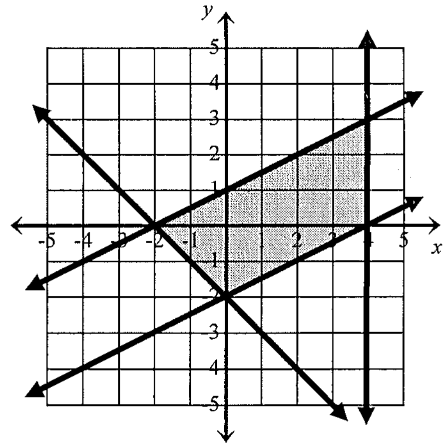
**1)** Given the graph of the constraints and the objective function, determine the vertices of the feasible region, the values of the objective function and the maximum and minimum values.

**a)** $C = 2x + 3y$



**i)** Find and list the vertices of the feasible region.

(-2,0) (0,-2) (4,0) (4,3)

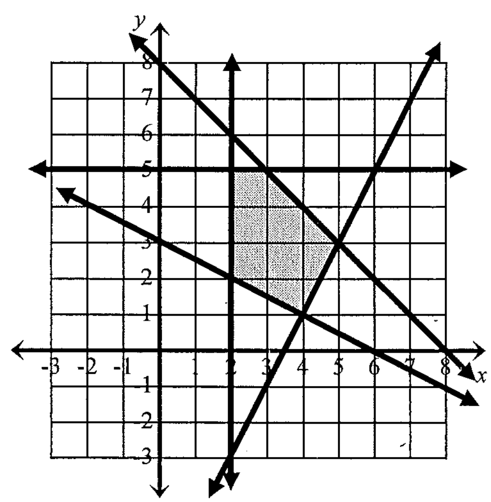
**ii)** Test each one of the vertices to determine the maximum and minimum values of the objective function.

| Ordered pair | Calculations | Value |
|---|---|---|
| (-2,0) | c = 2(-2) + 3(0) | -4 |
| (0,-2) | c = 2(0) + 3(-2) | -6 |
| (4,0) | c = 2(4) + 3(0) | 8 |
| (4,3) | c = 2(4) + 3(3) | 17 |

**iii)** Summarize your findings by identifying the maximum and minimum values of the objective function and the ordered pair that created each significant value.

maximum is 17 at (4,3)
minimum is -6 at (0,-2)

**b)** $C = 3x + 6y$



**i)** Find and list the vertices of the feasible region.

(2,2) (2,5) (4,1) (5,3) (3,5)

**ii)** Test each one of the vertices to determine the maximum and minimum values of the objective function.

| Ordered pair | Calculations | Value |
|---|---|---|
| (2,2) | c = 3(2) + 6(2) | 18 |
| (2,5) | c = 3(2) + 6(5) | 36 |
| (4,1) | c = 3(4) + 6(1) | 18 |
| (5,3) | c = 3(5) + 6(3) | 33 |
| (3,5) | c = 3(3) + 6(5) | 39 |

**iii)** Summarize your findings by identifying the maximum and minimum values of the objective function and the ordered pair that created each significant value.

maximum is 39 at (3,5) and minimum is 18 at (2,2)
this answer is wrong in the back of the book

**2)** Given the following objective function and the vertices of the feasible region, determine the maximum and minimum values.

Objective function: $C = 3x + y$

Vertices: (3, 0), (4, 5), (–1, 6), (–7, 5)

$C = 3(3) + 0 = 9$
$C = 3(4) + 5 = 17$
$C = 3(-1) + 6 = 3$
$C = 3(-7) + 5 = -16$

max. is 17 at (4, 5) min. is -16 at (-7, 5)

**3)** Given the constraints and the objective function:

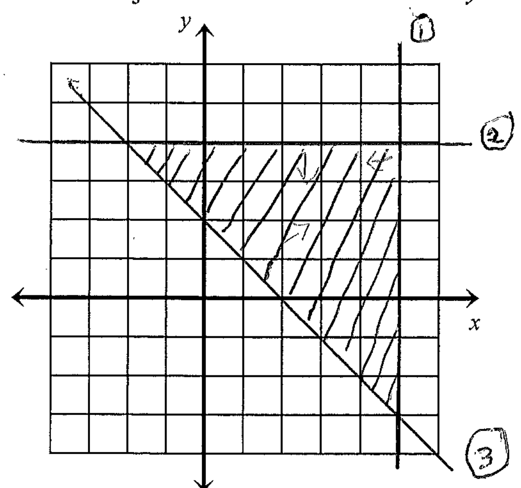
**i)** Graph the constraints.

**ii)** Find the vertices of the feasible region.

**iii)** Summarize your findings by identifying the maximum and minimum values of the objective function and the ordered pair that created each significant value.

**a)**
$$\begin{cases} x \le 5 & (1) \\ y \le 4 & (2) \\ x + y \ge 2 & (3) \end{cases}$$
$y \ge -x + 2$

Objective function: $C = 3x - 2y$



vertices:
(-2, 4) (5, 4) (5, -3)
$C = 3(-2) - 2(4) = -14$
$C = 3(5) - 2(4) = 7$
$C = 3(5) - 2(-3) = 21$
max is 21 at (5, -3)
min is -14 at (-2, 4)

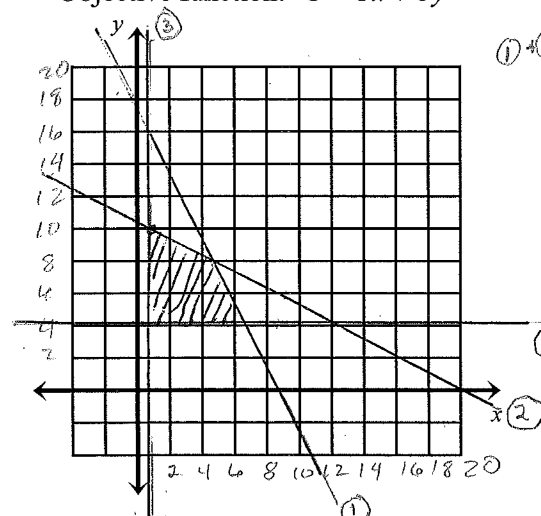
**b)**
$$\begin{cases} 2x + y \le 18 & (1) \\ x + 2y \le \cancel{12}\ 21 & (2) \\ x \ge 1 & (3) \\ y \ge 4 & (4) \end{cases}$$

(1) $y \le -2x + 18$
$2y \le -x + 21$
(2) $y \le -\frac{1}{2}x + 10.5$

| x | y |
|---|---|
| 1 | 10 |
| 9 | 6 |
| 5 | 8 |

Objective function: $C = 3x + 6y$

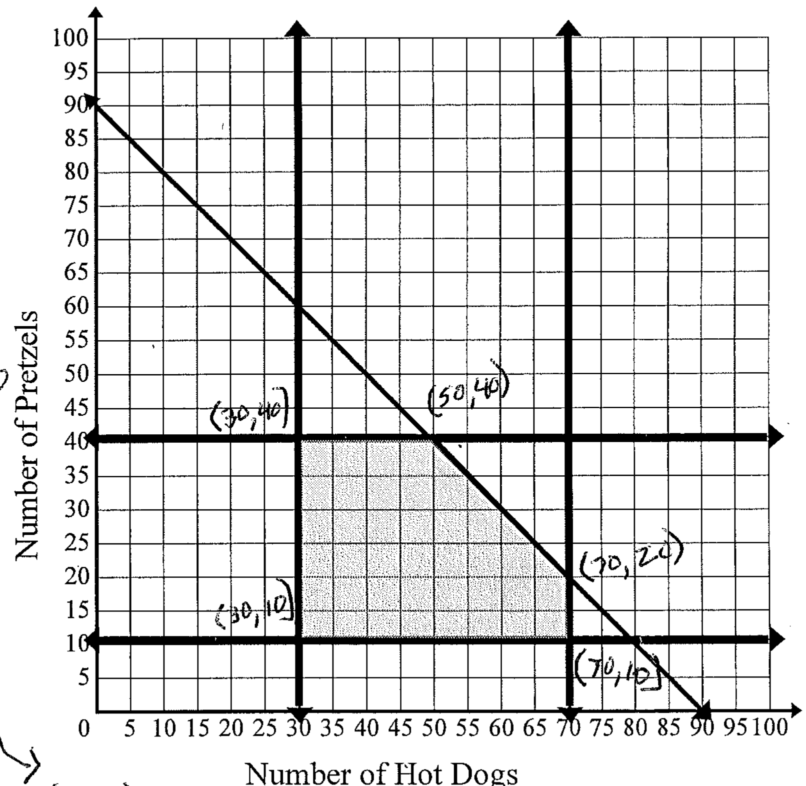
(1)+(4)
$2x + 4 = 18$
$2x = 14$
$x = 7$
(7, 4)



vertices:
(1, 4) (1, 10) (5, 8) (7, 4)
$C(1,4) = 3(1) + 6(4) = 27$
$C(1,10) = 3(1) + 6(10) = 63$
$C(5,8) = 3(5) + 6(8) = 63$
$C(7,4) = 3(7) + 6(4) = 45$

**4)** In **section 1.3B** we looked at the selling patterns of a local street vendor sells hotdogs and pretzels. To make a profit, the street vendor must sell at least 30 hotdogs but cannot prepare more than 70. The street vendor must also sell at least 10 pretzels but cannot prepare more than 40. The street vendor cannot prepare more than a total of 90 hotdogs and pretzels altogether.

**a)** Find the vertices of the feasible region and label them on the graph.




**b)** The profit is \$0.48 on a hotdog and \$0.25 on a pretzel. Write an objective function.

Profit = .48h + .25p

**c)** What combination of hotdogs and pretzels would maximize her profits?

P(30,40) = .48(30) + .25(40) = 24.40
P(30,10) = .48(30) + .25(10) = 16.90
P(50,40) = .48(50) + .25(40) = 34.00
P(70,20) = .48(70) + .25(20) = 38.60

max 70 hotdogs 20 pretzels for \$38.60 profit

P(70,10) = .48(70) + .25(10) = 36.10

**d)** She realized that pretzels were better sellers than hotdogs and wanted to increase her profits. She changed the profit of a hotdog to \$0.25 and the profit of a pretzel is now \$0.48. Write the revised objective function. What combination of hot dogs and pretzel sales gives the best profit now? Show the calculations and write a summary statement.

Profit = .25h + .48p

P(30,40) = .25(30) + .48(40) = 26.70
P(30,10) = .25(30) + .48(10) = 10.00
P(50,40) = .25(50) + .48(40) = 17.30
P(70,20) = .25(70) + .48(20) = 27.10
P(70,10) = .25(70) + .48(10) = 22.30

max is 30 hotdogs, 40 pretzels for profit of \$26.70

**e)** She decided that it would be easier if the sale of every product created a profit of \$0.40. Write an objective function for this situation. What combination gives the best profit now? Show the calculations and write a summary statement. Profit = .40h + .40p

P(30,40) = .40(30) + .40(40) = 28.00
P(30,10) = .40(30) + .40(10) = 16.00
P(50,40) = .40(50) + .40(40) = 36.00
P(70,20) = .40(70) + .40(20) = 36
P(70,10) = .40(70) + .40(10) = 32

Either 70 hotdogs and 20 pretzels or 50 hotdogs and 40 pretzels gives a profit of \$36

**f)** Taking into consideration all factors, what profit scenario would you recommend that the street vendor function with? Scenario C) gives the maximum profit of \$38.60 with 70 hotdogs at \$0.48 profit and 20 pretzels at \$0.25 profit

# 1.3E *Solving Linear Programming Problems*

**5)** Use Linear Programming to solve the problem.

A bakery is making whole-wheat bread and apple bran muffins. For each batch of bread they make \$35 profit. For each batch of muffins they make \$10 profit. The bread takes 4 hours to prepare and 1 hour to bake. The muffins take 2 hours to prepare and 2 hours to bake. The maximum preparation time available is 16 hours. The maximum baking time available is 10 hours. How many batches of bread and muffins should be made to maximize profits?

**a)** Define the variables $x =$ batches of bread $y =$ batches of muffins

**b)** Write the constraints as a system of inequalities. (Hint: There are 4 restrictions)

| | x | y | total |
|---|---|---|---|
| prep | 4 | 2 | 16 |
| bake | 1 | 2 | 10 |
| Profit | 35 | 10 | |

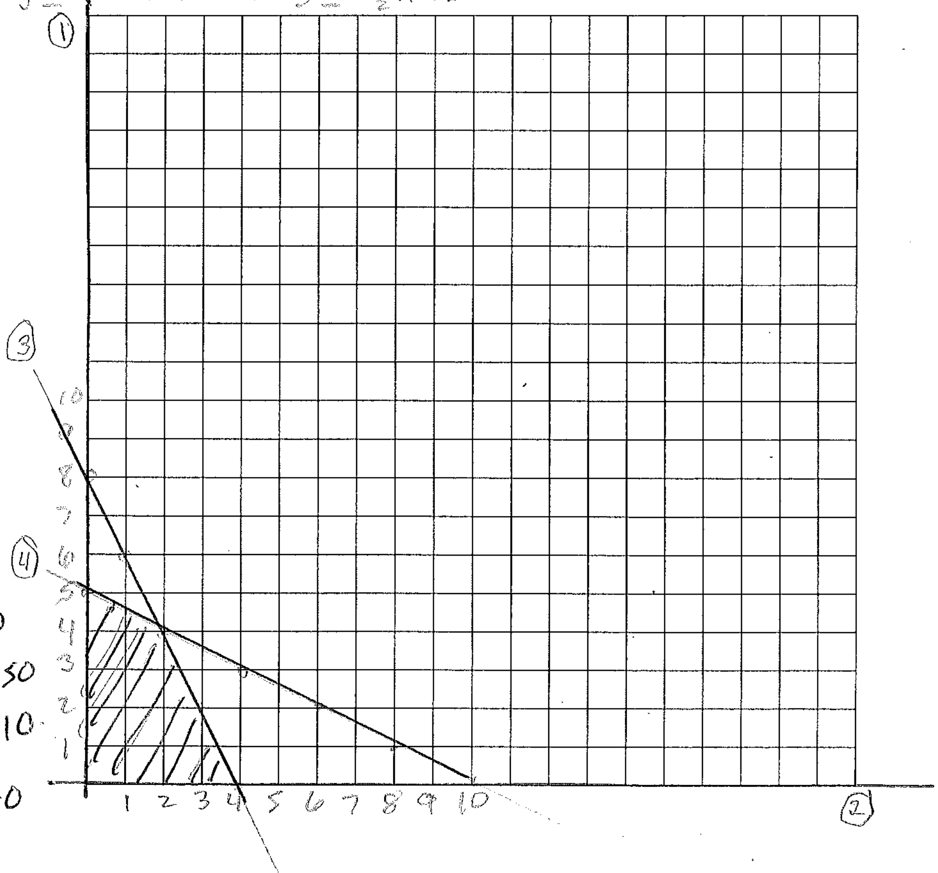
① $x \geq 0$

② $y \geq 0$

③ $4x + 2y \leq 16$ $\quad 2y \leq -4x + 16 \rightarrow y \leq -2x + 8$

④ $x + 2y \leq 10$ $\quad 2y \leq -x + 10 \rightarrow y \leq -\frac{1}{2}x + 5$

**c)** Graph the system of inequalities.



**d)** What are the vertices of the feasible region?

$(0,0)$ $(0,5)$

$(2,4)$ $(4,0)$

**e)** What is the profit for each of these combinations?

$P(0,0) = 35(0) + 10(0) = 0$

$P(0,5) = 35(0) + 10(5) = 50$

$P(2,4) = 35(2) + 10(4) = 110$

$P(4,0) = 35(4) + 10(0) = 140$

**f)** Summarize your findings to answer the question presented.

The bakery should bake 4 batches of bread and no muffins for a profit of \$140.


**6)** Larry is starting a yard care business this summer featuring lawn mowing Services or $25 per lawn and yard cleanup (trimming/raking) services for $40 per yard. He is on the summer baseball team so he can only work 24 hours a week. It takes him 1½ hours to mow a lawn and 3 hours to trim and rake a yard. Larry is borrowing the equipment from his uncle who is charging him $2 every time he uses the lawnmower and $1 every time he uses the weed wacker for trimming. He doesn't want to spend more than $20 each week on equipment.

He charges

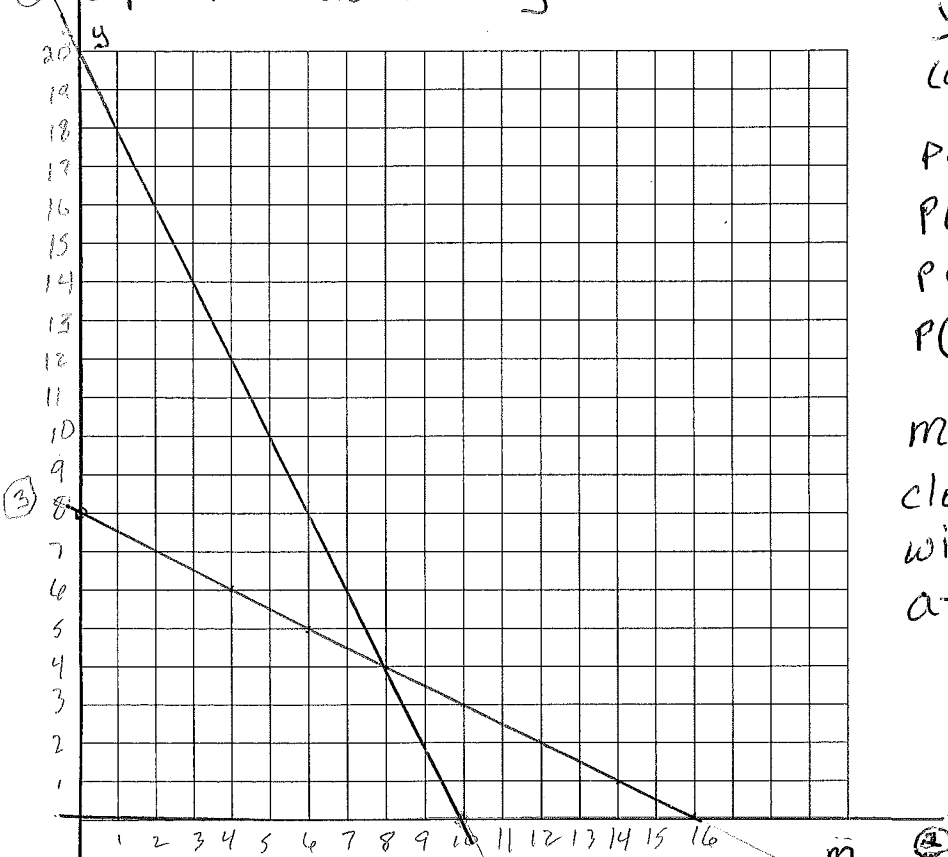
Larry wants to make as much money as possible on his new business. He has talked to his neighbors and is pretty confident that he will have plenty of customers. What combination of mowing lawns and yard trimming will give him the most profit?

| | m | y | total |
|---|---|---|---|
| hours | 1.5 | 3 | 24 |
| rent | 2 | 1 | 20 |
| Price | 25 | 40 | |

m = lawns mowed
y = yard clean up jobs

① $m \geq 0$
② $y \geq 0$
③ $1.5m + 3y \leq 24 \rightarrow 3y \leq -1.5m + 24 \rightarrow y \leq -0.5m + 8$
④ $2m + y \leq 20 \rightarrow y \leq -2m + 20$

profit $= 25m + 40y$




vertices
(0,0) (0,8) (10,0) (8,4)

$P(0,0) = 25(0) + 40(0) = 0$
$P(0,8) = 25(0) + 40(8) = 320$
$P(10,0) = 25(10) + 40(0) = 250$
$P(8,4) = 25(8) + 40(4) = 360$

mowing 8 lawns and cleaning up 4 yards will maximize his profits at $360.

7) The manager of a travel agency is printing brochures and fliers to advertise special discounts on vacation spots during the winter months. Each brochure costs \$0.08 to print, and each flier costs \$0.04 to print. A brochure requires 3 pages, and a flier requires 2 pages. The manager does not want to use more than 600 pages, and she needs at least 50 brochures and 150 fliers. How many of each should she print to minimize the cost?

b = # of brochures printed
f = # of fliers printed

| | b | f | total |
|---|---|---|---|
| #print | 50 | 150 | $b \geq 50$, $f \geq 150$ |
| pages | 3 | 2 | 600 |
| cost | .08 | .04 | |

① $b \geq 50$

② $f \geq 150$

③ $3b + 2f \leq 600 \rightarrow 2f \leq -3b + 600$

$f \leq -\frac{3}{2}b + 300$

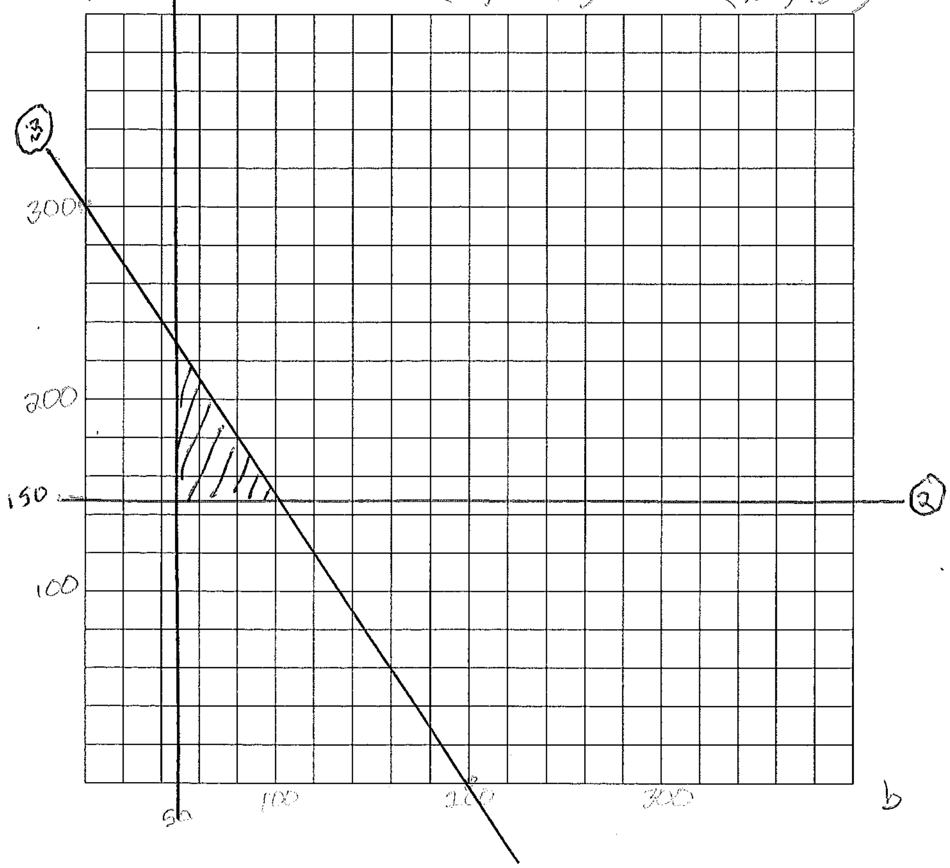
| b | f |
|---|---|
| 0 | 300 |
| 200 | 0 |

cost $= .08b + .04f$

① & ③
$3(50) + 2f = 600$
$f = 225$
$(50, 225)$

② & ③
$3b + 2(150) = 600$
$b = 100$
$(100, 150)$

f ①




Vertices:
$(50, 150)$ $(50, 225)$
$(100, 150)$

cost $(50, 150) = .08(50) + .04(150)$
$= 10$

cost $(50, 225) = .08(50) + .04(225)$
$= 13$

cost $(100, 150) = .08(100) + .04(150)$
$= 14$

The manager will minimize printing costs if she prints 50 brochures and 150 fliers for \$10.

Section 1.3E