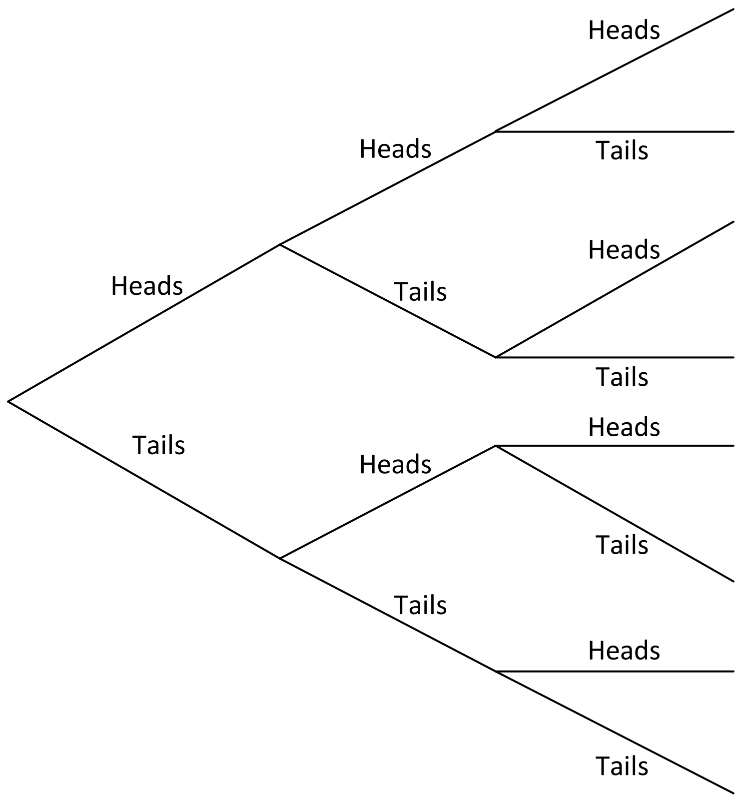
Probability and Statistics Solutions – Chapter 1

Problem Set 1.1 Exercises Page 4

1) a) S={HH, HT, TH, TT} b) There are four outcomes.



2) a) S={HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} b) There are eight outcomes.
3) a) S={HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT} b) There are sixteen outcomes.
4) S={1, 2, 3, 4}
5) a)

+	**1**	**2**	**3**	**4**
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

b) S={2, 3, 4, 5, 6, 7, 8}

6) a)

	1	**2**	**3**	**4**
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

b) S={1, 2, 3, 4, 6, 8, 9, 12, 16}

c) There are nine different outcomes.

d) Four is the most common outcome.

Problem Set 1.2 Exercises Pages 9-10

1) $3 \cdot 5 \cdot 4 = 60$
2) There are 10 digits possible for each of the five numbers in the zip code. This gives us a total of $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^5 = 100,000$ possible zip codes.
3) There are 10 choices for the first digit, but since there are no repeats there are only 9 choices for the second digit. This leads us to $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30,240$ possible zip codes.
4) There are 9 choices for the first batter. Once that player is selected, there are only 8 players available to bat second. This leads us to $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 9! = 362,880$ ways to arrange the batting order.
5) There are 6 choices for the first brand of soap. Once that brand is used, there are only 5 choices for the 2nd brand. This leads us to $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6! = 720$ ways to arrange the brands of soap.
6) $8 \cdot 6 \cdot 3 = 144$
7) There are 8 choices for the first film. Once this film is shown, there are only 7 choices for the 2nd film. This leads us to $8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 8! = 40,320$ orders that the films could be shown.
8) There are 2 choices for the first letter. Once that letter is chosen, there are only 25 letters to pick from for the second letter. This leads to $2 \cdot 25 \cdot 24 \cdot 23 = 27,600$ letter combinations possible for a radio station.
9) We have 2 choices for the first letter. Since letters may be repeated, there are 26 choices for the 2nd, 3rd, and 4th letters. This gives us $2 \cdot 26 \cdot 26 \cdot 26 = 35,152$ possible letter combinations for the radio stations.
10) There are 10 choices for each digit or $10 \cdot 10 \cdot 10 \cdot 10 = 10^4 = 10,000$ ID tags possible.
11) There is only one choice for the first digit. Since repeats are not allowed, there will be only 9 values left for the second digit and so on. This gives $1 \cdot 9 \cdot 8 \cdot 7 = 504$ possible ID tags.

12) There are seven choices for the first book. Once that book is selected, there are only 6 choices for the next book. This gives us $7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1=7!=5,040$ ways to arrange the books.

13) $5\cdot2=10$

14) There are two possible outcomes for each flip, heads or tails. This gives $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2=2^8=256$ different possible outcomes for a series of 8 flips.

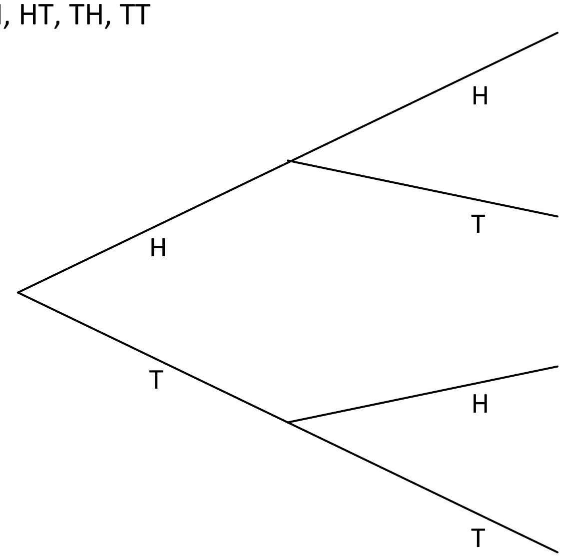
15) There are six colors available for the first tile, five for the second tile and so on. This gives us $6\cdot5\cdot4\cdot3=360$ tile patterns of four colors.

16) There are six colors available for each tile chosen. This gives $6\cdot6\cdot6\cdot6=1,296$ patterns.

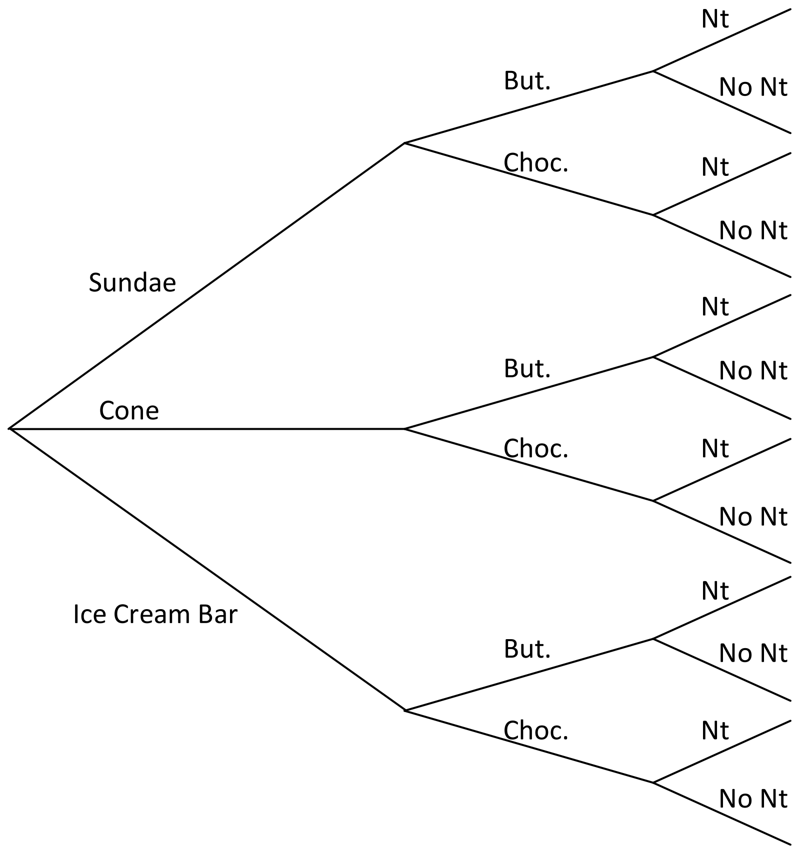
17) There will be four choices of suit for each card or $4\cdot4\cdot4\cdot4=256$ possible results.

18) Consider each ingredient separately. You could either have your pizza with pepperoni or without pepperoni. This means there are two choices for pepperoni. The same could be said of sausage, either it is on your pizza or it is not. Therefore, there are two options for each of the 6 possible pizza toppings or $2\cdot2\cdot2\cdot2\cdot2\cdot2=2^6=64$ possible pizzas.

19) HH, HT, TH, TT



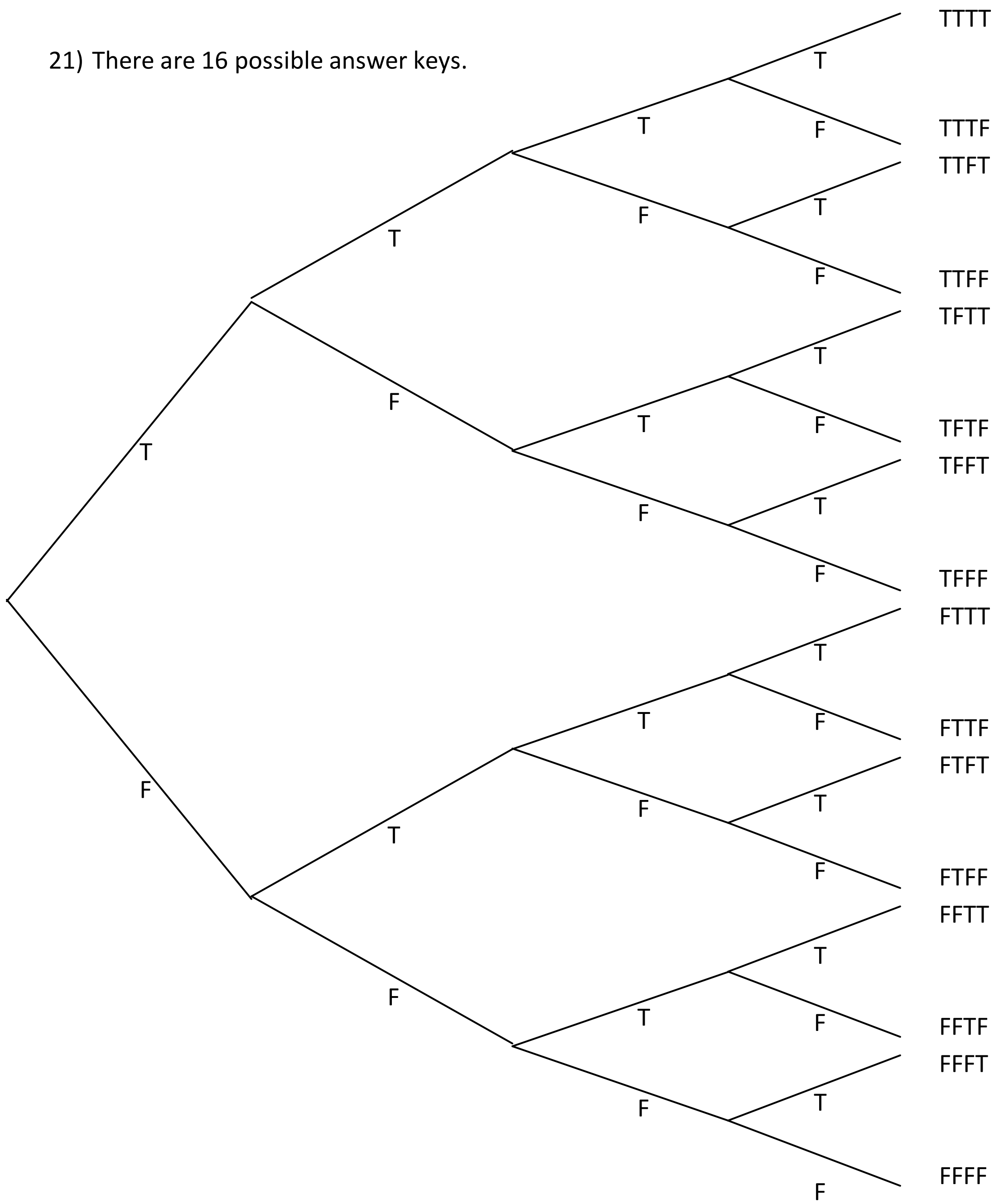
20) a)



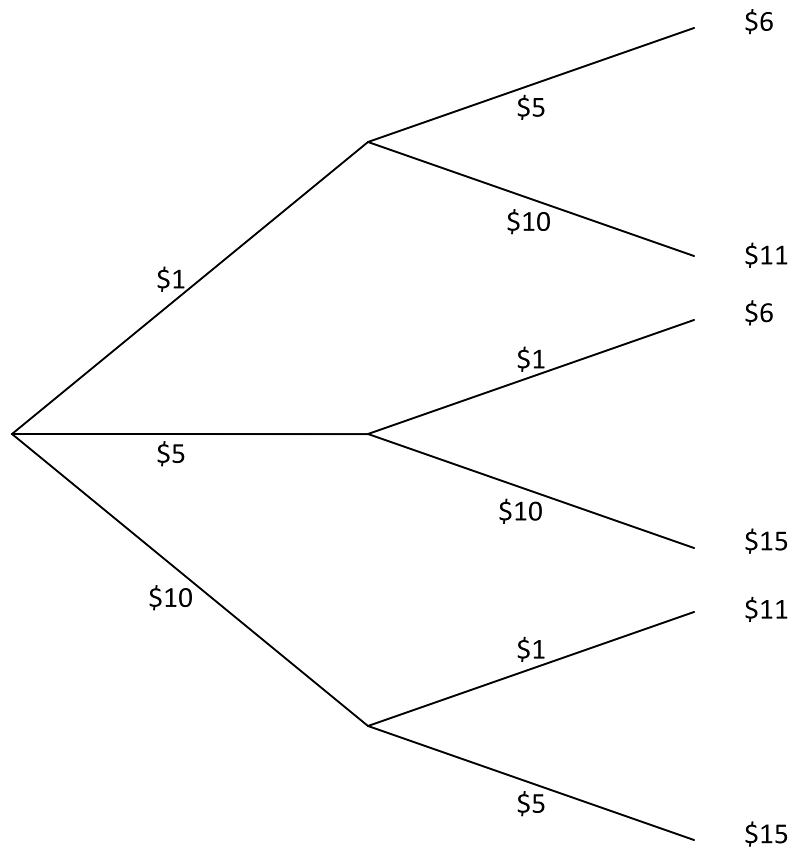
b) To find the number of treats possible, multiply the number of choices on each branch.

c) There are $3 \cdot 2 \cdot 2 = 12$ possible treats.

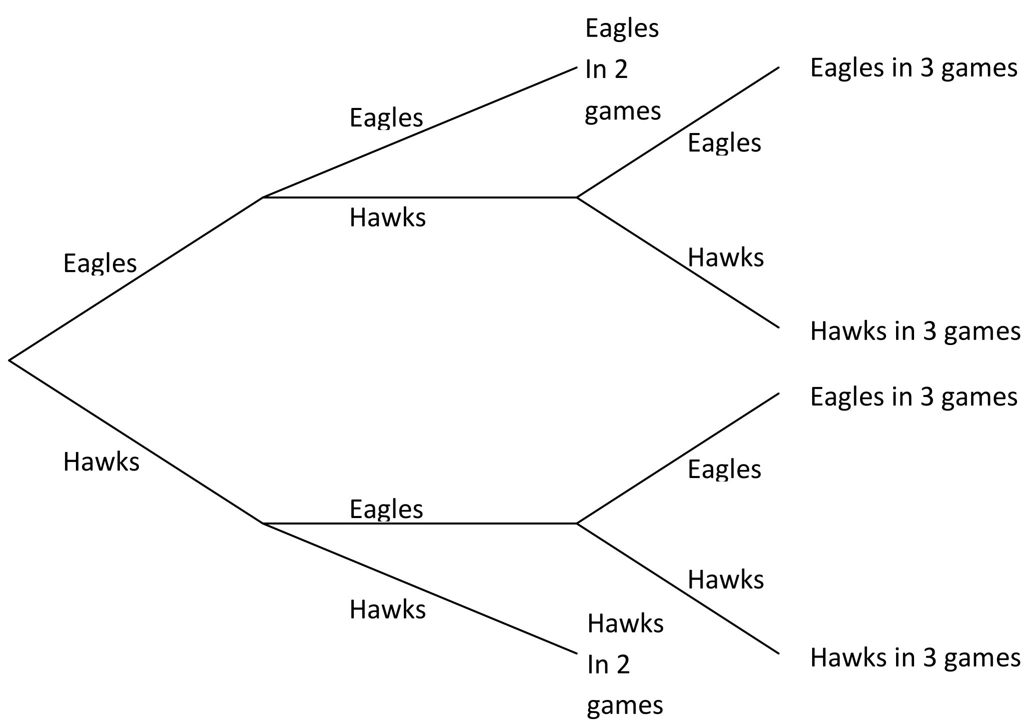
21) There are 16 possible answer keys.



22) The different amounts of money possible are \$6, \$11, and \$15 as shown.



23) Notice that a 3rd game is not always needed.



Problem Set 1.2 Review Exercises

24) a) S = { P1C1, P1C2, P2C1, P2C2, P3C1, P3C2, P4C1, P4C2} b) There are 8 outcomes.

Problem Set 1.3 Exercises Pages 14-15

1) a) $\frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{40,320}{120} = 336$

   b) $\frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{24}{1} = 24$

   c) $\frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{120}{2} = 60$

   d) $\frac{5!}{(5-0)!} = \frac{5!}{5!} = 1$

2) There are 7 letters available and we will be selecting 4 letters at a time. The order that we select these letters will make a difference so we get ${}_7P_4 = 840$ permutations.
3) Since the order that we place people in these three positions matters we have ${}_8P_3 = 336$ ways that we can fill the positions.
4) The order of the numbers will make a difference on an idea card. There are ten digits to select from and we will use six. ${}_{10}P_6 = 151,200$ different ID cards are possible.
5) The order that we put the brands on the shelf will make a difference. We have seven brands to pick from and we are going to use all seven. ${}_7P_7 = 5,040$ different ways to display the brands of soap.
6) The car will look different depending on how the child puts on the stickers so the order matters. ${}_4P_4 = 24$ different sticker patterns are possible.
7) Since it states that the inspector will specify the order, the order that the tests are performed will matter. ${}_7P_3 = 210$ different orders are possible.
8) Since the prizes are different for each winning raffle ticket, the order that the tickets are drawn will matter. There are ${}_{50}P_4 = 5,527,200$ ways to draw the four winning tickets.
9) Each rat receives a different treatment so order matters. ${}_5P_5 = 120$ ways to distribute the antibiotics to the rats.
10) Since there is a different part for each musician, order will matter. There are ${}_5P_3 = 60$ ways the trio could be selected.
11) Since there is a different part for each musician, order will matter. There are ${}_5P_4 = 120$ ways the quartet could be selected.

12) Since there is a different part for each musician, order will matter. There are ${}_5P_5 = 120$ ways the quartet could be selected.

13) The order the musicians are selected will matter. In this case there could be 3, 4, or 5 musicians selected to play. The total is ${}_5P_3 + {}_5P_4 + {}_5P_5 = 60 + 120 + 120 = 300$ ways to select the musicians. Notice that this is just the sum of the answers from problems 10, 11, and 12.

14) a) No. The child picking red, blue, and then green is the same as the child picking blue, green, and then red.

b) Yes. The order that the digits are entered will make a difference.

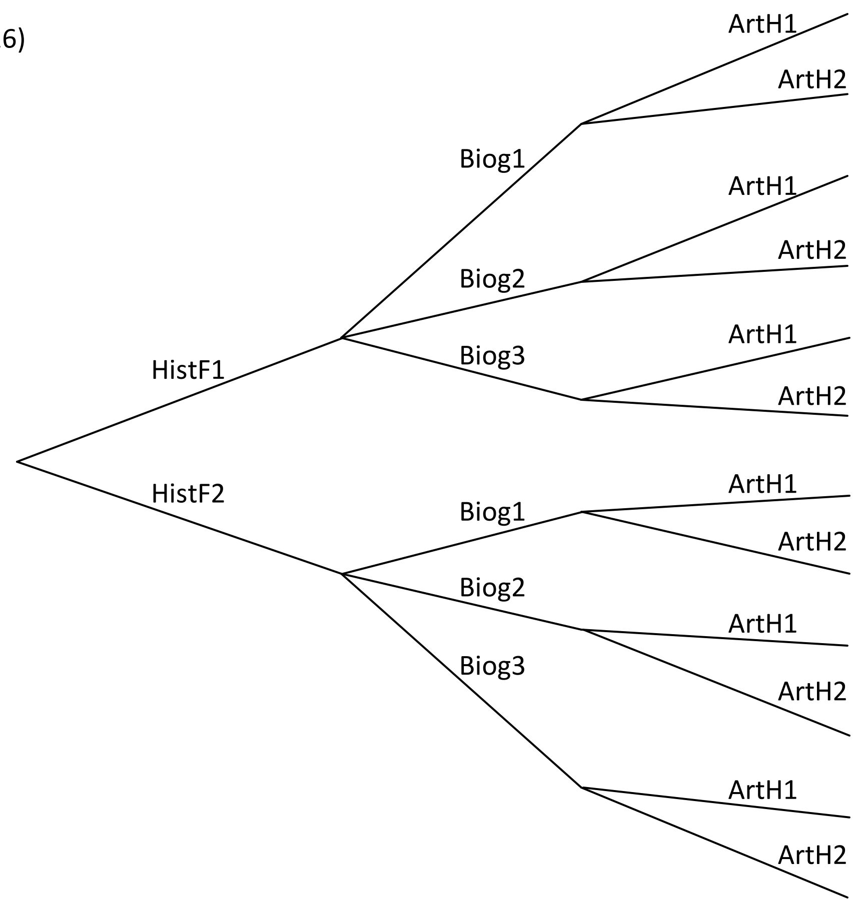
c) No. The order will not matter because the fastest drivers will still get the best times whether they complete their time trials early or late in the line of racers.

d) Yes. If the student doesn't follow the order on the instructions, the cookies might not turn out.

Problem Set 1.3 Review Exercises

15) The there are $4 \cdot 3 \cdot 4 = 48$ ways that a person could order a meal.

16)



These books can be checked out in 12 different ways.

17) Solve this problem by making a 4 by 6 grid as shown below.

+	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
6	7	8	9	10

The different totals are 2, 3, 4, 5, 6, 7, 8, 9, and 10. There are 9 different outcomes.

Problem Set 1.4 Pages 18-19

1) a) $\frac{5!}{5!(5-5)!} = \frac{5!}{5! \cdot 0!} = \frac{120}{120 \cdot 1} = 1$

b) $\frac{6!}{4!(6-4)!} = \frac{6!}{4! \cdot 2!} = \frac{720}{24 \cdot 2} = \frac{720}{48} = 15$

c) $\frac{3!}{0!(3-0)!} = \frac{3!}{0! \cdot 3!} = \frac{6}{1 \cdot 6} = \frac{6}{6} = 1$

d) $\frac{7!}{3!(7-3)!} = \frac{7!}{3! \cdot 4!} = \frac{5,040}{6 \cdot 24} = 35$

2) Order will not matter here. Getting dealt the ace of spades, 8 of diamonds, and then the queen of clubs is equivalent to getting the eight of diamonds, queen of clubs, and then the ace of spades. There are ${}_{52}C_3 = 22,100$ ways of getting dealt 3 cards.

3) It does not matter the order that the bracelets are pulled out of the box. There are ${}_{10}C_3 = 120$ ways to select the bracelets.

4) The order that the student answers their 5 questions will not matter. The teacher does not care which of the five questions was answered first. There are ${}_{9}C_5 = 126$ different combinations of questions possible.

5) In this case, the student is required to answer the first and last questions. As a result, there are only seven questions from which to choose three. There are ${}_{7}C_3 = 35$ different combinations of questions possible.

6) The order will not matter in how those 6 restaurants are chosen. There are ${}_{11}C_6 = 462$ ways that the manager can choose the 6 restaurants.
7) This is a committee so the order we place people on this committee will not matter. There are 12 people to select from in all. There are ${}_{12}C_4 = 495$ ways this can be done.
8) Since each raffle winner gets the same prize, the order that the winners are selected does not matter. There are ${}_{200}C_5 = 2,535,620,040$ ways to select the winners.
9) a) The order that the 2 students will be selected does not matter. There are ${}_{27}C_2 = 351$ ways to choose these two female students.
   b) The order that the 2 students will be selected does not matter. There are ${}_{19}C_2 = 171$ ways to choose these two male students.
   c) The order that 2 students does not matter. This time, however, there are 46 students to select from. There are ${}_{46}C_2 = 1,035$ ways to select 2 students. From part a), 351 of these ways are all female and from part b), 171 of these ways are all male. Therefore, there are 1,035 – 351 – 171 = 513 ways to select 2 students if one student is male and the other is female.

Problem Set 1.4 Review Exercises

10) There are 7 choices for the first kicker, 6 for the second, and so on. This gives us a total of 7! = 5,040 ways to arrange the kicking order.
11) a) There are 10 choices for each of the 4 digits or $10 \cdot 10 \cdot 10 \cdot 10 = 10^4$ = 10,000 ways that this can be done.
   b) There are 10 choices for the first digit, 9 for the second, 8 for the third, and 7 for the fourth or $10 \cdot 9 \cdot 8 \cdot 7 = 5,040$ ways this can be done.
12) S={HH, HT, TH, TT}
13) a) The order that the two students are picked does not matter because they will both be doing the same problem anyway. This does not involve permutations.
   b) The order that the child puts on their socks, shoes, and boots does make a difference so this situation does involve permutations.
   c) First of all, notice that there are 5 hours of tasks and 5 hours with which to complete the tasks. If all goes according to plan, all three of the tasks will be finished in time no matter what order the student decides to complete them. This does not involve permutations.

Problem Set 1.5 Pages 22-23

1) In a permutation problem, the order that we select our objects matters. Order does not matter in a combination problem.
2) Since we don't care which day of the week we wear the shoes, the order does not matter. There are ${}_{10}C_5 = 252$ ways to select the shoes.
3) Since we now care which day of the week we wear the shoes, the order now matters. There are ${}_{10}P_5 = 30,240$ ways to select the shoes.
4) The order will matter. A rainbow that has blue on top followed by green, purple, red, and yellow looks different than a rainbow that has green on top followed by blue, purple, red, and yellow. There are ${}_{24}P_5 = 5,100,480$ ways to draw the rainbow.
5) We are now asking how many ways we can pick the five colors. We are no longer saying which color goes where so the order now does not matter. There are ${}_{24}C_5 = 42,504$ ways to select the five colors.
6) a) Since we are only concerned with how many of these 5-card hands are possible, the order that these five cards are dealt will not matter. There are ${}_{52}C_5 = 2,598,960$ unique 5 card poker hands.

   b) This question now asks about how many different orders we could receive 5 cards. This time, we are paying attention to order. There are ${}_{52}P_5 = 311,875,200$ orders that 5 cards can be dealt.
7) The order does make a difference. In other words calling party member 'A' the prime minister and member 'B' the secretary of state is different than if 'A' is the secretary of state and 'B' is the prime minister. There are ${}_{36}P_2 = 1,260$ ways to select the two party members.
8) For all three parts of this question, we are dealing with combinations because order will not matter on a committee.

   a) ${}_{12}C_4 = 495$

   b) We need two men **and** two women from 5 men and 7 women. There are ${}_5C_2 \cdot {}_7C_2 = 10 \cdot 21 = 210$ ways to choose this committee.

   c) Since we need at least 2 women on the committee we could have either 2 women **&** 2 men **or** 3 women **&** 1 man **or** 4 women **&** no men. There are ${}_5C_2 \cdot {}_7C_2 + {}_5C_1 \cdot {}_7C_3 + {}_5C_0 \cdot {}_7C_4 = 10 \cdot 21 + 5 \cdot 35 + 1 \cdot 35 = 210 + 175 + 35 = 420$ ways to select a committee with at least 2 women.
9) The inspector will select 3 cars **&** 4 trucks. The order these vehicles are picked will not matter. There are ${}_8C_3 \cdot {}_{11}C_4 = 56 \cdot 330 = 18,480$ ways to select the vehicles.

10) There is no indication that the order that train cars are picked matters. We must pick tankers **and** boxcars **and** flatcars. There are ${}_4C_2 \cdot {}_{12}C_5 \cdot {}_7C_3 = 6 \cdot 792 \cdot 35 = 166,320$ ways to pick the cars for the train.

11) There is no indication that the order that the boxes of cereal are selected matters. We must pick two boxes of Sugar Sweet **and** two boxes of Touch O' Honey. There are ${}_{10}C_2 \cdot {}_{10}C_2 = 45 \cdot 45 = 2,025$ ways to pick the boxes of cereal.

12) The order that members of a jury are selected does not matter so we are working with a combination. With 22 people to select from, there will be ${}_{22}C_{12} = 646,646$ ways to select this jury.

13) We must select 6 women **&** 6 men for this jury. There are ${}_{10}C_6 \cdot {}_{12}C_6 = 210 \cdot 924 = 194,040$ ways to select this jury.

14) The order will matter as to who the president appoints as manager and assistant manager. The manager must pick two people for the first store **&** two people for the second store. There are ${}_9P_2 \cdot {}_7P_2 = 72 \cdot 42 = 3,024$ ways to fill these positions.

15) There is no indication that the order that the students select the teachers matters so we will use combinations. A student will need to have 2 math teachers **&** 2 social studies teachers **&** 1 reading teacher. There are ${}_9C_2 \cdot {}_{12}C_2 \cdot {}_4C_1 = 36 \cdot 66 \cdot 4 = 9,504$ ways to select these teachers.

16) One way to interpret this question is that we must select two people from 6 for the first office, **and** then we select two people from 4 for the second office, **and** finally we select two people from 2. There is no indication that being selected first or second for an office is advantageous so order does not matter for each office. The word 'and' suggests that we need to multiply. We have ${}_6C_2 \cdot {}_4C_2 \cdot {}_2C_2 = 15 \cdot 6 \cdot 1 = 90$ ways to select office arrangements for these six people.

Problem Set 1.5 Review Exercises

17) ${}_7C_3 = \dfrac{7!}{3!(7-3)!} = \dfrac{7!}{3! \cdot 4!} = \dfrac{5,040}{6 \cdot 24} = 35$

18) ${}_6P_4 = \dfrac{6!}{(6-4)!} = \dfrac{6!}{2!} = \dfrac{720}{2} = 360$

19) There are 5 choices for the 1st letter, 4 for the 2nd and so on. This gives 5! = 120 ways that the letters can be arranged.

20) The possible totals are 5, 6, 7, and 8. There are 4 outcomes possible.

21) The order the students are selected matters. There are ${}_{34}P_3 = 35,904$ ways that the three students can be selected.

Chapter 1 Review Exercises   Pages 24-26

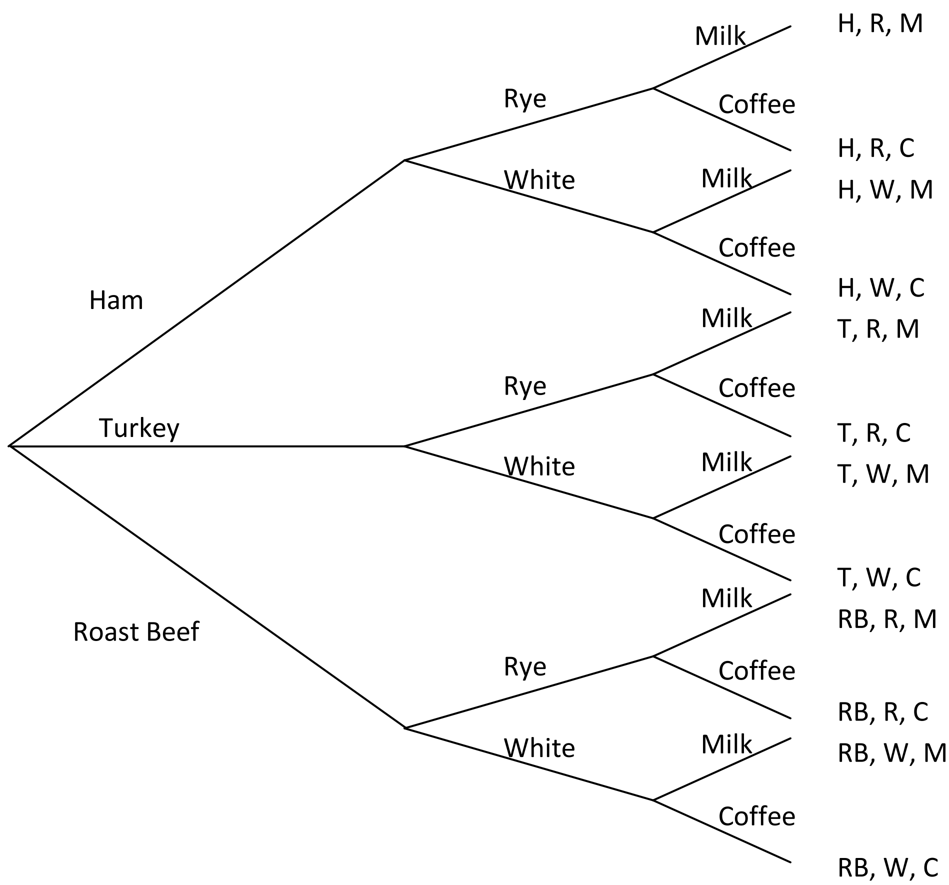
1) a)

	1	**2**	**3**	**4**	**5**
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

b) S={2, 3, 4, 5, 6, 7, 8, 9, 10}

c) There are 6 spaces that are divisible by 4 that are each worth $4 to you. There are 19 spaces that would cost you $2. One way to think of this is that there are $24 worth of spaces in your favor and $38 worth of spaces against you. If you decide to play, you would expect to lose money.

2)



3) a) $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

b) ${}_6P_3 = \dfrac{6!}{(6-3)!} = \dfrac{6!}{3!} = \dfrac{720}{6} = 120$

c) ${}_7C_5 = \dfrac{7!}{5!(7-5)!} = \dfrac{7!}{5! \cdot 2!} = \dfrac{5,040}{120 \cdot 2} = \dfrac{5,040}{240} = 21$

d) $(5-2)! = 3! = 6$

e) $4! - 2! = 24 - 2 = 22$

4) The order that the runners finish does make a difference so we have ${}_4P_4 = 24$ ways that the runners can finish the race.

5) Since the order that the 6 outfits are selected won't matter, we are dealing with a combination. There are ${}_{18}C_6 = 18,564$ ways to select the outfits.

6) $4 \cdot 3 = 12$

7) There are 26 choices for each letter and 10 choices for each digit. This gives us $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^3 \cdot 10^3 = 17,576 \cdot 1,000 = 17,576,000$ different possible license plates.

8) There are no repeats allowed in this situation so we have $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 = 11,232,000$ possible license plates.

9) The order matters in this situation. There are ${}_{20}P_3 = 6,840$ ways to select the three candidates.

10) There is no indication that the order that the photos are selected matters so we will treat this as a combination problem. We must select 2 juniors **&** 2 seniors. There are ${}_{42}C_2 \cdot {}_{45}C_2 = 861 \cdot 990 = 852,390$ ways to select the photos.

11) Using the Fundamental Counting Principle we have 7 choices for the oven, 6 choices for refrigerators, and 5 choices for dishwashers. There are $7 \cdot 6 \cdot 5 = 210$ ways to select the appliances.

12) There are 8 choices for the first scoop and 8 choices for the second or $8 \cdot 8 = 64$ ways to make the ice cream cone.

13) There are now 8 choices for the first scoop but only 7 choices for the second or $8 \cdot 7 = 56$ ways to make the ice cream cone.

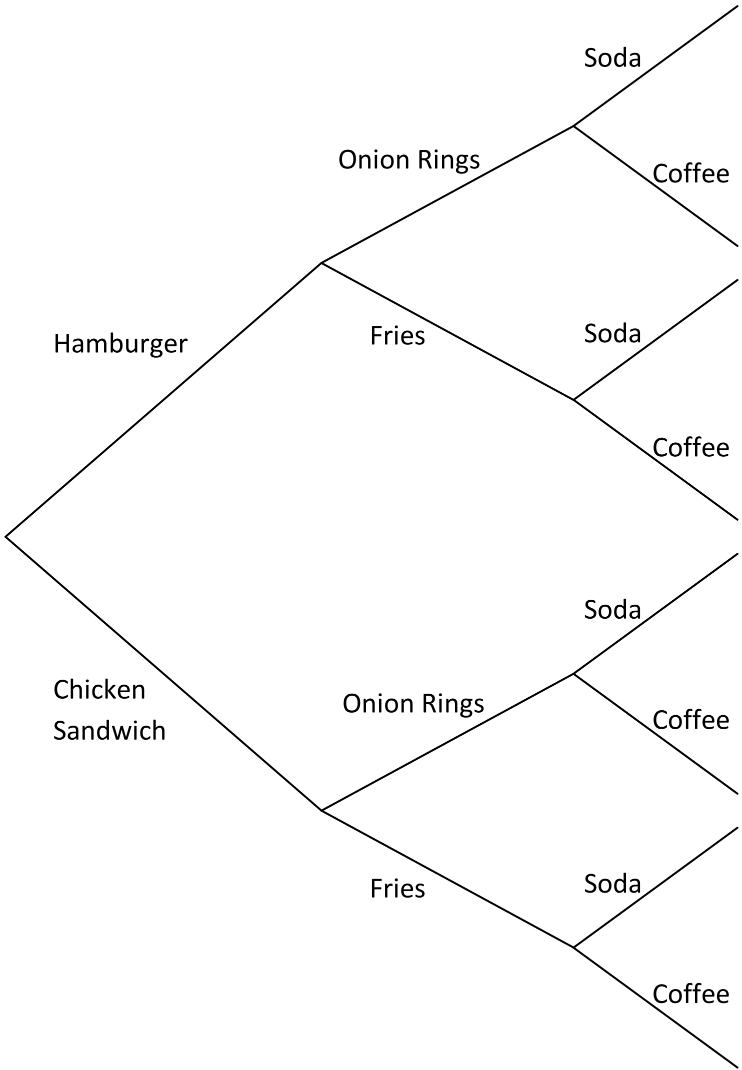
14) Order does not matter for a jury so we have ${}_{20}C_{12} = 125,970$ ways to select this jury.

15) We now must select exactly 5 women **and** 7 men. This gives us ${}_{13}C_7 \cdot {}_7C_5 = 1,716 \cdot 21 = 36,036$ ways to select this jury.

16) In this case we are allowed to have 5 women **&** 7 men **or** 6 women **&** 6 men **or** 7 women **&** 5 men. This gives ${}_7C_5 \cdot {}_{13}C_7 + {}_7C_6 \cdot {}_{13}C_6 + {}_7C_7 \cdot {}_{13}C_5 = 21 \cdot 1,716 + 7 \cdot 1,716 + 1 \cdot 1,287$. Adding this up gives 36,036+12,012+1,287=49,335.

17) $7 \cdot 5 \cdot 4 = 140$

18) a)



b) There are 8 outcomes total. There are two choices for the sandwich, two choices for the side order, and two choices for the drink. There are $2 \cdot 2 \cdot 2 = 8$ ways to order the special.