

Opener

$$\int x e^{2x} dx =$$

(A) $\frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + C$

(B) $\frac{x e^{2x}}{2} - \frac{e^{2x}}{2} + C$

(C) $\frac{x e^{2x}}{2} + \frac{e^{2x}}{4} + C$

(D) $\frac{x e^{2x}}{2} + \frac{e^{2x}}{2} + C$

(E) $\frac{x^2 e^{2x}}{4} + C$

$$\int e^{\cos x} \sin x dx =$$

(A) $-e^{\cos(x)+1} + C$

(B) $e^{\cos x} + C$

(C) $-e^{\cos x} + C$

(D) $e^{\sin x} + C$

(E) $e^{\sin x} \cos(x) + C$



6-5 day 1 Decomposing into Partial Fractions

Learning Objectives:

I can evaluate an integral using the decomposing into partial fractions technique.

I. Proper Fractions

Ex1. Evaluate

$$1.) \int \frac{x-13}{2x^2-7x+3} dx = \int \frac{x-13}{(2x-1)(x-3)} dx$$

$$\left[\frac{x-13}{(2x-1)(x-3)} = \frac{A}{2x-1} + \frac{B}{x-3} \right] (2x-1)(x-3)$$

$$x-13 = A(x-3) + B(2x-1)$$

$$x=3 \quad -10 = A(0) + B(5)$$

$$-10 = 5B \Rightarrow B = -2$$

$$x = 1/2 \quad -12\frac{1}{2} = A(-2\frac{1}{2}) + B(0)$$

$$-12\frac{1}{2} = -2\frac{1}{2}A \Rightarrow A = 5$$

$$\int \frac{x-13}{2x^2-7x+3} dx = \int \frac{5}{2x-1} + \frac{-2}{x-3} dx$$

$$= 5 \int \frac{1}{2x-1} dx - 2 \int \frac{1}{x-3} dx$$

$$= \frac{5}{2} \ln|2x-1| - 2 \ln|x-3| + C$$

$$= \ln|2x-1|^{5/2} - \ln|x-3|^2 + C$$

$$= \ln \frac{|2x-1|^{5/2}}{|x-3|^2} + C$$

$$2.) \int \frac{x+3}{x^2-4} dx$$

$$\int \frac{x+3}{x^2-4} dx = \int \frac{x+3}{(x+2)(x-2)} \left[\frac{x+3}{x^2-4} = \frac{A}{x+2} + \frac{B}{x-2} \right] (x+2)(x-2)$$

$$x+3 = A(x-2) + B(x+2)$$

$$x=2 \quad 5=4B \quad B=\frac{5}{4}$$

$$x=-2 \quad 1=-4A \quad A=-\frac{1}{4}$$

$$= \int \frac{x+3}{x^2-4} dx = \int \frac{\frac{5}{4}}{x-2} + \frac{-\frac{1}{4}}{x+2} \quad \frac{5}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| + C$$

$$3.) \int \frac{3x^2 - 3x + 18}{x(x^2 - 9)} dx = \int \frac{3x^2 - 3x + 18}{x(x+3)(x-3)} dx$$

$$\frac{3x^2 - 3x + 18}{x(x+3)(x-3)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-3}$$

$$3x^2 - 3x + 18 = A(x-3)(x+3) + Bx(x+3) + Cx(x-3)$$

$$3x^2 - 3x + 18 = A(x-3)(x+3) + Bx(x+3) + Cx(x-3)$$

let $x=0$ $18 = A(-3)(3) + B(0)(3) + C(0)(-3)$
 $\frac{18}{-9} = \frac{-9A}{-9}$
 $-2 = A$

let $x=3$ $36 = A(0)(6) + B(3)(6) + C(0)(3)$
 $\frac{36}{18} = \frac{18B}{18}$
 $2 = B$

let $x=-3$ $54 = A(-6)(0) + B(-3)(0) + C(-3)(-6)$
 $\frac{54}{18} = \frac{18C}{18}$
 $3 = C$

$$\int \left(\frac{-2}{x} + \frac{2}{x-3} + \frac{3}{x+3} \right) dx = -2 \ln|x| + 2 \ln|x-3| + 3 \ln|x+3| + C$$

OR

$$= \boxed{-2 \ln|x| + 2 \ln|x-3| + 3 \ln|x+3| + C}$$

$$= -\ln|x|^2 + \ln|x-3|^2 + \ln|x+3|^3 + C$$

$$= -\ln x^2 + \ln (x-3)^2 + \ln |x+3|^3 + C$$

$$\Rightarrow = \boxed{\ln \frac{(x-3)^2 |x+3|^3}{x^2}} + C \quad \text{tricky AP}$$

II. Improper Fractions

Ex2. Evaluate

$$1.) \int \frac{x^3}{x^2+x-2} dx$$

$$\int (x-1) + \frac{3x-2}{x^2+x-2} dx$$

$$\begin{array}{r} x-1 + \frac{3x-2}{x^2+x-2} \\ x^2+x-2 \overline{) x^3 + 0x^2 + 0x + 0} \\ \underline{-x^3 - x^2 + 2x} \\ -x^2 + 2x + 0 \\ \underline{+x^2 + x - 2} \\ 3x - 2 \end{array}$$

$$\int x-1 + \frac{3x-2}{x^2+x-2} dx \quad \frac{3x-2}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$\begin{array}{l} 3x-2 = A(x-1) + B(x+2) \\ x=1 \quad 1-3B \quad B=1/3 \\ x=-2 \quad -8=-3A \quad A=8/3 \end{array} = \frac{8/3}{x+2} + \frac{1/3}{x-1}$$

$$\int x-1 + \frac{8}{3} \frac{1}{x+2} + \frac{1}{3} \frac{1}{x-1} dx$$

$$\frac{1}{2}x^2 - x + \frac{8}{3} \ln|x+2| + \frac{1}{3} \ln|x-1| + C$$

$$\frac{1}{2}x^2 - x + \frac{\ln|x+2|^{8/3}}{\ln|x-1|^{1/3}} + C$$

$$+ \ln|(x+2)^{8/3} \cdot (x-1)^{1/3}|$$

$$\frac{1}{2}x^2 - x + \ln|\sqrt[3]{(x+2)^8(x-1)}| + C$$

$$2.) \int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8} dx$$

$$2.) \int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8}$$

$$\int 2x + \frac{x+5}{x^2-2x-8}$$

$$\begin{array}{r} 2x \quad \frac{x+5}{x^2-2x-8} \\ x^2-2x-8 \overline{) 2x^3-4x^2-15x+5} \\ \underline{2x^3+4x^2+16x} \\ x+5 \end{array}$$

$$\frac{x+5}{x^2-2x-8} = \frac{A}{(x-4)} + \frac{B}{(x+2)}$$

$$x+5 = A(x+2) + B(x-4)$$

$$-2+5 = -6B \quad 4+5 = 6A$$

$$3 = -6B \quad 9 = 6A$$

$$B = -1/2$$

$$A = 3/2$$

$$\boxed{x^2 + \frac{1}{2} \ln|x+2| + \frac{3}{2} \ln|x-4| + C}$$

But sometimes you have to be careful because these look like partial fractions problems but they are not:

1.) $\int \frac{dx}{4x^2+9}$

① $\int \frac{dx}{4x^2+9}$ this resembles $\int \frac{dx}{x^2+1} = \tan^{-1}x + C$

Let $4x^2 = 9u^2$ choose $2x = 3u \Rightarrow u = \frac{2}{3}x$
 $2dx = 3du$
 $dx = \frac{3}{2}du$

$\int \frac{\frac{3}{2}du}{9u^2+9} = \frac{\frac{3}{2}}{9} \int \frac{du}{u^2+1} = \frac{1}{6} \int \frac{du}{u^2+1}$

$= \frac{1}{6} \tan^{-1}u + C = \boxed{\frac{1}{6} \tan^{-1}\left(\frac{2}{3}x\right) + C}$

2.) $\int \frac{6x+2}{3x^2+2x+1} dx$

② $\int \frac{6x+2}{3x^2+2x+1} dx$ let $u = 3x^2+2x+1$
 $du = 6x+2 dx$

$\int \frac{du}{u} = \ln|u| + C$

$= \boxed{\ln|3x^2+2x+1| + C}$

Homework

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