$

$\lim_{x\to -\infty} f(x) = \lim_{x\to 0^-} f\left(\frac{1}{x}\right) = 1$

53. **(a)** $\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty}\left(\frac{1}{x}\right) = 0$

(b) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty}(-1) = -1$

(c) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{1}{x} = -\infty$

(d) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(-1) = -1$

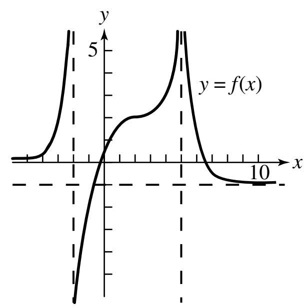
54. **(a)** $\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty}\frac{x-2}{x-1} = \lim_{x\to -\infty}\frac{x}{x} = 1$

(b) $\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{1}{x^2} = 0$

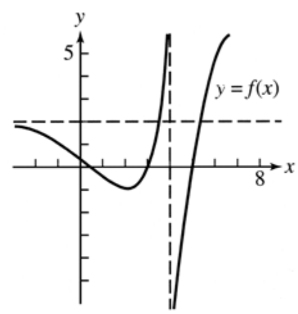
(c) $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\frac{x-2}{x-1} = \frac{0-2}{0-1} = 2$

(d) $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\frac{1}{x^2} = \infty$

55. One possible answer:



56. One possible answer:



57. Note that $\frac{f_1(x)/f_2(x)}{g_1(x)/g_2(x)} = \frac{f_1(x)g_2(x)}{g_1(x)f_2(x)} = \frac{f_1(x)/g_1(x)}{f_2(x)/g_2(x)}$.

As x becomes large, $\frac{f_1}{g_1}$ and $\frac{f_2}{g_2}$ both approach 1.

Therefore, using the above equation, $\frac{f_1/f_2}{g_1/g_2}$ must also approach 1.

58. Yes. The limit of $(f+g)$ will be the same as the limit of g. This is because adding numbers that are very close to a given real number L will not have a significant effect on the value of $(f+g)$ since the values of g are becoming arbitrarily large.

59. True. For example, $f(x) = \frac{x}{\sqrt{x^2+1}}$ has $y = \pm 1$ as horizontal asymptotes.

60. False. Consider $f(x) = 1/x$.

61. A. $\lim_{x\to 2^-}$ approaches zero, so $\lim_{x\to 2^-}\frac{x}{x-2}$ approaches $-\infty$.

62. E. $\lim_{x\to 0}\frac{\cos(2x)}{x}=\frac{\cos(0)}{0}=\frac{1}{0}$ undefined

63. C.

64. D. $\frac{2x^3}{x^3}=-2$

65. (a) Note that $fg=f(x)g(x)=1$.

$f\to -\infty$ as $x\to 0^-$, $f\to\infty$ as $x\to 0^+$, $g\to 0, fg\to 1$

(b) Note that $fg=f(x)g(x)=-8$.

$f\to\infty$ as $x\to 0^-$, $f\to -\infty$ as $x\to 0^+$, $g\to 0$,
$fg\to -8$

(c) Note that $fg=f(x)g(x)=3(x-2)^2$.

$f\to -\infty$ as $x\to 2^-$, $f\to\infty$ as $x\to 2^+$, $g\to 0, fg\to 0$

(d) Note that $fg=f(x)g(x)=\frac{5}{(x-3)^2}$.

$f\to\infty, g\to 0, fg\to\infty$

(e) Nothing – you need more information to decide.

66. (a) This follows from $x-1<\text{int}\,x\le x$, which is true for all x. Dividing by x gives the result.

(b, c) Since $\lim_{x\to\pm\infty}\frac{x-1}{x}=\lim_{x\to\pm\infty}1=1$, the Sandwich Theorem gives $\lim_{x\to\infty}\frac{\text{int}\,x}{x}=\lim_{x\to-\infty}\frac{\text{int}\,x}{x}=1$.

67. For $x>0, 0<e^{-x}<1$, so $0<\frac{e^{-x}}{x}<\frac{1}{x}$. Since both 0 and $\frac{1}{x}$ approach zero as $x\to\infty$, the Sandwich Theorem states that $\frac{e^{-x}}{x}$ must also approach zero.

68. This is because as x approaches infinity, sin x continues to oscillate between 1 and –1 and doesn't approach any given real number.

69. $\lim_{x\to\infty}\frac{\ln x^2}{\ln x}=2$, because $\frac{\ln x^2}{\ln x}=\frac{2\ln x}{\ln x}=2$.

70. $\lim_{x\to\infty}\frac{\ln x}{\log x}=\ln(10)$, Since $\frac{\ln x}{\log x}=\frac{\ln x}{(\ln x)/(\ln 10)}$
$=\ln 10$.

71. $\lim_{x\to\infty}\frac{\ln(x+1)}{\ln x}=1$

Since $\ln(x+1)=\ln\left[x\left(1+\frac{1}{x}\right)\right]=\ln x+\ln\left(1+\frac{1}{x}\right)$,

$$\frac{\ln(x+1)}{\ln x}=\frac{\ln x+\ln\left(1+\frac{1}{x}\right)}{\ln x}=1+\frac{\ln\left(1+\frac{1}{x}\right)}{\ln x}$$

But as $x\to\infty$, $1+1/x$ approaches 1, so $\ln(1+1/x)$ approaches $\ln(1)=0$. Also, as $x\to\infty$, ln x approaches infinity. This means the second term above approaches 0 and the limit is 1.

Quick Quiz Sections 2.1 and 2.2

1. D. $\lim_{x\to 3}\frac{x^2-x-6}{x-3}$ where $x=3.1$

$$\frac{(3.1)^2-3.1-6}{3.1-3}=\frac{9.61-3.1-6}{0.1}=\frac{0.5}{0.1}=5$$

2. A. $3x+1$ $x<2$ is not done since $\lim_{x\to 2^+}$

$\frac{5}{x+1}$ where $x=2.1$

$\frac{5}{2.1+1}=\frac{5}{3.1}\approx\frac{5}{3}$

3. E. $\frac{3x^3}{2x^3}=\frac{3}{2}$

4. (a) $f(x)=\frac{\cos x}{x}=\frac{\cos\infty}{\infty}=0$

(b) For $x>0$, $-1\le\cos x\le 1$. Thus $\frac{-1}{x}\le\frac{\cos x}{x}\le\frac{1}{x}$ and

$\lim_{x\to\infty}-1/x\le\lim_{x\to\infty}\frac{\cos x}{x}\le\lim_{x\to\infty}\frac{1}{x}$. Because

$\lim_{x\to\infty}\frac{-1}{x}=\lim_{x\to\infty}\frac{1}{x}=0$, $\lim_{x\to\infty}\frac{\cos x}{x}=0$

(c) $\lim_{x\to\infty}\frac{\cos x}{x}$, $-1\le\cos x\le 1$

$\lim_{x\to\infty}\frac{1}{x}=0$

(d) For all $x>0, -1\le\cos x\le 1$.

Therefore, $-\frac{1}{x}\le\frac{\cos x}{x}\le\frac{1}{x}$.

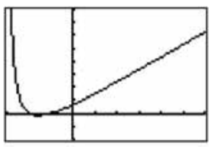
Since $\lim_{x\to\infty}\left(-\frac{1}{x}\right)=\lim_{x\to\infty}\left(\frac{1}{x}\right)=0$, it follows by the Sandwich Theorem that $\lim_{x\to\infty}\frac{\cos x}{x}=0$.

Section 2.3 Continuity (pp. 78–86)

Exploration 1 Removing a Discontinuity

1. $x^2 - 9 = (x-3)(x+3)$. The domain of f is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ or all $x \neq \pm 3$.

2. It appears that the limit of f as $x \to 3$ exists and is a little more than 3.



[−3, 6] by [−2, 8]

3. $f(3)$ should be defined as $\dfrac{10}{3}$.

4. $x^3 - 7x - 6 = (x-3)(x+1)(x+2)$, $x^2 - 9$
$= (x-3)(x+3)$, so $f(x) = \dfrac{(x+1)(x+2)}{x+3}$ for $x \neq 3$.

Thus, $\lim\limits_{x\to 3} \dfrac{(x+1)(x+2)}{x+3} = \dfrac{20}{6} = \dfrac{10}{3}$.

5. $\lim\limits_{x\to 3} g(x) = \dfrac{10}{3} = g(3)$, so g is continuous at $x = 3$.

Quick Review 2.3

1. $\lim\limits_{x\to -1} \dfrac{3x^2 - 2x + 1}{x^3 + 4} = \dfrac{3(-1)^2 - 2(-1) + 1}{(-1)^3 + 4} = \dfrac{6}{3} = 2$

2. (a) $\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-} \text{int}(x) = -2$

(b) $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} f(x) = -1$

(c) $\lim\limits_{x\to -1} f(x)$ does not exist, because the left- and right-hand limits are not equal.

(d) $f(-1) = \text{int}(-1) = -1$

3. (a) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} (x^2 - 4x + 5) = 2^2 - 4(2) + 5 = 1$

(b) $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} (4 - x) = 4 - 2 = 2$

(c) $\lim\limits_{x\to 2} f(x)$ does not exist, because the left- and right-hand limits are not equal.

(d) $f(2) = 4 - 2 = 2$

4. $(f\circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x} + 1\right) = \dfrac{2\left(\dfrac{1}{x}+1\right) - 1}{\left(\dfrac{1}{x}+1\right) + 5}$

$= \dfrac{2(1+x) - x}{(1+x) + 5x} = \dfrac{x+2}{6x+1}$, $x \neq 0$

$(g\circ f)(x) = f(g(x)) = g\left(\dfrac{2x-1}{x+5}\right) = \dfrac{1}{\dfrac{2x-1}{x+5}} + 1$

$= \dfrac{x+5}{2x-1} + \dfrac{2x-1}{2x-1} = \dfrac{3x+4}{2x-1}$, $x \neq -5$

5. Note that $\sin x^2 = (g\circ f)(x) = g(f(x)) = g(x^2)$.

Therefore: $g(x) = \sin x$, $x \geq 0$

$(f\circ g)(x) = f(g(x)) = f(\sin x) = (\sin x^2)$ or $\sin^2 x$, $x \geq 0$

6. Note that $\dfrac{1}{x} = (g\circ f)(x)\ g(f(x)) = \sqrt{f(x) - 1}$. Therefore,

$\sqrt{f(x)-1} = \dfrac{1}{x}$ for $x > 0$. Squaring both sides gives

$f(x) - 1 = \dfrac{1}{x^2}$. Therefore, $f(x) = \dfrac{1}{x^2} + 1$, $x > 0$.

$(f\circ g)(x) = f(g(x)) = \dfrac{1}{(\sqrt{x-1})^2} + 1 = \dfrac{1}{x-1} + 1$

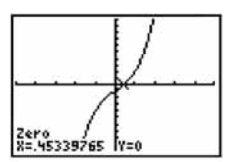
$= \dfrac{1 + x - 1}{x-1} = \dfrac{x}{x-1}$, $x > 1$

7. $2x^2 + 9x - 5 = 0$

$(2x-1)(x+5) = 0$

Solutions: $x = \dfrac{1}{2}$, $x = -5$

8.



[−5, 5] by [−10, 10]

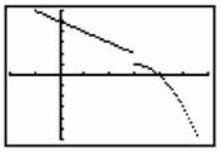
Solution: $x \approx 0.453$

9. For $x \leq 3$, $f(x) = 4$ when $5 - x = 4$, which gives $x = 1$. (Note that this value is, in fact, ≤ 3.)

For $x > 3$, $f(x) = 4$ when $-x^2 + 6x - 8 = 4$, which gives $x^2 - 6x + 12 = 0$. The discriminant of this equation is $b^2 - 4ac = (-6)^2 - 4(1)(12) = -12$. Since the discriminant is negative, the quadratic equation has no solution.

The only solution to the original equation is $x = 1$.

10.



[−2.7, 6.7] by [−6, 6]

A graph of $f(x)$ is shown. The range of $f(x)$ is $(-\infty, 1) \cup [2, \infty)$. The values of c for which $f(x) = c$ has no solution are the values that are excluded from the range. Therefore, c can be any value in $[1, 2)$.

Section 2.3 Exercises

1. The function $y = \dfrac{1}{(x+2)^2}$ is continuous because it is a quotient of polynomials, which are continuous. Its only point of discontinuity occurs where it is undefined. There is an infinite discontinuity at $x = -2$.

2. The function $y = \dfrac{x+1}{x^2 - 4x + 3}$ is continuous because it is a quotient of polynomials, which are continuous. Its only points of discontinuity occur where it is undefined, that is, where the denominator $x^2 - 4x + 3 = (x-1)(x-3)$ is zero. There are infinite discontinuities at $x = 1$ and at $x = 3$.

3. The function $y = \dfrac{1}{x^2+1}$ is continuous because it is a quotient of polynomials, which are continuous. Furthermore, the domain is all real numbers because the denominator, $x^2 + 1$, is never zero. Since the function is continuous and has domain $(-\infty, \infty)$, there are no points of discontinuity.

4. The function $y = |x-1|$ is a composition $(f \circ g)(x)$ of the continuous functions $f(x) = |x|$ and $g(x) = x - 1$, so it is continuous. Since the function is continuous and has domain $(-\infty, \infty)$, there are no points of discontinuity.

5. The function $y = \sqrt{2x+3}$ is a composition $(f \circ g)(x)$ of the continuous functions $f(x) = \sqrt{x}$ and $g(x) = 2x + 3$, so it is continuous. Its points of discontinuity are the points not in the domain, i.e., all $x < -\dfrac{3}{2}$.

6. The function $y = \sqrt[3]{2x-1}$ is a composition $(f \circ g)(x)$ of the continuous functions $f(x) = \sqrt[3]{x}$ and $g(x) = 2x - 1$, so it is continuous. Since the function is continuous and has domain $(-\infty, \infty)$, there are no points of discontinuity.

7. The function $y = \dfrac{|x|}{x}$ is equivalent to

$$y = \begin{cases} -1, & x < 0 \\ 1, & x > 0 \end{cases}.$$

It has a jump discontinuity at $x = 0$.

8. The function $y = \cot x$ is equivalent to $y = \dfrac{\cos x}{\sin x}$, a quotient of continuous functions, so it is continuous. Its only points of discontinuity occur where it is undefined. It has infinite discontinuities at $x = k\pi$ for all integers k.

9. The function $y = e^{1/x}$ is a composition $(f \circ g)(x)$ of the continuous functions $f(x) = e^x$ and $g(x) = \dfrac{1}{x}$, so it is continuous. Its only point of discontinuity occurs at $x = 0$, where it is undefined. Since $\lim_{x\to 0^+} e^{1/x} = \infty$, this may be considered an infinite discontinuity.

10. The function $y = \ln(x+1)$ is a composition $(f \circ g)(x)$ of the continuous functions $f(x) = \ln x$ and $g(x) = x + 1$, so it is continuous. Its points of discontinuity are the points not in the domain, i.e., $x < -1$.

11. **(a)** Yes, $f(-1) = 0$.

(b) Yes, $\lim_{x\to -1^+} = 0$.

(c) Yes

(d) Yes, since -1 is a left endpoint of the domain of f and $\lim_{x\to -1^+} f(x) = f(-1)$, f is continuous at $x = -1$.

12. **(a)** Yes, $f(1) = 1$.

(b) Yes, $\lim_{x\to 1} f(x) = 2$.

(c) No

(d) No

13. **(a)** No

(b) No, since $x = 2$ is not in the domain.

14. Everywhere in $[-1, 3)$ except for $x = 0, 1, 2$.

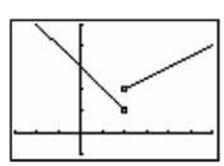
15. Since $\lim_{x\to 2} f(x) = 0$, we should assign $f(2) = 0$.

16. Since $\lim_{x\to 1} f(x) = 2$, we should reassign $f(1) = 2$.

17. No, because the right-hand and left-hand limits are not the same at zero.

18. Yes, Assign the value 0 to $f(3)$. Since 3 is a right endpoint of the extended function and $\lim_{x\to 3^-} f(x) = 0$, the extended function is continuous at $x = 3$.

19.

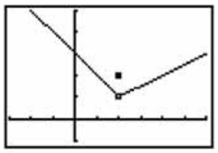


[−3, 6] by [−1, 5]

(a) $x = 2$

(b) Not removable, the one-sided limits are different.

20.

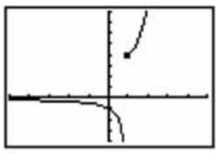


[−3, 6] by [−1, 5]

(a) $x = 2$

(b) Removable, assign the value 1 to $f(2)$.

21.

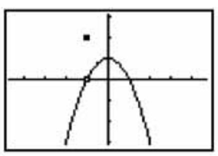


[−5, 5] by [−4, 8]

(a) $x = 1$

(b) Not removable, it's an infinite discontinuity.

22.



[−4.7, 4.7] by [−3.1, 3.1]

(a) $x = -1$

(b) Removable, assign the value 0 to $f(-1)$.

23. **(a)** All points not in the domain along with $x = 0, 1$

(b) $x = 0$ is a removable discontinuity, assign $f(0) = 0$. $x = 1$ is not removable, the one-sided limits are different.

24. **(a)** All points not in the domain along with $x = 1, 2$

(b) $x = 1$ is not removable, the one-sided limits are different.
$x = 2$ is a removable discontinuity, assign $f(2) = 1$.

25. For $x \neq -3, f(x) = \dfrac{x^2-9}{x+3} = \dfrac{(x+3)(x-3)}{x+3} = x-3$.
The extended function is $y = x - 3$.

26. For $x \neq 1, f(x) = \dfrac{x^3-1}{x^2-1}$

$$= \frac{(x-1)(x^2+x+1)}{(x+1)(x-1)} = \frac{x^2+x+1}{x+1}.$$

The extended function is $y = \dfrac{x^2+x+1}{x+1}$.

27. Since $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$, the extended function is

$$y = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0. \end{cases}$$

28. Since $\lim\limits_{x\to 0} \dfrac{\sin 4x}{x} = 4 \lim\limits_{x\to 0} \dfrac{\sin 4x}{4x} = 4(1) = 4$, the extended function is

$$y = \begin{cases} \dfrac{\sin 4x}{x}, & x \neq 0 \\ 4, & x = 0. \end{cases}$$

29. For $x \neq 4$ (and $x > 0$),

$$f(x) = \frac{x-4}{\sqrt{x}-2} = \frac{(\sqrt{x}+2)(\sqrt{x}-2)}{\sqrt{x}-2} = \sqrt{x}+2.$$

The extended function is $y = \sqrt{x}+2.$.

30. For $x \neq 2$ (and $x \neq -2$),

$$f(x) = \frac{x^3-4x^2-11x+30}{x^2-4} = \frac{(x-2)(x-5)(x+3)}{(x-2)(x+2)} = \frac{(x-5)(x+3)}{x+2} = \frac{x^2-2x-15}{x+2}.$$

The extended function is $y = \dfrac{x^2-2x-15}{x+2}$.

31. The domain of f is all real numbers $x \neq 3$. f is continuous at all those points so f is a continuous function.

32. The domain of g is all real numbers $x > 1$. f is continuous at all those points so g is a continuous function.

33. f is the composite of two continuous functions $g \circ h$ where

$g(x) = \sqrt{x}$ and $h(x) = \dfrac{x}{x+1}$.

34. f is the composite of two continuous functions $g \circ h$ where

$g(x) = \sin x$ and $h(x) = x^2 + 1$.

35. f is the composite of three continuous functions $g \circ h \circ k$

where $g(x) = \cos x$, $h(x) = \sqrt[3]{x}$, and $k(x) = 1 - x$.

36. f is the composite of two continuous functions $g \circ h$ where

$g(x) = \tan x$ and $h(x) = \dfrac{x^2}{x^2+4}$.

37. One possible answer:

Assume $y = x$, constant functions, and the square root function are continuous.

By the sum theorem, $y = x + 2$ is continuous.

By the composite theorem, $y = \sqrt{x+2}$ is continuous.

By the quotient theorem, $y = \dfrac{1}{\sqrt{x+2}}$ is continuous.

Domain: $(-2, \infty)$

38. One possible answer:

Assume $y = x$, constant functions, and the cube root function are continuous.

By the difference theorem, $y = 4 - x$ is continuous.

By the composite theorem, $y = \sqrt[3]{4-x}$ is continuous.

By the product theorem, $y = x^2 = x \cdot x$ is continuous.

By the sum theorem, $y = x^2 + \sqrt[3]{4-x}$ is continuous.

Domain: $(-\infty, \infty)$

39. Possible answer:

Assume $y = x$ and $y = |x|$ are continuous.

By the product theorem, $y = x^2 = x \cdot x$ is continuous.

By the constant multiple theorem, $y = 4x$ is continuous.

By the difference theorem, $y = x^2\ 4x$ is continuous.

By the composite theorem, $y = |x^2 - 4x|$ is continuous.

Domain: $(-\infty, \infty)$

40. One possible answer:

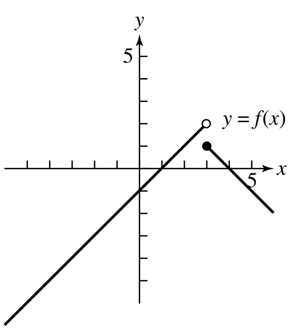
Assume $y = x$ and $y = 1$ are continuous.

Use the product, difference, and quotient theorems. One also needs to verify that the limit of this function as x approaches 1 is 2.

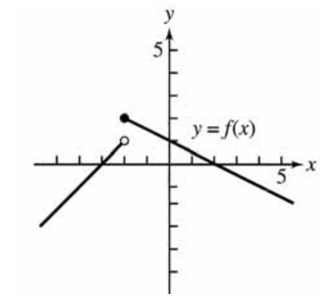
Alternately, observe that the function is equivalent to $y = x + 1$ (for all x), which is continuous by the sum theorem.

Domain: $(-\infty, \infty)$

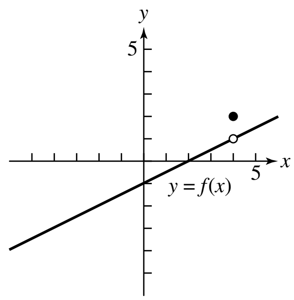
41. One possible answer:



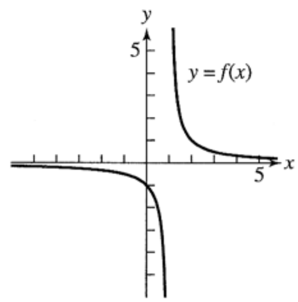
42. One possible answer:



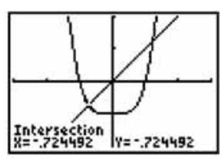
43. One possible answer:



44. One possible answer:



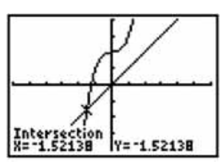
45.



[−3, 3] by [−2, 2]

Solving $x = x^4 - 1$, we obtain the solutions $x \approx -0.724$ and $x \approx 1.221$.

46.



[−6, 6] by [−4, 4]

Solving $x = x^3 + 2$, we obtain the solution $x \approx -1.521$.

47. We require that $\lim_{x\to 3^+} 2ax = \lim_{x\to 3^-}(x^2-1)$:

$$2a(3) = 3^2 - 1$$
$$6a = 8$$
$$a = \frac{4}{3}$$

48. Solve at $x = 2$

$f(x) = 2x+3 = 2(2)+3 = 4+3 = 7$
$f(x) = ax+1$ at $x = 2.1$
$7 = 2a+1$
$6 = 2a$
$a = 3$

49. Solve at $x = -1$

$f(x) = 4 - x^2 = 4-(-1)^2 = 4+1 = 5$
$f(x) = ax+1$ at $x = 1$
$5 = a(1)+1$
$a = 4$

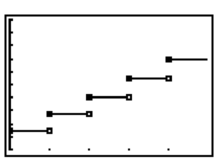
50. Solve at $x = 1$

$f(x) = x^3 = 1^3 = 1$
$f(x) = x^2 + x + a$
$1 = 1^2 + 1 + a$
$a = -1$

51. Consider $f(x) = x - e^{-x}$. f is continuous, $f(0) = -1$, and $f(1) = 1 - \frac{1}{e} > 0.5$. By the Intermediate Value Theorem, for some c in (0, 1), $f(c) = 0$ and $e^{-c} = c$.

52. **(a)** Sarah's salary is \$36,500 = \$36,500 $(1.035)^0$ for the first year ($0 \le t < 1$), \$36,500(1.035) for the second year ($1 \le t < 2$), \$36,500 $(1.035)^2$ for the third year ($2 \le t < 3$), and so on. This corresponds to $y = 36{,}500\,(1.035)^{\text{int}\,t}$.

(b)



[0, 4.8] by [35000, 45000]

The function is continuous at all points in the domain [0, 5) except at t = 1, 2, 3, 4.

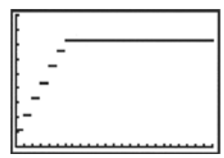
53. **(a)** We require:

$$f(x) = \begin{cases} 0 & x = 0 \\ 1.10, & 0 < x \le 1 \\ 2.20, & 1 < x \le 2 \\ 3.30, & 2 < x \le 3 \\ 4.40, & 3 < x \le 4 \\ 5.50, & 4 < x \le 5 \\ 6.60, & 5 < x \le 6 \\ 7.25, & 6 < x \le 24. \end{cases}$$

This may be written more compactly as

$$f(x) = \begin{cases} -1.10\ \text{int}(-x), & 0 \le x \le 6 \\ 7.25, & 6 < x \le 24 \end{cases}$$

(b)



[0, 24] by [0, 9]

This is continuous for all valyes of xin the domain [0, 24] except for x = 0, 1, 2, 3, 4, 5, 6.

54. False. Consider $f(x) = 1/x$ which is continuous and has a point of discontinuity at $x = 0$.

55. True. If f has a jump discontinuity at $x = a$, then $\lim_{x\to a^-} f(x) \quad \lim_{x\to a^+} f(x)$ so f is not continuous at $x = a$.

56. B. $f(x) = \frac{1}{\sqrt{0}}$ is not defined.

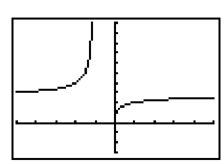
57. E. $f(x) = \sqrt{x-1} = \sqrt{1-1} = \sqrt{0}$ is the only defined option.

58. A. $f(1) = 1$.

59. E. $x = 3$ causes a zero to be in the denominator.

60. **(a)** The function is defined when $1 + \frac{1}{x} > 0$, that is, on $(-\infty, -1) \cup (0, \infty)$. (It can be argued that the domain should also include certain values in the interval (–1, 0), namely, those rational numbers that have odd denominators when expressed in lowest terms.)

(b)



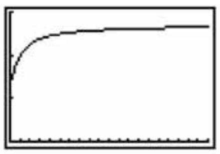
[–5, 5] by [–3, 10]

60. Continued

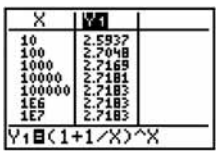
(c) If we attempt to evaluate $f(x)$ at these values, we obtain $f(-1) = \left(1 + \frac{1}{-1}\right)^{-1} = 0^{-1} = \frac{1}{0}$ (undefined) and $f(0) = \left(1 + \frac{1}{0}\right)^{0}$ (undefined). Since f is undefined at these values due to division by zero, both values are points of discontinuity.

(d) The discontinuity at $x = 0$ is removable because the right-hand limit is 0. The discontinuity at $x = -1$ is not removable because it is an infinite discontinuity.

(e)



[0, 20] by [0, 3]



X	Y1
10	2.5937
100	2.7048
1000	2.7169
10000	2.7181
100000	2.7183
1E6	2.7183
1E7	2.7183

Y1=(1+1/X)^X

The limit is about 2.718, or e.

61. This is because $\lim_{h\to 0} f(a+h) = \lim_{x\to a} f(x)$.

62. Suppose not. Then f would be negative somewhere in the interval and positive somewhere else in the interval. So, by the Intermediate Value Theorem, it would have to be zero somewhere in the interval, which contradicts the hypothesis.

63. Since the absolute value function is continuous, this follows from the theorem about continuity of composite functions.

64. For any real number a, the limit of this function as x approaches a cannot exist. This is because as x approaches a, the values of the function will continually oscillate between 0 and 1.

Section 2.4 Rates of Change and Tangent Lines (pp. 87–94)

Quick Review 2.4

1. $\Delta x = 3 - (-5) = 8$
$\Delta y = 5 - 2 = 3$

2. $\Delta x = a - 1$
$\Delta y = b - 3$

3. $m = \frac{-1-3}{5-(-2)} = \frac{-4}{7} = -\frac{4}{7}$

4. $m = \frac{3-(-1)}{3-(-3)} = \frac{4}{6} = \frac{2}{3}$

5. $y = \frac{3}{2}[x - (-2)] + 3$
$y = \frac{3}{2}x + 6$

6. $m = \frac{-1-6}{4-1} = \frac{-7}{3} = -\frac{7}{3}$
$y = -\frac{7}{3}(x-1) + 6$
$y = -\frac{7}{3}x + \frac{25}{3}$

7. $y = -\frac{3}{4}(x-1) + 4$
$y = -\frac{3}{4}x + \frac{19}{4}$

8. $m = -\frac{1}{-3/4} = \frac{4}{3}$
$y = \frac{4}{3}(x-1) + 4$
$y = \frac{4}{3}x + \frac{8}{3}$

9. Since $2x + 3y = 5$ is equivalent to $y = -\frac{2}{3}x + \frac{5}{3}$, we use $m = -\frac{2}{3}$.
$y = -\frac{2}{3}[x - (-1)] + 3$
$y = -\frac{2}{3}x + \frac{7}{3}$

10. $\frac{b-3}{4-2} = \frac{5}{3}$
$b - 3 = \frac{10}{3}$
$b = \frac{19}{3}$

Section 2.4 Exercises

1. (a) $\frac{\Delta f}{\Delta x} = \frac{f(3) - f(2)}{3-2} = \frac{28-9}{1} = 19$

(b) $\frac{\Delta f}{\Delta x} = \frac{f(1) - f(-1)}{1-(-1)} = \frac{2-0}{2} = 1$

2. (a) $\frac{\Delta f}{\Delta x} = \frac{f(2) - f(0)}{2-0} = \frac{3-1}{2} = 1$

(b) $\frac{\Delta f}{\Delta x} = \frac{f(12) - f(10)}{12-10} = \frac{7-\sqrt{41}}{2} \approx 0.298$

3. (a) $\frac{\Delta f}{\Delta x} = \frac{f(0) - f(-2)}{0-(-2)} = \frac{1-e^{-2}}{2} \approx 0.432$

(b) $\frac{\Delta f}{\Delta x} = \frac{f(3) - f(1)}{3-1} = \frac{e^3 - e}{2} \approx 8.684$

4. (a) $\frac{\Delta f}{\Delta x}=\frac{f(4)-f(1)}{4-1}=\frac{\ln 4-0}{3}=\frac{\ln 4}{3}\approx 0.462$

(b) $\frac{\Delta f}{\Delta x}=\frac{f(103)-f(100)}{103-100}=\frac{\ln 103-\ln 100}{3}=\frac{1}{3}\ln\frac{103}{100}$
$=\frac{1}{3}\ln 1.03\approx 0.0099$

5. (a) $\frac{\Delta f}{\Delta x}=\frac{f(3\pi/4)-f(\pi/4)}{(3\pi/4)-(\pi/4)}=\frac{-1-1}{\pi/2}=-\frac{4}{\pi}\approx -1.273$

(b) $\frac{\Delta f}{\Delta x}=\frac{f(\pi/2)-f(\pi/6)}{(\pi/2)-(\pi/6)}=\frac{0-\sqrt{3}}{\pi/3}=-\frac{3\sqrt{3}}{\pi}\approx -1.654$

6. (a) $\frac{\Delta f}{\Delta x}=\frac{f(\pi)-f(0)}{\pi-0}=\frac{1-3}{\pi}=-\frac{2}{\pi}\approx -0.637$

(b) $\frac{\Delta f}{\Delta x}=\frac{f(\pi)-f(-\pi)}{\pi-(-\pi)}=\frac{1-1}{2\pi}=0$

7. We use $Q_1=(10, 225)$, $Q_2=(14, 375)$, $Q_3=(16.5, 475)$, $Q_4=(18, 550)$, and $P=(20, 650)$.

(a) Slope of PQ_1: $\frac{650-225}{20-10}\approx 43$
Slope of PQ_2: $\frac{650-375}{20-14}\approx 46$
Slope of PQ_3: $\frac{650-475}{20-16.5}=50$
Slope of PQ_4: $\frac{650-550}{20-18}=50$

Secant	Slope
PQ_1	43
PQ_2	46
PQ_3	50
PQ_4	50

The appropriate units are meters per second.

(b) Approximately 50 m/sec

8. We use $Q_1=(5, 20)$, $Q_2=(7, 38)$, $Q_3=(8.5, 56)$, $Q_4=(9.5, 72)$, and $P=(10, 80)$.

(a) Slope of PQ_1: $\frac{80-20}{10-5}=12$
Slope of PQ_2: $\frac{80-38}{10-7}=14$
Slope of PQ_3: $\frac{80-56}{10-8.5}=16$
Slope of PQ_4: $\frac{80-72}{10-9.5}=16$

Secant	Slope
PQ_1	12
PQ_2	14
PQ_3	16
PQ_4	16

The appropriate units are meters per second.

(b) Approximately 16 m/sec

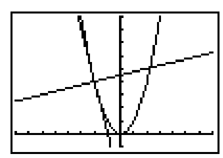
9. (a) $\lim_{h\to 0}\frac{y(-2+h)-y(-2)}{h}=\lim_{h\to 0}\frac{(-2+h)^2-(-2)^2}{h}$
$=\lim_{h\to 0}\frac{4-4h+h^2-4}{h}$
$=\lim_{h\to 0}\frac{-4h+h^2}{h}$
$=\lim_{h\to 0}(-4+h)$
$=-4$

(b) The tangent line has slope –4 and passes through
$(-2, y(-2))=(-2, 4)$
$y=-4[x-(-2)]+4$
$y=-4x-4$

(c) The normal line has slope $-\frac{1}{-4}=\frac{1}{4}$ and passes through $(-2, y(-2))=(-2, 4)$.

$y=\frac{1}{4}[x-(-2)]+4$
$y=\frac{1}{4}x+\frac{9}{2}$

(d)



[–8, 7] by [–1, 9]

10. (a) $\lim_{h\to 0}\frac{y(1+h)-y(1)}{h}$
$=\lim_{h\to 0}\frac{[(1+h)^2-4(1+h)]-[1^2-4(1)]}{h}$
$=\lim_{h\to 0}\frac{1+2h+h^2-4-4h+3}{h}$
$=\lim_{h\to 0}\frac{h^2-2h}{h}$
$=\lim_{h\to 0}(h-2)$
$=-2$

(b) The tangent line has slope –2 and passes through $(1, y(1))=(1, -3)$.

$y=-2(x-1)-3$
$y=-2x-1$

10. Continued

(c) The normal line has slope $-\frac{1}{-2}=\frac{1}{2}$ and passes through $(1, y(1)) = (1, -3)$.

$$y=\frac{1}{2}(x-1)-3$$
$$y=\frac{1}{2}x-\frac{7}{2}$$

(d)

[–6, 6] by [–6, 2]

11. (a) $$\lim_{h\to 0}\frac{y(2+h)-y(2)}{h}=\lim_{h\to 0}\frac{\frac{1}{(2+h)-1}-\frac{1}{2-1}}{h}=\lim_{h\to 0}\frac{\frac{1}{h+1}-1}{h}=\lim_{h\to 0}\frac{1-(h+1)}{h(h+1)}=\lim_{h\to 0}\left(-\frac{1}{h+1}\right)=-1$$

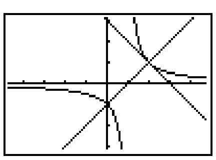
(b) The tangent line has slope –1 and passes through $(2, y(2)) = (2, 1)$.

$$y=-(x-2)+1$$
$$y=-x+3$$

(c) The normal line has slope $-\frac{1}{-1}=1$ and passes through $(2, y(2)) = (2, 1)$.

$$y=1(x-2)+1$$
$$y=x-1$$

(d)



[–4.7, 4.7] by [–3.1, 3.1]

12. (a) $$\lim_{h\to 0}\frac{y(0+h)-y(0)}{h}=\lim_{h\to 0}\frac{(h^2-3h-1)-(-1)}{h}=\lim_{h\to 0}\frac{h^2-3h}{h}=\lim_{h\to 0}(h-3)=-3$$

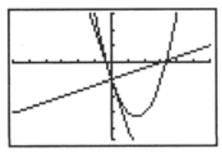
(b) The tangent line has slope –3 and passes through $(0, y(0)) = (0, -1)$.

$$y=-3(x-0)-1$$
$$y=-3x-1$$

(c) The normal line has slope $-\frac{1}{-3}=\frac{1}{3}$ and passes through $(0, y(0)) = (0, -1)$.

$$y=\frac{1}{3}(x-0)-1$$
$$y=\frac{1}{3}x-1$$

(d)



[–6, 6] by [–5, 3]

13. (a) Near $x = 2$, $f(x) = |x| = x$.

$$\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}=\lim_{h\to 0}\frac{(2+h)-2}{h}=\lim_{h\to 0}1=1$$

(b) Near $x = -3$, $f(x) = |x| = -x$.

$$\lim_{h\to 0}\frac{f(-3+h)-f(-3)}{h}=\lim_{h\to 0}\frac{(3-h)-3}{h}=\lim_{h\to 0}-1=-1$$

14. Near $x = 1$, $f(x) = |x-2| = -(x-2) = 2-x$.

$$\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{[2-(1+h)]-(2-1)}{h}=\lim_{h\to 0}\frac{1-h-1}{h}=\lim_{h\to 0}-1=-1$$

15. First, note that $f(0) = 2$.

$$\begin{aligned}\lim_{h\to 0^-}\frac{f(0+h)-f(0)}{h} &= \lim_{h\to 0^-}\frac{(2-2h-h^2)-2}{h}\\ &= \lim_{h\to 0^-}\frac{-2h-h^2}{h}\\ &= \lim_{h\to 0^-}(-2-h)\\ &= -2\end{aligned}$$

$$\begin{aligned}\lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h} &= \lim_{h\to 0^+}\frac{(2h+2)-2}{h}\\ &= \lim_{h\to 0^+}2\\ &= 2\end{aligned}$$

No, the slope from the left is –2 and the slope from the right is 2. The two-sided limit of the difference quotient does not exist.

16. First, note that $f(0) = 0$.

$$\lim_{h\to 0^-}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-}\frac{-h-0}{h} = -1$$

$$\lim_{h\to 0^+}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+}\frac{(h^2-h)-0}{h}$$

$$= \lim_{h\to 0^+}(h-1) = -1$$

Yes. The slope is –1.

17. First, note that $f(2) = \frac{1}{2}$

$$\begin{aligned}\lim_{h\to 0^-}\frac{f(2+h)-f(2)}{h} &= \lim_{h\to 0^-}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}\\ &= \lim_{h\to 0^-}\frac{2-(2+h)}{2h(2+h)}\\ &= \lim_{h\to 0^-}\frac{-h}{2h(2+h)}\\ &= \lim_{h\to 0^-}-\frac{1}{2(2+h)}\\ &= -\frac{1}{4}\end{aligned}$$

$$\begin{aligned}\lim_{h\to 0^+}\frac{f(2+h)-f(2)}{h} &= \lim_{h\to 0^+}\frac{\frac{4-(2+h)}{4}-\frac{1}{2}}{h}\\ &= \lim_{h\to 0^+}\frac{[4-(2+h)]-2}{4h}\\ &= \lim_{h\to 0^+}\frac{-h}{4h}\\ &= -\frac{1}{4}\end{aligned}$$

Yes. The slope is $-\frac{1}{4}$.

18. No. The function is discontinuous at $x = \frac{3\pi}{4}$ because

$\lim_{x\to(3\pi/4)^-} f(x) = \lim_{x\to(3\pi/4)^-}\sin x = \sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ but $f\left(\frac{3\pi}{4}\right) = \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$.

19. (a)

$$\begin{aligned}\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0}\frac{[(a+h)^2+2]-(a^2+2)}{h}\\ &= \lim_{h\to 0}\frac{a^2+2ah+h^2+2-a^2-2}{h}\\ &= \lim_{h\to 0}\frac{2ah+h^2}{h}\\ &= \lim_{h\to 0}(2a+h)\\ &= 2a\end{aligned}$$

(b) The slope of the tangent steadily increases as a increases.

20. (a)

$$\begin{aligned}\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0}\frac{\frac{2}{a+h}-\frac{2}{a}}{h}\\ &= \lim_{h\to 0}\frac{2a-2(a+h)}{ah(a+h)}\\ &= \lim_{h\to 0}\frac{-2}{a(a+h)}\\ &= -\frac{2}{a^2}\end{aligned}$$

(b) The slope of the tangent is always negative. The tangents are very steep near $x = 0$ and nearly horizontal as a moves away from the origin.

21. (a)

$$\begin{aligned}\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0}\frac{\frac{1}{a+h-1}-\frac{1}{a-1}}{h}\\ &= \lim_{h\to 0}\frac{(a-1)-(a+h-1)}{h(a-1)(a+h-1)}\\ &= \lim_{h\to 0}-\frac{1}{(a-1)(a+h-1)}\\ &= -\frac{1}{(a-1)^2}\end{aligned}$$

(b) The slope of the tangent is always negative. The tangents are very steep near $x = 1$ and nearly horizontal as a moves away from the origin.

22. (a)

$$\begin{aligned}\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0}\frac{[9-(a+h)^2]-(9-a^2)}{h}\\ &= \lim_{h\to 0}\frac{9-a^2-2ah-h^2-9+a^2}{h}\\ &= \lim_{h\to 0}\frac{-2ah-h^2}{h}\\ &= \lim_{h\to 0}(-2a-h)\\ &= -2a\end{aligned}$$

(b) The slope of the tangent steadily decreases as a increases.

23. Let $f(t) = 100 - 4.9t^2$.

$$\begin{aligned}&\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}\\&=\lim_{h\to 0}\frac{[100-4.9(2+h)^2]-[100-4.9(2)^2]}{h}\\&=\lim_{h\to 0}\frac{100-19.6-19.6h-4.9h^2-100+19.6}{h}\\&=\lim_{h\to 0}(-19.6-4.9h)\\&=-19.6\end{aligned}$$

The object is falling at a speed of 19.6 m/sec.

24. Let $f(t) = 3t^2$.

$$\begin{aligned}\lim_{h\to 0}\frac{f(10+h)-f(10)}{h}&=\lim_{h\to 0}\frac{3(10+h)^2-300}{h}\\&=\lim_{h\to 0}\frac{300+60h+3h^2-300}{h}\\&=\lim_{h\to 0}(60+3h)\\&=60\end{aligned}$$

The rocket's speed is 60 ft/sec.

25. Let $f(r)=\pi r^2$, the area of a circle of radius r.

$$\begin{aligned}\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}&=\lim_{h\to 0}\frac{\pi(3+h)^2-\pi(3)^2}{h}\\&=\lim_{h\to 0}\frac{9\pi+6\pi h+\pi h^2-9\pi}{h}\\&=\lim_{h\to 0}(6\pi+\pi h)\\&=6\pi\end{aligned}$$

The area is changing at a rate of 6π in^2/in., that is, 6π square inches of area per inch of radius.

26. Let $f(r)=\frac{4}{3}\pi r^3$.

$$\begin{aligned}\lim_{h\to 0}\frac{f(2+h)-f(2)}{h}&=\lim_{h\to 0}\frac{\frac{4}{3}\pi(2+h)^3-\frac{4}{3}\pi(2)^3}{h}\\&=\frac{4}{3}\pi\lim_{h\to 0}\frac{(2+h)^3-2^3}{h}\\&=\frac{4}{3}\pi\lim_{h\to 0}\frac{8+12h+6h^2+h^3-8}{h}\\&=\frac{4}{3}\pi\lim_{h\to 0}(12+6h+h^2)\\&=\frac{4}{3}\pi\cdot 12\\&=16\pi\end{aligned}$$

The volume is changing at a rate of 16π in^3/in., that is, 16π cubic inches of volume per inch of radius.

27.
$$\begin{aligned}\lim_{h\to 0}\frac{s(1+h)-s(1)}{h}&=\lim_{h\to 0}\frac{1.86(1+h)^2-1.86(1)^2}{h}\\&=\lim_{h\to 0}\frac{1.86+3.72h+1.86h^2-1.86}{h}\\&=\lim_{h\to 0}(3.72+1.86h)\\&=3.72\end{aligned}$$

The speed of the rock is 3.72 m/sec.

28.
$$\begin{aligned}\lim_{h\to 0}\frac{s(2+h)-s(2)}{h}&=\lim_{h\to 0}\frac{11.44(2+h)^2-11.44(2)^2}{h}\\&=\lim_{h\to 0}\frac{45.76+45.76h+11.44h^2-45.76}{h}\\&=\lim_{h\to 0}(45.76+11.44h)\\&=45.76\end{aligned}$$

The speed of the rock is 45.76m/sec.

29. First, fin the slope of the tangent at $x = a$.

$$\begin{aligned}&\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}\\&=\lim_{h\to 0}\frac{[(a+h)^2+4(a+h)-1]-(a^2+4a-1)}{h}\\&=\lim_{h\to 0}\frac{a^2+2ah+h^2+4a+4h-1-a^2-4a+1}{h}\\&=\lim_{h\to 0}\frac{2ah+h^2+4h}{h}\\&=\lim_{h\to 0}(2a+h+4)\\&=2a+4\end{aligned}$$

The tangent at $x = a$ is horizontal when $2a + 4 = 0$, or $a = -2$. The tangent line is horizontal at $(-2, f(-2)) = (-2, -5)$.

30. First, find the slope of the tangent at $x = a$.

$$\begin{aligned}&\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}\\&=\lim_{h\to 0}\frac{[3-4(a+h)-(a+h)^2]-(3-4a-a^2)}{h}\\&=\lim_{h\to 0}\frac{3-4a-4h-a^2-2ah-h^2-3+4a+a^2}{h}\\&=\lim_{h\to 0}\frac{-4h-2ah-h^2}{h}\\&=\lim_{h\to 0}(-4-2a-h)\\&=-4-2a\end{aligned}$$

The tangent at $x = a$ is horizontal when $-4 - 2a = 0$, or $a = -2$. The tangent line is horizontal at $(-2, f(-2)) = (-2, 7)$.

31. (a) From Exercise 21, the slope of the curve at $x = a$, is

$-\frac{1}{(a-1)^2}$. The tangent has slope -1 when

$-\frac{1}{(a-1)^2} = -1,$ which gives $(a-1)^2 = 1$, so $a = 0$ or

$a = 2$. Note that $y(0) = \frac{1}{0-1} = -1$ and $y(2) = \frac{1}{2-1} = 1,$

so we need to find the equations of lines of slope -1 passing through $(0, -1)$ and $(2, 1)$, respectively.

$$\begin{aligned}\text{At } x = 0\text{: } & y = -1(x-0)-1\\ & y = -x-1\\ \text{At } x = 2\text{: } & y = -1(x-2)+1\\ & y = -x+3\end{aligned}$$

(b) The normal has slope 1 when the tangent has slope

$\frac{-1}{1} = -1,$ so we again need to find lines through

$(0, -1)$ and $(2, 1)$, this time using slope 1.

$$\begin{aligned}\text{At } x = 0: & y = 1(x-0)-1\\ & y = x-1\\ \text{At } x = 2: & y = 1(x-2)+1\\ & y = x-1\end{aligned}$$

There is only one such line. It is normal to the curve at two points and its equation is $y = x - 1$.

32. Consider a line that passes through $(1, 12)$ and a point $(a, 9 - a^2)$ on the curve. Using the result of Exercise 22, this line will be tangent to the curve at a if its slope is $-2a$.

$$\begin{aligned}\frac{(9-a^2)-12}{a-1} &= -2a\\ 9-a^2-12 &= -2a(a-1)\\ -a^2-3 &= -2a^2+2a\\ a^2-2a-3 &= 0\\ (a+1)\,(a-3) &= 0\\ a &= -1 \text{ or } a = 3\end{aligned}$$

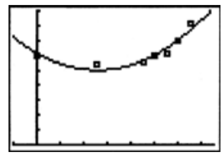
$$\begin{aligned}&\text{At } a = -1 (\text{or } x = -1), \text{ the slope is } -2(-1) = 2.\\ &\quad y = 2(x-1)+12\\ &\quad y = 2x+10\\ &\text{At } a = 3 (\text{or } x = 3), \text{ the slope is } -2(3) = -6.\\ &\quad y = -6(x-1)+12\\ &\quad y = -6x+18\end{aligned}$$

33. (a) $\frac{(272.1-299.3)\text{ billion}}{(1995-1990)\text{ years}} = \frac{-27.2\text{ billion}}{5\text{ years}} = -\$5.4\frac{\text{billion}}{\text{year}}$

(b) $\frac{(305.5-294.5)\text{ billion}}{(2001-2000)\text{ years}} = \$11.0\frac{\text{billion}}{\text{year}}$

(c) $\frac{(404.9-348.6)\text{ billion}}{(2003-2002)\text{ years}} = \$56.3\frac{\text{billion}}{\text{year}}$

(d) $y \approx 2.177x^2 - 22.315x + 306.443$



[–2, 15] by [0, 450]

(e)

1990 to 1995: $\frac{(2.177(5)^2-22.315(5)+306.443)-(2.177(0)^2-22.315(0)+306.443)}{5-0}$

$= \frac{54.425-111.575+306.443-306.443}{5}$

$= \frac{-57.15}{5} = -\$11.4$ billion

2000 to 2001: $\frac{(2.177(11)^2-22.315(11)+306.443)-(2.177(10)^2-22.315(10)+306.443)}{11-10}$

$= \frac{263.417-2700.115-217.7+2231.5}{1}$

$= \$23.4\frac{\text{billion}}{\text{year}}$

2002 to 2003: $\frac{(2.177(13)^2-22.315(13)+306.443)-(2.177(12)^2-22.313(12)+306.443)}{13-12}$

$= 367.913-290.095-313.488+267.756$

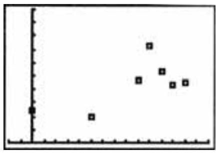
$= \$32.1\frac{\text{billion}}{\text{year}}$

(f) $\frac{2.177(13.1)^2-22.315(13.1)+306.443-(2.177(13.)^2-22.315(13)+306.443)}{13.1-13}$

$= \frac{373.595-292.327-367.913+290.095}{0.1} = \$34.3\frac{\text{billion}}{\text{year}}$

(g) One possible reason is that the war in Iraq and increased spending to prevent terrorist attacks in the U.S. caused an unusual increase in defense spending.

34. (a)



[–2, 15] by [0, 50]

(b)

$$PQ_1 = \frac{36.6 - 22.6}{2000 - 2003} = -4.7 \qquad PQ_2 = \frac{26.4 - 22.6}{2003 - 2001} = -1.9$$

$$PQ_3 = \frac{22.6 - 22.0}{2003 - 2002} = 0.6$$

35. True. The normal line is perpendicular to the tangent line at the point.

36. False. There's no tangent at $x = 0$ because f is undefined at $x = 0$.

37. D. $\frac{-3-5}{-1-2} = \frac{8}{3}$

38. E. $\frac{f(3) - f(1)}{3-1} = \frac{3^2 + 3 - 1^2 - 1}{2} = \frac{10}{2} = 5$

39. C. $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

$$= \left(\frac{2}{x+h} - \frac{2}{x}\right)\frac{1}{h}$$

$$= \frac{2x - 2x - 2h}{x^2 + hx}\left(\frac{1}{h}\right)$$

$$= -\frac{2}{x^2}$$

$$y = -\frac{2}{(1)^2} = -2$$

$$y = m(x - x_1) + y_1$$

$$y = -2(x-1) + 2$$

$$y = -2x + 4$$

40. A. From 39, $m_2 = -\frac{1}{m_1}$

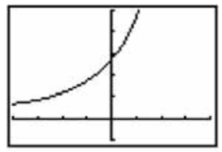
$$m_2 = \frac{1}{2}$$

$$y = \frac{1}{2}(x-1) + 2$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

41. (a) $\frac{f(1+h) - f(1)}{h} = \frac{e^{1+h} - e}{h}$

(b)



[−4, 4] by [−1, 5]

X	Y1
-.01	2.7047
-5E-4	2.7176
-1E-4	2.7181
0	ERROR
1E-4	2.7184
5E-4	2.719
.01	2.7319

Y1■(e^(1+X)−e^(...

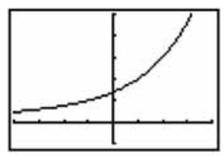
Limit ≈ 2.718

(c) They're about the same.

(d) Yes, it has a tangent whose slope is about e.

42. (a) $\frac{f(1+h) - f(1)}{h} = \frac{2^{1+h} - 2}{h}$

(b)



[−4, 4] by [−1, 5]

X	Y1
-.01	1.3815
-5E-4	1.3861
-1E-4	1.3862
0	ERROR
1E-4	1.3863
5E-4	1.3865
.01	1.3911

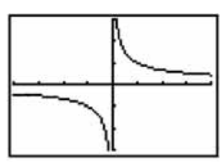
X=-.01

Limit ≈ 1.386

(c) They're about the same.

(d) Yes, it has a tangent whose slope is about ln 4.

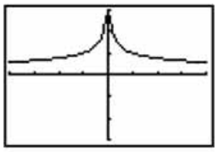
43. Let $f(x) = x^{2/5}$. The graph of $y = \frac{f(0+h) - f(0)}{h} = \frac{f(h)}{h}$ is shown.



[−4, 4] by [−3, 3]

The left- and right-hand limits are –∞ and ∞, respectively. Since they are not the same, the curve does not have a vertical tangent at $x = 0$. No.

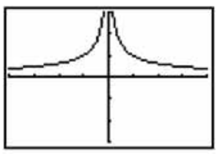
44. Let $f(x) = x^{3/5}$. The graph of $y = \frac{f(0+h)-f(0)}{h} = \frac{f(h)}{h}$ is shown.



[−4, 4] by [−3, 3]

Yes, the curve has a vertical tangent at $x = 0$ because $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \infty$.

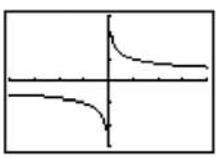
45. Let $f(x) = x^{1/3}$. The graph of $y = \frac{f(0+h)-f(0)}{h} = \frac{f(h)}{h}$ is shown.



[−4, 4] by [−3, 3]

Yes, the curve has a vertical tangent at $x = 0$ because $\lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \infty$.

46. Let $f(x) = x^{2/3}$. The graph of $y = \frac{f(0+h)-f(0)}{h} = \frac{f(h)}{h}$ is shown.



[−4, 4] by [−3, 3]

The left- and right-hand limits are $-\infty$ and ∞, respectively. Since they are not the same, the curve does not have a vertical tangent at $x = 0$. No.

47. This function has a tangent with slope zero at the origin. It is sandwiched between two functions, $y = x^2$ and $y = -x^2$, both of which have slope zero at the origin.

Looking at the difference quotient,

$$-h \le \frac{f(0+h)-f(0)}{h} \le h,$$

so the Sandwich Theorem tells us the limit is 0.

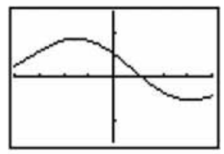
48. This function does not have a tangent line at the origin. As the function oscillates between $y = x$ and $y = -x$ infinitely often near the origin, there are an infinite number of difference quotients (secant line slopes) with a value of 1 and with a value of –1. Thus the limit of the difference quotient doesn't exist.

The difference quotient is $\frac{f(0+h)-f(0)}{h} = \sin\frac{1}{h}$ which oscillates between 1 and –1 infinitely often near zero.

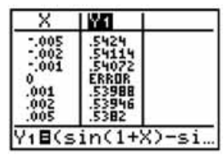
49. Let $f(x) = \sin x$. The difference quotient is

$$\frac{f(1+h)-f(1)}{h} = \frac{\sin(1+h)-\sin(1)}{h}.$$

A graph and table for the difference quotient are shown.



[−4, 4] by [−1.5, 1.5]



X	Y1
-.005	.5424
-.002	.54114
-.001	.54072
0	ERROR
.001	.53988
.002	.53946
.005	.5382

Y1■(sin(1+X)-si…

Since the limit as $h \to 0$ is about 0.540, the slope of $y = \sin x$ at $x = 1$ is about 0.540.

Quick Quiz Sections 2.3 and 2.4

1. D. $\frac{f(3)-f(0)}{3-0} = \frac{\sqrt{3+1}-\sqrt{0+1}}{3} = \frac{2-1}{3} = \frac{1}{3}$

2. E. $f(4-h) \approx \frac{3}{4}(4) \approx 3$ where $h \to 0$

$f(4) = 2$
$f(4+h) \approx -4+7 \approx 3$

3. B. $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$\lim_{h\to 0} \frac{9-(x+h)^2-(9-x^2)}{h}$

$\lim_{h\to 0} \frac{-2xh-h^2}{h} = \lim_{h\to 0} -2x-h = -2x$

$y = 9-x^2 = 9-(2)^2$
$y = 5$
$y^1 = -2(2) = -4$
$y = -4(x-2)+5$
$y = -4x+13$

4. (a) $f(3) = 2(3) - (3)^2$
$= 6 - 9 = -3$

(b) $f(3+h) = 2(3+h) - (3+h)^2$
$= 6 + 2h - (9 + 6h + h^2)$
$= -3 - 4h - h^2$

(c) $\dfrac{f(3+h) - f(3)}{h}$
$= \dfrac{-3 - 4h - h^2 - (-3)}{h}$
$= -4 - h$

(d) $-4 - h$ where $h \to 0$ -4

Chapter 2 Review Exercises (pp. 95–97)

1. $\lim_{x\to -2}(x^3 - 2x^2 + 1) = (-2)^3 - 2(-2)^2 + 1 = -15$

2. $\lim_{x\to -2} \dfrac{x^2+1}{3x^2 - 2x + 5} = \dfrac{(-2)^2 + 1}{3(-2)^2 - 2(-2) + 5} = \dfrac{5}{21}$

3. No limit, because the expression $\sqrt{1-2x}$ is undefined for values of x near 4.

4. No limit, because the expression $\sqrt[4]{9-x^2}$ is undefined for values of x near 5.

5. $\lim_{x\to 0} \dfrac{\frac{1}{2+x} - \frac{1}{2}}{x} = \lim_{x\to 0} \dfrac{2-(2+x)}{2x(2+x)} = \lim_{x\to 0} \dfrac{-x}{2x(2+x)}$
$= \lim_{x\to 0}\left(-\dfrac{1}{2(2+x)}\right) = -\dfrac{1}{2(2+0)} = -\dfrac{1}{4}$

6. $\lim_{x\to \pm\infty} \dfrac{2x^2+3}{5x^2+7} = \lim_{x\to\pm\infty} \dfrac{2x^2}{5x^2} = \dfrac{2}{5}$

7. An end behavior model for $\dfrac{x^4 + x^3}{12x^3 + 128}$ is $\dfrac{x^4}{12x^3} = \dfrac{1}{12}x.$

Therefore

$\lim_{x\to\infty} \dfrac{x^4+x^3}{12x^3+128} = \lim_{x\to\infty} \dfrac{1}{12}x = \infty$
$\lim_{x\to -\infty} \dfrac{x^4+x^3}{12x^3+128} = \lim_{x\to -\infty} \dfrac{1}{12}x = -\infty$

8. $\lim_{x\to 0} \dfrac{\sin 2x}{4x} = \dfrac{1}{2}\lim_{x\to 0} \dfrac{\sin 2x}{2x} = \dfrac{1}{2}(1) = \dfrac{1}{2}$

9. Multiply the numerator and denominator by $\sin x$.

$\lim_{x\to 0} \dfrac{x\csc x + 1}{x \csc x} = \lim_{x\to 0} \dfrac{x + \sin x}{x} = \lim_{x\to 0}\left(1 + \dfrac{\sin x}{x}\right)$
$= \left(\lim_{x\to 0} 1\right) + \left(\lim_{x\to 0} \dfrac{\sin x}{x}\right) = 1 + 1 = 2$

10. $\lim_{x\to 0} e^x \sin x = e^0 \sin 0 = 1\cdot 0 = 0$

11. Let $x = \dfrac{7}{2} + h,$ where h is in $\left(0, \dfrac{1}{2}\right).$ Then

$\text{int}(2x-1) = \text{int}\left[2\left(\dfrac{7}{2}\right) + 2h - 1\right] = \text{int}\,(6 + 2h) = 6,$ because 6 + $2h$ is in (6, 7).

Therefore, $\lim_{x\to 7/2^+} \text{int}(2x-1) = \lim_{x\to 7/2^+} 6 = 6.$

12. Let $x = \dfrac{7}{2} + h,$ where h is in $\left(-\dfrac{1}{2}, 0\right).$ Then

$\text{int}(2x-1) = \text{int}\left[2\left(\dfrac{7}{2}\right) + 2h - 1\right] = \text{int}\,(6 + 2h) = 5,$ because 6 + $2h$ is in (5, 6).

Therefore, $\lim_{x\to 7/2^-} \text{int}(2x-1) = \lim_{x\to 7/2^-} 5 = 5$

13. Since $\lim_{x\to\infty}(-e^{-x}) = \lim_{x\to\infty} e^{-x} = 0,$ and $-e^{-x} \le e^{-x}\cos x \le e^{-x}$ for all x, the Sandwich Theorem gives $\lim_{x\to\infty} e^{-x}\cos x = 0.$

14. Since the expression x is an end behavior model for both $x + \sin x$ and $x + \cos x$, $\lim_{x\to\infty} \dfrac{x + \sin x}{x + \cos x} = \lim_{x\to\infty} \dfrac{x}{x} = 1.$

15. Limit exists.

16. Limit exists.

17. Limit exists.

18. Limit does not exist.

19. Limit exists.

20. Limit exists.

21. Yes

22. No

23. No

24. Yes

25. (a) $\lim_{x\to 3^-} g(x) = 1$

(b) $g(3) = 1.5$

(c) No, since $\lim_{x\to 3^-} g(x) \ne g(3).$

(d) g is discontinuous at $x = 3$ (and at points not in the domain).

(e) Yes, the discontinuity at $x = 3$ can be removed by assigning the value 1 to $g(3)$.

26. **(a)** $\lim_{x\to 1^-} k(x) = 1.5$

(b) $\lim_{x\to 1^+} k(x) = 0$

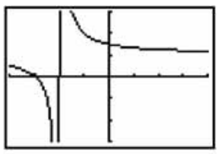
(c) $k(1) = 0$

(d) No, since $\lim_{x\to 1^-} k(x) \neq k(1)$

(e) k is discontinuous at $x = 1$ (and at points not in the domain).

(f) No, the discontinuity at $x = 1$ is not removable because the one-sided limits are different.

27.



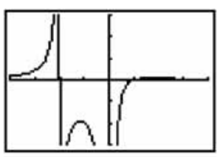
[−4, 4] by [−3, 3]

(a) Vertical asymptote: $x = -2$

(b) Left-hand limit $= \lim_{x\to -2^-} \dfrac{x+3}{x+2} = -\infty$

Right-hand limit: $\lim_{x\to -2^+} \dfrac{x+3}{x+2} = \infty$

28.



[−4, 4] by [−3, 3]

(a) Vertical asymptotes: $x = 0$, $x = -2$

(b) At $x = 0$:

Left-hand limit $= \lim_{x\to 0^-} \dfrac{x-1}{x^2(x+2)} = -\infty$

Right-hand limit $= \lim_{x\to 0^+} \dfrac{x-1}{x^2(x+2)} = -\infty$

At $x = -2$:

Left-hand limit $= \lim_{x\to -2^-} \dfrac{x-1}{x^2(x+2)} = \infty$

Right-hand limit $= \lim_{x\to -2^+} \dfrac{x-1}{x^2(x+2)} = -\infty$

29. **(a)** At $x = -1$:

Left-hand limit $= \lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (1) = 1$

Right-hand limit $= \lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (-x) = 1$

At $x = 0$:

Left-hand limit $= \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (-x) = 0$

Right-hand limit $= \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (-x) = 0$

At $x = 1$:

Left-hand limit $= \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (-x) = -1$

Right-hand limit $= \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (1) = 1$

(b) At $x = -1$: Yes, the limit is 1.

At $x = 0$: Yes, the limit is 0.

At $x = 1$: No, the limit doesn't exist because the two one-sided limits are different.

(c) At $x = -1$: Continuous because $f(-1)$ = the limit.

At $x = 0$: Discontinuous because $f(0) \neq$ the limit.

At $x = 1$: Discontinuous because the limit does not exist.

30. **(a)** Left-hand limit $= \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \left|x^3 - 4x\right|$

$= \left|(1)^3 - 4(1)\right| = \left|-3\right| = 3$

Right-hand limit $= \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x^2 - 2x - 2)$

$= (1)^2 - 2(1) - 2 = -3$

(b) No, because the two one-sided limits are different.

(c) Every place except for $x = 1$

(d) At $x = 1$

31. Since $f(x)$ is a quotient of polynomials, it is continuous and its points of discontinuity are the points where it is undefined, namely $x = -2$ and $x = 2$.

32. There are no points of discontinuity, since $g(x)$ is continuous and defined for all real numbers.

33. **(a)** End behavior model: $\dfrac{2x}{x^2}$, or $\dfrac{2}{x}$

(b) Horizontal asymptote: $y = 0$ (the x-axis)

34. **(a)** End behavior model: $\dfrac{2x^2}{x^2}$, or 2

(b) Horizontal asymptote: $y = 2$

35. **(a)** End behavior model: $\dfrac{x^3}{x}$, or x^2

(b) Since the end behavior model is quadratic, there are no horizontal asymptotes.

36. **(a)** End behavior model: $\dfrac{x^4}{x^3}$, or x

(b) Since the end behavior model represents a nonhorizontal line, there are no horizontal asymptotes.

37. **(a)** Since $\lim_{x\to\infty} \dfrac{x+e^x}{e^x} = \lim_{x\to\infty} \left(\dfrac{x}{e^x} + 1\right) = 1$, a right end behavior model is e^x.

(b) Since $\lim_{x\to -\infty} \dfrac{x+e^x}{x} = \lim_{x\to -\infty} \left(1 + \dfrac{e^x}{x}\right) = 1$, a left end behavior model is x.

38. (a, b) Note that $\lim_{x\to\pm\infty}\left(-\frac{1}{\ln|x|}\right)=\lim_{x\to\pm\infty}\left(\frac{1}{\ln|x|}\right)=0$ and

$-\frac{1}{\ln|x|}<\frac{\sin x}{\ln|x|}<\frac{1}{\ln|x|}$ for all $x\neq 0$.

Therefore, the Sandwich Theorem gives

$\lim_{x\to\pm\infty}\frac{\sin x}{\ln|x|}=0$. Hence

$\lim_{x\to\pm\infty}\frac{\ln|x|+\sin x}{\ln|x|}=\lim_{x\to\pm\infty}\left(1+\frac{\sin x}{\ln|x|}\right)=1+0=1,$

so $\ln|x|$ is both a right end behavior model and a left end behavior model.

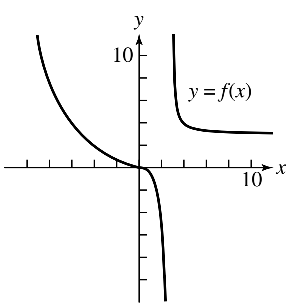
39. $\lim_{x\to 3}f(x)=\lim_{x\to 3}\frac{x^2+2x-15}{x-3}=\lim_{x\to 3}\frac{(x-3)(x+5)}{x-3}$
$=\lim_{x\to 3}(x+5)=3+5=8.$

Assign the value $k = 8$.

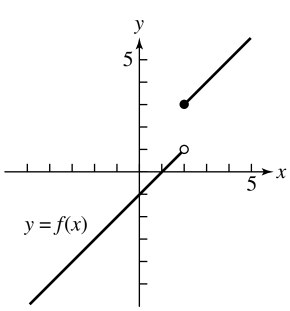
40. $\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{\sin x}{2x}=\frac{1}{2}\lim_{x\to 0}\frac{\sin x}{x}=\frac{1}{2}(1)=\frac{1}{2}$

Assign the value $k=\frac{1}{2}$.

41. One possible answer:



42. One possible answer:



43. $\frac{f(\pi/2)-f(0)}{\pi/2-0}=\frac{2-1}{\pi/2}=\frac{2}{\pi}$

44. $\lim_{h\to 0}\frac{V(a+h)-V(a)}{h}=\lim_{h\to 0}\frac{\frac{1}{3}\pi(a+h)^2H-\frac{1}{3}\pi a^2H}{h}$
$=\frac{1}{3}\pi H\lim_{h\to 0}\frac{a^2+2ah+h^2-a^2}{h}$
$=\frac{1}{3}\pi H\lim_{h\to 0}(2a+h)$
$=\frac{1}{3}\pi H(2a)$
$=\frac{2}{3}\pi aH$

45. $\lim_{h\to 0}\frac{S(a+h)-S(a)}{h}=\lim_{h\to 0}\frac{6(a+h)^2-6a^2}{h}$
$=\lim_{h\to 0}\frac{6a^2+12ah+6h^2-6a^2}{h}$
$=\lim_{h\to 0}(12a+6h)$
$=12a$

46. $\lim_{h\to 0}\frac{y(a+h)-y(a)}{h}$
$=\lim_{h\to 0}\frac{[(a+h)^2-(a+h)-2]-(a^2-a-2)}{h}$
$=\lim_{h\to 0}\frac{a^2+2ah+h^2-a-h-2-a^2+a+2}{h}$
$=\lim_{h\to 0}\frac{2ah+h^2-h}{h}$
$=\lim_{h\to 0}(2a+h-1)$
$=2a-1$

47. (a) $\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}=\lim_{h\to 0}\frac{[(1+h)^2-3(1+h)]-(-2)}{h}$
$=\lim_{h\to 0}\frac{1+2h+h^2-3-3h+2}{h}$
$=\lim_{h\to 0}(-1+h)$
$=-1$

(b) The tangent at P has slope –1 and passes through (1, –2).

$y=-1(x-1)-2$
$y=-x-1$

(c) The normal at P has slope 1 and passes through (1, –2).

$y=1(x-1)-2$
$y=x-3$

48. At $x = a$, the slope of the curve is

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to 0}\frac{[(a+h)^2-3(a+h)]-(a^2-3a)}{h}$$
$$= \lim_{h\to 0}\frac{a^2+2ah+h^2-3a-3h-a^2+3a}{h}$$
$$= \lim_{h\to 0}\frac{2ah-3h+h^2}{h}$$
$$= \lim_{h\to 0}(2a-3+h)$$
$$= 2a-3$$

The tangent is horizontal when $2a - 3 = 0$, at $a = \frac{3}{2}\left(\text{or } x = \frac{3}{2}\right)$. Since $f\left(\frac{3}{2}\right) = -\frac{9}{4}$, the point where this occurs is $\left(\frac{3}{2}, -\frac{9}{4}\right)$.

49. **(a)** $p(0) = \frac{200}{1+7e^{-0.1(0)}} = \frac{200}{8} = 25$

Perhaps this is the number of bears placed in the reserve when it was established.

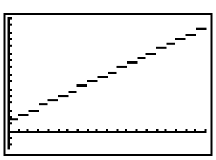
(b) $\lim_{t\to\infty} p(t) = \lim_{t\to\infty}\frac{200}{1+7e^{-0.1t}} = \frac{200}{1} = 200$

(c) Perhaps this is the maximum number of bears which the reserve can support due to limitations of food, space, or other resources. Or, perhaps the number is capped at 200 and excess bears are moved to other locations.

50. **(a)** $f(x) = \begin{cases} 3.20 - 1.35\,\text{int}(-x+1), & 0 < x \le 20 \\ 0, & x = 0 \end{cases}$

(Note that we cannot use the formula $f(x) = 3.20 + 1.35$ int x, because it gives incorrect results when x is an integer.)

(b)



[0, 20] by [–5, 32]

f is discontinuous at integer values of x: 0, 1, 2, . . . , 19.

51. **(a)**

[5, 20] by [15000, 18000]

(b) $PQ_1 = \frac{17{,}019-15{,}487}{2003-1998} = \frac{1532}{5} = 306.4$

$PQ_2 = \frac{17{,}019-15{,}759}{2003-1999} = \frac{1260}{4} = 315$

$PQ_3 = \frac{17{,}019-15{,}983}{2003-2000} = \frac{1036}{3} = 345.3$

$PQ_4 = \frac{17{,}019-16{,}355}{2003-2001} = \frac{664}{2} = 332$

$PQ_5 = \frac{17{,}019-16{,}692}{2003-2002} = \frac{327}{1} = 327$

(c) We use the average rate of change in the population from 2002 to 2003 which is 327,000

(d) $y = 309.457\,x + 12966.533$, rate of change is 309 thousand because rate of change of a linear function is its slope.

52. Let $A = \lim_{x\to c} f(x)$ and $B = \lim_{x\to c} g(x)$. Then $A + B = 2$ and $A - B = 1$. Adding, we have $2A = 3$, so $A = \frac{3}{2}$, whence $\frac{3}{2} + B = 2$, which gives $B = \frac{1}{2}$. Therefore, $\lim_{x\to c} f(x) = \frac{3}{2}$ and $\lim_{x\to c} g(x) = \frac{1}{2}$.

53. **(a)** $x - 9 \ne 0$

All x not equal to –3 or 3.

(b) $x = -3$, $x = 3$

(c) $\lim_{x\to\infty} = \frac{x}{|x^2-9|} = 0$

$y = 0$

(d) When $x - 9 \to 0, f(x) \to \infty$.

$x = -3$ and $x = 3$ are discontinuous.

(e) Yes. It is continuous at every point in its domain.

54. (a) $f(2) = x^2 - a^2 - x$

$= (2)^2 - a^2 2$

$= 4 - 2a^2$

(b) $f(2) = 4 - 2x^2$

$= 4 - 2(2)^2$

$= 4 - 8 = -4$

(c) For $x \neq 2, f$ is continuous. For $x = 2$, we have $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2) = -4$ as long as $a = \pm 2$.

55. (a) $g(x) = \frac{x^3}{x^2} = x$

(b) $\frac{f(x)}{g(x)} = \frac{x^3 - 2x^2 + 1}{x^2 + 3} = \frac{1}{x}$

$= \frac{x^3 - 2x^2 + 1}{x^3 + 3x}$

$\frac{x^3}{x^3} = 1$

Chapter 3
Derivatives

Section 3.1 Derivative of a Function (pp. 99–108)

Exploration 1 Reading the Graphs

1. The graph in Figure 3.3b represents the rate of change of the depth of the water in the ditch with respect to time. Since y is measured in inches and x is measured in days, the derivative $\frac{dy}{dx}$ would be measured in inches per day. Those are the units that should be used along the y-axis in Figure 3.3b.

2. The water in the ditch is 1 inch deep at the start of the first day and rising rapidly. It continues to rise, at a gradually decreasing rate, until the end of the second day, when it achieves a maximum depth of 5 inches. During days 3, 4, 5, and 6, the water level goes down, until it reaches a depth of 1 inch at the end of day 6. During the seventh day it rises again, almost to a depth of 2 inches.

3. The weather appears to have been wettest at the beginning of day 1 (when the water level was rising fastest) and driest at the end of day 4 (when the water level was declining the fastest).

4. The highest point on the graph of the derivative shows where the water is rising the fastest, while the lowest point (most negative) on the graph of the derivative shows where the water is declining the fastest.

5. The y-coordinate of point C gives the maximum depth of the water level in the ditch over the 7-day period, while the x-coordinate of C gives the time during the 7-day period that the maximum depth occurred. The derivative of the function changes sign from positive to negative at C', indicating that this is when the water level stops rising and begins falling.

6. Water continues to run down sides of hills and through underground streams long after the rain has stopped falling. Depending on how much high ground is located near the ditch, water from the first day's rain could still be flowing into the ditch several days later. Engineers responsible for flood control of major rivers must take this into consideration when they predict when floodwaters will "crest," and at what levels.

Quick Review 3.1

1. $\lim_{h\to 0} \frac{(2+h)^2 - 4}{4} = \lim_{h\to 0} \frac{(4 + 4h + h^2) - 4}{h}$

$= \lim_{h\to 0} 4 + h$

$= 4 + 0 = 4$

2. $\lim_{x\to 2^+} \frac{x+3}{2} = \frac{2+3}{2} = \frac{5}{2}$

3. Since $\frac{|y|}{y} = -1$ for $y < 0$, $\lim_{y\to 0^-} \frac{|y|}{y} = -1$.

4. $\lim_{x\to 4} \frac{2x - 8}{\sqrt{x} - 2} = \lim_{x\to 4} \frac{2(\sqrt{x}+2)(\sqrt{x}-2)}{\sqrt{x} - 2}$

$= \lim_{h\to 4} 2(\sqrt{x} + 2) = 2(\sqrt{4} + 2) = 8$

5. The vertex of the parabola is at (0, 1). The slope of the line through (0, 1) and another point $(h, h^2 + 1)$ on the parabola is $\frac{(h^2+1) - 1}{h - 0} = h$. Since $\lim_{h\to 0} h = 0$, the slope of the line tangent to the parabola at its vertex is 0.

6. Use the graph of f in the window [–6, 6] by [–4, 4] to find that (0, 2) is the coordinate of the high point and (2, –2) is the coordinate of the low point. Therefore, f is increasing on $(-\infty, 0]$ and $[2, \infty)$.

7. $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x-1)^2 = (1-1)^2 = 0$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x+2) = 1 + 2 = 3$

8. $\lim_{h\to 0^+} f(1+h) = \lim_{x\to 1^+} f(x) = 0$

9. No, the two one-sided limits are different (see Exercise 7).

10. No, f is discontinuous at $x = 1$ because $\lim_{x\to 1} f(x)$ does not exist.