CW: 4.2.1 Name: ________________________

Exploring Rules of Exponents: How can you use patterns to discover rules for multiplying with exponents?

Product of Powers Property:

Expression	Expanded form	# of factors	Product as a power
$2^4 \bullet 2^3$	$2\cdot2\cdot2\cdot2 \bullet 2\cdot2\cdot2$		2^7
$5^3 \bullet 5^5$	$5\cdot5\cdot5 \bullet 5\cdot5\cdot5\cdot5\cdot5$		5^8
$a^3 \bullet a^6$	$a\cdot a\cdot a \bullet a\cdot a\cdot a\cdot a\cdot a\cdot a$		a^9

To multiply powers having the same base, add the exponents.

$a^m \cdot a^n = a^{m+n}$ $\quad 3^2 \cdot 3^7 = 3^{2+7} = 3^9$

Examples:

1. $4^5 \cdot 4^3$ — 4^8
2. $y^3 \cdot y^4 \cdot y^5$ — $y\cdot y\cdot y \bullet y\cdot y\cdot y\cdot y \bullet y\cdot y\cdot y\cdot y\cdot y$ — $\boxed{y^{12}}$
3. $2^1 \cdot 2^6$ — $2 \bullet 2\cdot2\cdot2\cdot2\cdot2\cdot2$ — $\boxed{2^7}$
4. $(-5)^1(-5)^3$ — $(-5)\bullet(-5)(-5)(-5)$ — $(-5)^4$
5. $3^5 \cdot 3^2$ — 3^7
6. $z \cdot z^2 \cdot z^4$ — z^{7} ($1+2+4$)
7. $5^2 \cdot 5^6$ — 5^8
8. $(-3)^2 \cdot (-3)^5$ — $(-3)^7$

Exploring Rules of Exponents: How can you use patterns to discover rules for multiplying with exponents?

Power of a Power Property:

Expression	Expanded form	# of factors	Product as a power
$(4^2)^3$	$4^2 \cdot 4^2 \cdot 4^2$ $= 4\cdot4 \bullet 4\cdot4 \bullet 4\cdot4$		4^6
$(2^3)^5$	$2^3\cdot2^3\cdot2^3\cdot2^3\cdot2^3$ $= 2\cdot2\cdot2 \bullet 2\cdot2\cdot2 \bullet 2\cdot2\cdot2 \bullet 2\cdot2\cdot2 \bullet 2\cdot2\cdot2$		2^{15}
$(x^3)^4$	$x^3\cdot x^3\cdot x^3\cdot x^3$ $x\cdot x\cdot x \bullet x\cdot x\cdot x \bullet x\cdot x\cdot x \bullet x\cdot x\cdot x$		x^{12}

To find a power of a power, multiply the exponents

$(a^m)^n = a^{m\cdot n}$ $\quad (5^2)^4 = 5^{2\cdot4} = 5^8$

Examples:

1. $(5^2)^3$ — 5^6
2. $(x^3)^2$ — $x^3\cdot x^3$ — x^6
3. $[(-2)^3]^4$ — $(-2)^{12}$
4. $[(a-2)^3]^2$ — $(a-2)^6$
5. $(4^2)^4$ — 4^8
6. $(y^3)^5$ — y^{15}
7. $[(-4)^3]^5$ — $(-4)^{15}$
8. $[(y+2)^4]^2$ — $(y+2)^8$

Further exploration:

a. Is there a difference between -2^2 and $(-2)^2$?

$= -4$ $\quad$ $-(2)(2)$ $\qquad$ $= 4$ $\quad$ $(-2)(-2)$

b. Is there a difference between -2^3 and $(-2)^3$?

$-(2)(2)(2)$ $\quad$ $= -8$ $\qquad$ $= -8$ $\quad$ $(-2)(-2)(-2)$

What conclusions can you draw from a and b above?

Evaluating expressions. Simplify the expression.
Then, evaluate the expression when a=1 and b = 2.

a. $b \bullet b^4$

b^5

$2^5 = 32$

b. $a^2 \bullet a^3$

a^5

$1^5 = 1$

c. $(-a)^3 \bullet b^2$

$(-1)^3 \cdot 2^2$

$-1 \cdot 4 = -4$