Key

IB Math SL
Chapter 6 Review for Test

Name Key

1. Find the sum of the arithmetic series

$$17 + 27 + 37 + \ldots + 417.$$

$417 = 17 + (n-1)10$

$41 = n$

$S_{41} = \frac{41}{2}(2 \cdot 17 + (41-1)10)$

$= 8897$

2. In an arithmetic sequence, the fifth term is 5 and the tenth term is 40. Find the second term.

$\frac{40-5}{10-5} = \frac{35}{5} = 7 = d$

$-23 + 7 = -16 = u_2$

$5 = u_1 + (5-1)(7)$

$u_1 = -23$

3. In an arithmetic sequence, the first three terms are x+5, 3x + 8, 4x+14. Find the value of x.

$(4x+14) - (3x+8) = (3x+8) - (x+5)$

$x + 6 = 2x + 3$

$x = 3$

4. In a geometric sequence, the 5th term is 8 and the 9th term is 40.5. Find the possible value(s) for the common ratio.

$40.5 = u_1 r^{9-1}$ $\quad 8 = u_1 r^{5-1}$

$40.5 = u_1 r^8$ $\quad 8 = u_1 r^4$

$40.5 = \left(\frac{8}{r^4}\right) r^8 \leftarrow u_1 = \frac{8}{r^4}$

$40.5 = 8r^4$ $\quad r^4 = 5.0625$

$r = \pm 1.5$

5. Find the sum of the infinite geometric series $\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \frac{16}{81} + \ldots$

$\frac{-\frac{4}{9}}{\frac{2}{3}}$ $\quad -\frac{4}{9} \cdot \frac{3}{2} = -\frac{12}{18} = -\frac{2}{3}$

$S_\infty = \frac{\frac{2}{3}}{1 - (-\frac{2}{3})} = \frac{2}{5} = 0.4$

**6.** Evaluate the expression:

$$\sum_{i=5}^{8} 2(-x)^i$$

$2(-x)^5 + 2(-x)^6 + 2(-x)^7 + 2(-x)^8$

$-2x^5 + 2x^6 - 2x^7 + 2x^8$

**7.** The diagrams below show the first three squares in a sequence of squares in which a third of one row is split into three more rows and the top one is shaded. The area A of the shaded region in figure 1 is $\frac{1}{3}$.

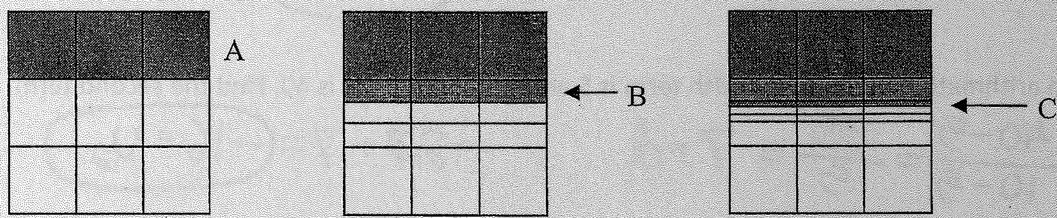


Figure 1 Figure 2 Figure 3

(a) (i) Find the area of regions B and C. (These are not the total areas, just the newly added shaded regions.)

$B = \frac{1}{9}, \ C = \frac{1}{27}$

(ii) Show that the areas of regions A, B and C are in geometric progression.

$\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

$\times 1/3 \quad \times \frac{1}{3}$

(iii) Write down the common ratio of the progression.

$\frac{1}{3}$

(b) (i) Find the **total** area shaded in figure 2.

$\frac{1}{3} + \frac{1}{9} = \frac{4}{9}$

(ii) Find the **total** area shaded in the $8^{th}$ figure of this sequence. Give your answer correct to six significant figures.

$S_8 = \dfrac{\frac{1}{3}\left(\left(\frac{1}{3}\right)^8 - 1\right)}{\frac{1}{3} - 1} = 0.499924$

(c) The dividing and shading process illustrated is continued indefinitely. Find the total area shaded.

$S_\infty = \dfrac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$

**8.** Ashley and Billie are swimmers training for a competition.

(a) Ashley trains for 12 hours in the first week. She decides to increase the amount of time she spends training by 2 hours each week. Find the total number of hours she spends training during the first 15 weeks.

$S_{15} = \frac{15}{2}[2 \cdot 12 + 14 \cdot 2] = 390$ hrs

(b) Billie also trains for 12 hours in the first week. She decides to train for 10% longer each week than the previous week.

(i) Show that in the third week she trains for 14.52 hours.

$U_3 = 12(1.1)^{3-1} = 14.52$

(ii) Find the total number of hours she spends training during the first 15 weeks.

$S_{15} = \frac{12(1-1.1^{15})}{1-1.1} = 381$ hrs

(c) In which week will the time Billie spends training first exceed 50 hours?

$U_n = 50$

$12(1.1)^{n-1} > 50$

$n = 15.97$, so 16 weeks

**9.** The *Acme* insurance company sells two savings plans, Plan A and Plan B. For Plan A, an investor starts with an initial deposit of \$1000 and increases this by \$80 each month, so that in the second month, the deposit is \$1080, the next month it is \$1160 and so on.

For Plan B, the investor again starts with \$1000 and each month deposits 6% more than the previous month.

(a) Write down the amount of money invested under Plan B in the second and third months.

\$1060, \$1123.60

*Give your answers to parts (b) and (c) correct to the nearest dollar.*

(b) Find the amount of the 12th deposit for each Plan.

A $U_{12} = 1000 + (12-1)(80) = \$1880$  B $U_{12} = 1000(1.06)^{12-1} = \$1898$

(c) Find the total amount of money invested during the first 12 months

(i) under Plan A;

$S_{12} = \frac{12}{2}[2000 + (12-1)(80)] = \$17{,}280$

(ii) under Plan B.

$S_{12} = \frac{1000(1.06^{12}-1)}{1.06-1} = \$16{,}870$