

2008 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

| | | | | | |
|---------------------------------------|---|---|----|----|----|
| Distance from the river's edge (feet) | 0 | 8 | 14 | 22 | 24 |
| Depth of the water (feet) | 0 | 7 | 8 | 2 | 0 |

3. A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin(\sqrt{t+10})$ for $0 \leq t \leq 120$ minutes.
- (a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.
- (b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.
- (c) The scientist proposes the function f , given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.
- (d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \leq t \leq 60$ minutes. Does this value indicate that the water must be diverted?
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WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

2001 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

| t (days) | $W(t)$ (°C) |
|---------------|----------------|
| 0 | 20 |
| 3 | 31 |
| 6 | 28 |
| 9 | 24 |
| 12 | 22 |
| 15 | 21 |

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t . The table above shows the water temperature as recorded every 3 days over a 15-day period.
- Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.
 - Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
 - A student proposes the function P , given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.
 - Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.
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2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

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|------------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| t (minutes) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| $v(t)$ (miles per minute) | 7.0 | 9.2 | 9.5 | 7.0 | 4.5 | 2.4 | 2.4 | 4.3 | 7.3 |

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t . Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table above.
- (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t) dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t) dt$ in terms of the plane's flight.
- (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.
- (c) The function f , defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3 \sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.
- (d) According to the model f , given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \leq t \leq 40$?

END OF PART A OF SECTION II

2004 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B

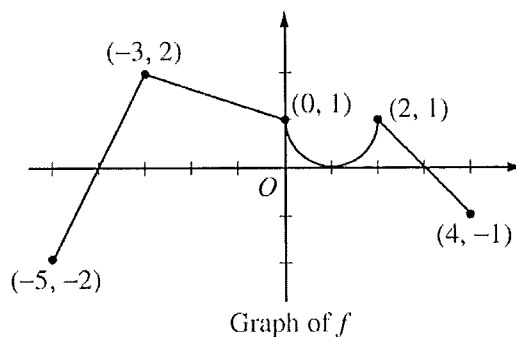
Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

4. Consider the curve given by $x^2 + 4y^2 = 7 + 3xy$.

- (a) Show that $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$.
- (b) Show that there is a point P with x -coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y -coordinate of P .
- (c) Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at the point P ? Justify your answer.
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5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^x f(t) dt$.
- (a) Find $g(0)$ and $g'(0)$.
- (b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.
- (c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.
- (d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.
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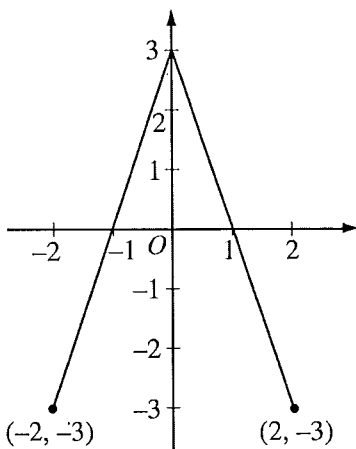
2002 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB SECTION II, Part B

Time—45 minutes

Number of problems—3

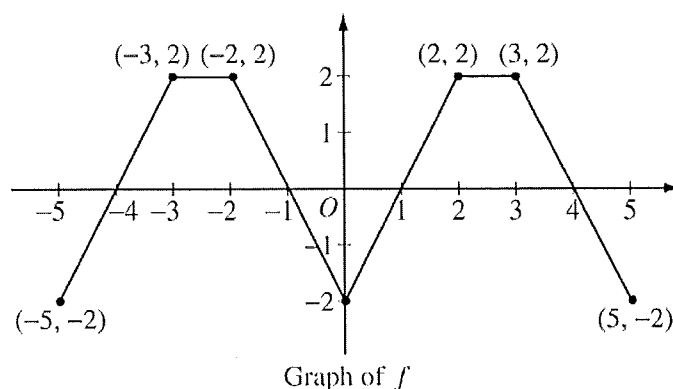
No calculator is allowed for these problems.



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t) dt$.
- (a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.
 - (b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.
 - (c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.
 - (d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
(Note: The axes are provided in the pink test booklet only.)

2006 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



3. The graph of the function f shown above consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t) dt$.
- (a) Find $g(4)$, $g'(4)$, and $g''(4)$.
- (b) Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.
- (c) Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph above shows two periods of f . Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at $x = 108$.
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF PART A OF SECTION II

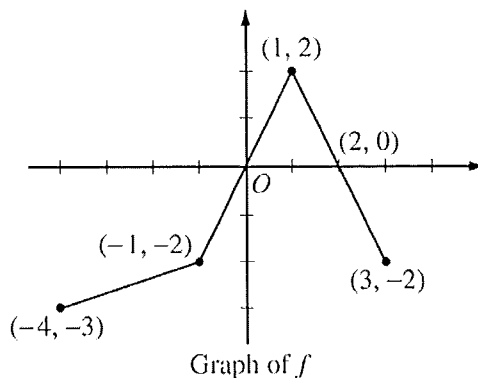
2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

**CALCULUS AB
SECTION II, Part B**

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



4. The graph of the function f above consists of three line segments.

- (a) Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.
- (b) For the function g defined in part (a), find the x -coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.
- (c) Let h be the function given by $h(x) = \int_x^3 f(t) dt$. Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.
- (d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

WRITE ALL WORK IN THE TEST BOOKLET.

1999 CALCULUS AB

| t (hours) | $R(t)$ (gallons per hour) |
|----------------|------------------------------|
| 0 | 9.6 |
| 3 | 10.4 |
| 6 | 10.8 |
| 9 | 11.2 |
| 12 | 11.4 |
| 15 | 11.3 |
| 18 | 10.7 |
| 21 | 10.2 |
| 24 | 9.6 |

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table above shows the rate as measured every 3 hours for a 24-hour period.
- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$. Using correct units, explain the meaning of your answer in terms of water flow.
- (b) Is there some time t , $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.
- (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$.
Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period.
Indicate units of measure.
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