5-4 day 3 The FUNdamental Theorem of Calculus

Learning Objectives:

I can evaluate the derivative of an integral using the Fundamental Theorem of Calculus Part 1.

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Ex1. Evaluate. Check your answer on the graphing calculator

1.) $\int_{1}^{5} \left(3x^2 + 4x + 1\right) dx$

2.) $\int_{0}^{7} \frac{dx}{3x+2}$

3.) $\int_{0}^{\pi/4} \sec^2 x dx$

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The FUNdamental Theorem of Calculus Part 1

If f(x) is continuous on[a,b], then the

function $F(x)=\int_0^x f(t)dt$ has a derivative

at every point x in [a,b] and

$$\frac{dF}{dx}=\frac{d}{dx}\left(\int_0^x f(t)dt\right)=f(x)$$

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$\frac{d}{dx}\int (4x+2)dx =$ $4x+2$

Is true only if: $\frac{d}{dx}\left[\int_0^x (4t+2)dt\right] = 4x+2$

$\frac{d}{dx}\; 2t^2+2t \Big|_0^x$

$\frac{d}{dx}\left[(2x^2+2x)-(2\cdot 0^2+2\cdot 0)\right]$

$\frac{d}{dx}\left[2x^2+2x\right] = 4x+2$

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Show: $$\frac{d}{dx}\int_0^x (4t+2)dt = 4x+2$$

What happens if you change the lower limit?

$$\frac{d}{dx}\int_{2}^{x}(4t+2)\,dt = \frac{d}{dx}\left[2t^2+2t\Big|_2^x\right]$$

$$= \frac{d}{dx}\left[(2x^2+2x)-(8+4)\right]$$

$$= \frac{d}{dx}\left[2x^2+2x-12\right]$$

$$= 4x+2$$

What happens if you change the upper limit?

$$\frac{d}{dx}\int_0^{x^2}(4t+2)\,dt = \frac{d}{dx}\left[2t^2+2t\,\Big|_0^{x^2}\right]$$

$$= \frac{d}{dx}\left[\left(2(x^2)^2+2(x^2)\right)-(0)\right]$$

$$= \frac{d}{dx}\left[2x^4+2x^2\right] = 8x^3+4x$$

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Consequence of FTC Part 1

$$\frac{d}{dx}\int_a^{f(x)} g(t)\,dt = g(f(x))\cdot f'(x)$$

chain Rule

Why?

Ex2. Evaluate

1.) $\frac{d}{dx}\int_a^{3x}\left(t^2+t+1\right)dt$ ← der

$= \left[(3x)^2+(3x)+1\right]\cdot 3$

$= (9x^2+3x+1)\cdot 3 = 27x^2+9x+3$

2.) $\frac{d}{dx}\int_{x^2}^{2}\left(\sqrt{3t-4}\right)dt$

$-\frac{d}{dx}\int_2^{x^2}\sqrt{3t-4}\,dt$ $\quad -2x\sqrt{3x^2-4}$

3.) $\frac{d}{dx}\int_0^{3x^4\sec x}\frac{\sqrt{3\ln t-4e^t\sin t}}{\tan^3 t\cos^{-1} t}dt$

$\frac{\sqrt{3\ln(3x^4\sec x)-4e^{3x^4\sec x}\sin(3x^4\sec x)}}{\tan^3(3x^4\sec x)\cos^{-1}(3x^4\sec x)}\cdot\left[12x^3\sec x+3x^4\cdot\sec x\tan x\right]$

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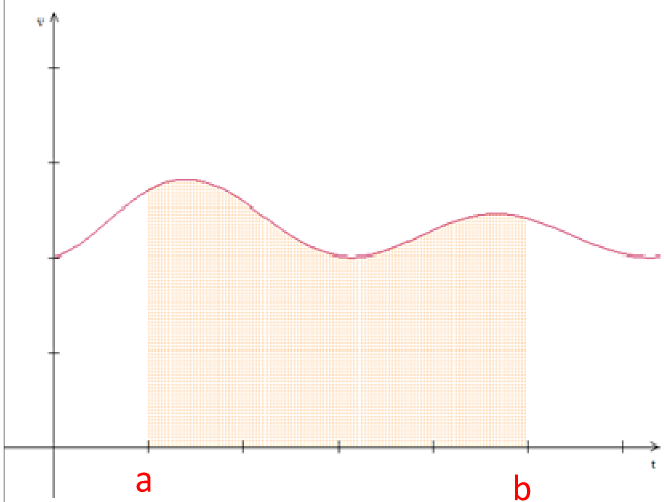
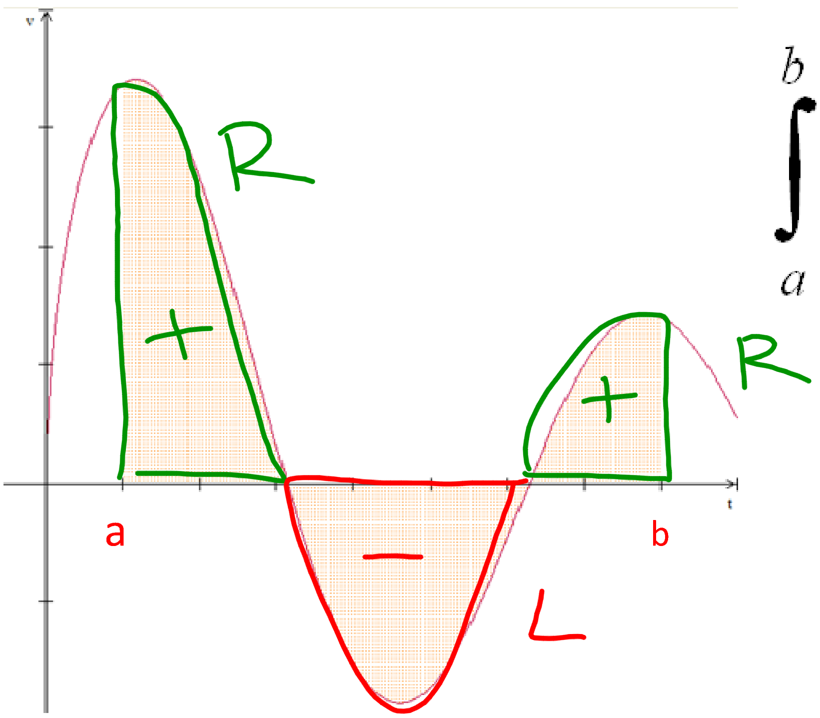


$$\int_a^b v(t)\,dt$$

Explain the meaning of this integral.



$$\int_a^b v(t)\,dt$$

Explain the meaning of this integral.

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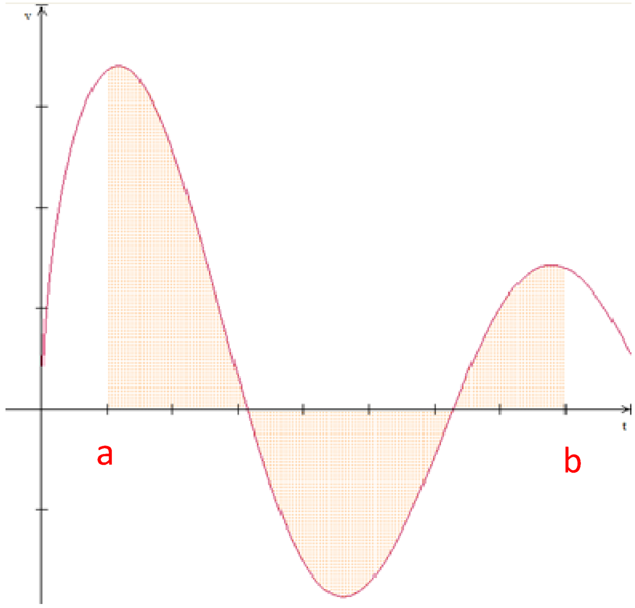
Then how would we find the total distance traveled?



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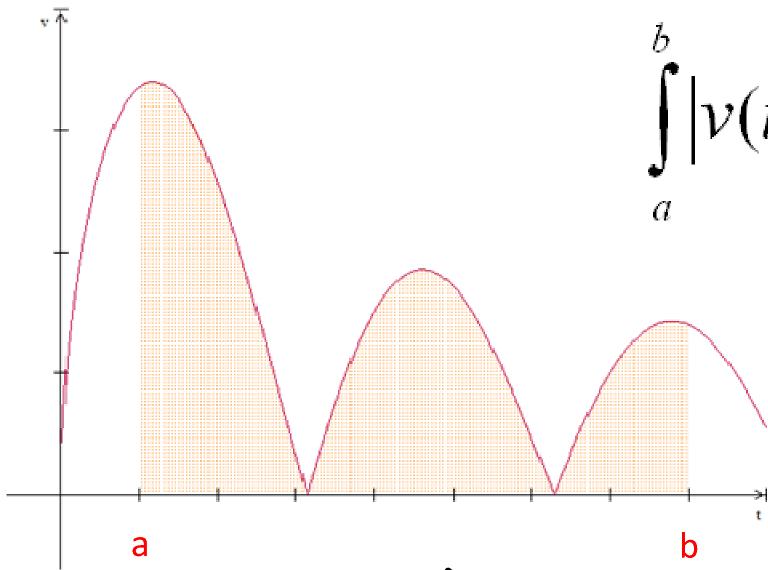

$$\int_a^b |v(t)|\,dt$$

~~$$\left|\int_a^b v(t)\,dt\right|$$~~

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Displacement $\int_a^b v(t)dt$

From time t=a to time t=b

Total Distance Traveled $\int_a^b |v(t)|dt$

From time t=a to time t=b

Homework

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