The variables have been defined, the objective and constraints have been written. Now, you must finish each of the linear programming problems below. Start by graphing the constraints, shading, and testing vertices.

1) You need to buy some filing cabinets. You know that Cabinet X costs $10 per unit, requires six square feet of floor space, and holds eight cubic feet of files. Cabinet Y costs $20 per unit, requires eight square feet of floor space, and holds twelve cubic feet of files. You have been given $140 for this purchase, though you don't have to spend that much. The office has room for no more than 72 square feet of cabinets. How many of which model should you buy, in order to maximize storage volume?

Variables:
- X is the number of Cabinet X
- Y is the number of Cabinet Y

Objective:
\[ V = 8X + 12Y \]

Constraints:
\[ 10X + 20Y \leq 140 \]
\[ 6X + 8Y \leq 72 \]
2) A snack bar cooks and sells hamburgers and hot dogs during football games. To stay in business, it must sell at least 10 hamburgers but can not cook more than 40. It must also sell at least 30 hot dogs, but can not cook more than 70. The snack bar can not cook more than 90 items total. The profit on a hamburger is 33 cents, and the profit on a hot dog is 21 cents. How many of each item should it sell to make the maximum profit?

Variables:
H is the number of hamburgers
D is the number of hot dogs

Objective:
\[ P = 0.33H + 0.21D \]

Constraints:
\[ H \geq 10, \ H \leq 40 \]
\[ D \geq 30, \ D \leq 70 \]
\[ H + D \leq 90 \]
3) In order to ensure optimal health (and thus accurate test results), a lab technician needs to feed the rabbits a daily diet containing a minimum of 24 grams (g) of fat, 36 g of carbohydrates, and 4 g of protein. But the rabbits should be fed no more than five ounces of food a day. Rather than order rabbit food that is custom blended, it is cheaper to order Food X and Food Y, and blend them for an optimal mix. Food X contains 8 g of fat, 12 g of carbohydrates, and 2 g of protein per ounce, and costs $0.20 per ounce. Food Y contains 12 g of fat, 12 g of carbohydrates, and 1 g of protein per ounce, at a cost of $0.30 per ounce. What is the optimal blend of Food X and Food Y?

Variables:
- X is the number of ounces of Food X
- Y is the number of ounces of Food Y

Objective:
- \[ C = 0.2X + 0.3Y \]

Constraints:
- \[ 8X + 12Y \geq 24 \]
- \[ 12X + 12Y \geq 36 \]
- \[ 2X + Y \geq 4 \]
- \[ X + Y \leq 5 \]
4) You are about to take a test that contains questions of type A worth 4 points and type B worth 7 points. You must answer at least 4 of type A and 3 of type B, but time restricts answering more than 10 of either type. In total, you can answer no more than 18. How many of each type of question must you answer, assuming all of your answers are correct, to maximize your score? What is your maximum score?

Variables:
A is the number of type A questions
B is the number of type B questions

Objective:
\[ S = 4A + 7B \]

Constraints:
\[ A \geq 4, \ A \leq 10 \]
\[ B \geq 3, \ B \leq 10 \]
\[ A + B \leq 18 \]
5) Marcus is creating a low-fat pie crust recipe for his pie shop. Butter has six grams of saturated fat and one gram of unsaturated fat per tablespoon. Vegetable shortening has one gram of saturated fat and four grams of unsaturated fat per tablespoon. In the recipe, the butter and vegetable shortening will not be more than 25 tablespoons. The butter and vegetable shortening combine for at least 34 grams of saturated fat and at least 44 grams of polyunsaturated fat. Minimize the number of calories in the recipe if butter has 100 calories per tablespoon and vegetable shortening has 115 calories per tablespoon.

Variables:
- B is the number of tablespoons of Butter
- V is the number of tablespoons of Vegetable Shortening

Objective:
\[ C = 100B + 115V \]

Constraints:
\[ B + V \leq 25 \]
\[ 6B + V \geq 34 \]
\[ B + 4V \geq 44 \]
Answers to Target 1.3 - Section 2 Skill WS - Linear Programming

1) \((8, 3)\) is the solution, which means 8 Cabinet X's and 3 Cabinet Y's for a maximum volume of \(100 \text{ ft}^3\).

2) \((40, 50)\) is the solution, which means that 40 hamburgers and 50 hot dogs should be sold to make the maximum profit of $23.70.

3) \((3, 0)\) is the solution, which means 3 ounces of Food X and 0 ounces of Food Y for a minimum cost of $.60.
4) \((8, 10)\) is the solution, which means you should choose to answer 8 questions of type A and 10 questions of type B for a maximum score of 102 points.

5) \((4, 10)\) is the solution, which means that 4 tablespoons of Butter and 10 tablespoons of Vegetable Shortening should be used for a minimum of 1550 calories.