Name ______________________________ Period __________

5.1A *Solving Quadratic Equations by Graphing*

1. What does "find the zeros of the function" mean?

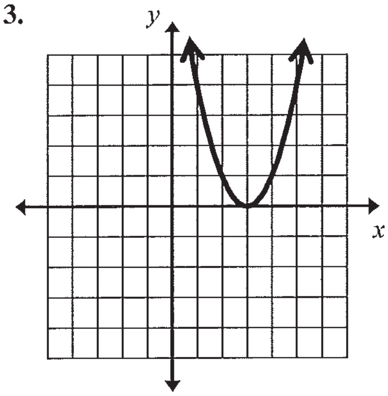
To find the x-value(s) that make the function $f(x) = 0$.

2. When you are solving a quadratic equation by graphing, what do you look for on the graph?

The x-intercepts

#3 – 5: Determine whether the quadratic functions have two real roots, one real root, or no real roots. If possible, list the zeros of the function.

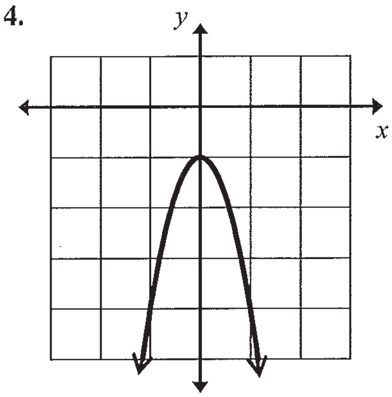
3.



Number and type of roots: 1 real root

Zeros: $x = 3$

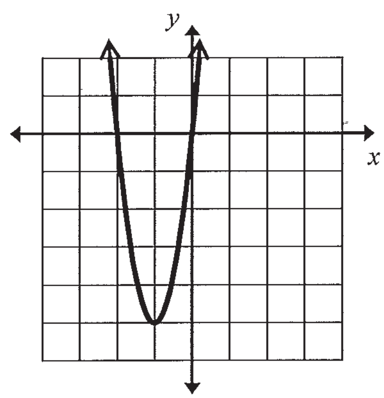
4.



Number and type of roots: no real roots

Zeros: none

5.

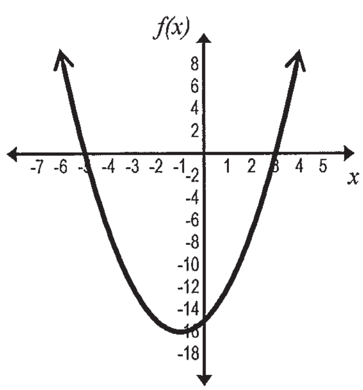


Number and type of roots: 2 real roots

Zeros: $x = 0$ or $x = -2$

#6 – 7: Use the graph to find the zeros of the following quadratic functions. Check that the solutions work.

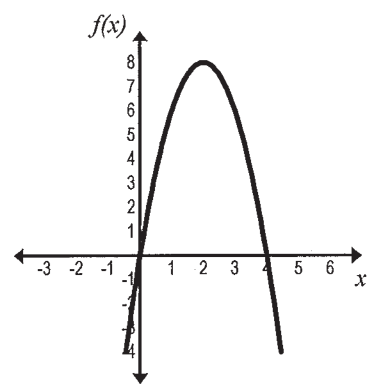
6. $f(x) = x^2 + 2x - 15$



Solution(s): $x = 3$ or $x = -5$

Check:
$f(3) = (3)^2 + 2(3) - 15 =$
$9 + 6 - 15 = 0$ ✓
$f(-5) = (-5)^2 + 2(-5) - 15$
$25 - 10 - 15 = 0$ ✓

7. $f(x) = -2x^2 + 8x$



Solution(s): $x = 0$ or $x = 4$

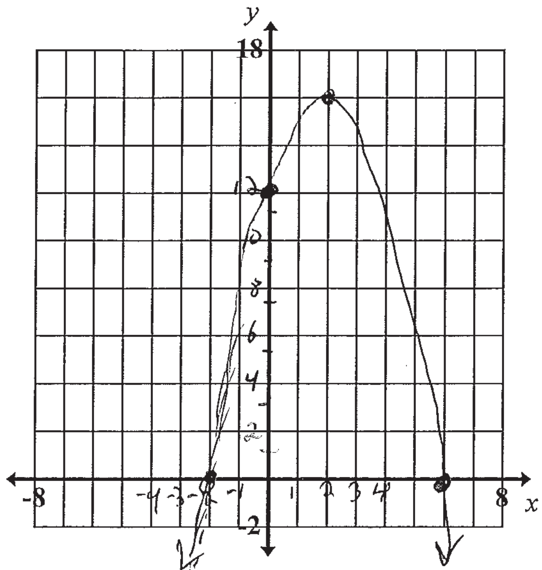
Check:
$f(0) = -2(0)^2 + 8(0) = 0$ ✓
$0 + 0$
$f(4) = -2(4)^2 + 8(4)$
$-32 + 32 = 0$ ✓

Name ______________________________ Period __________

5.1A *Solving Quadratic Equations by Graphing*

#8 – 9: Graph each of the following quadratic functions and use the graph to find the zeros. Create a table of values if necessary. Verify that the values truly are solutions.

8. $f(x) = -x^2 + 4x + 12$



Solution(s): $\boxed{x = -2, 6}$

Verify: $f(-2) = -(-2)^2 + 4(-2) + 12$

$= -(4) + -8 + 12$

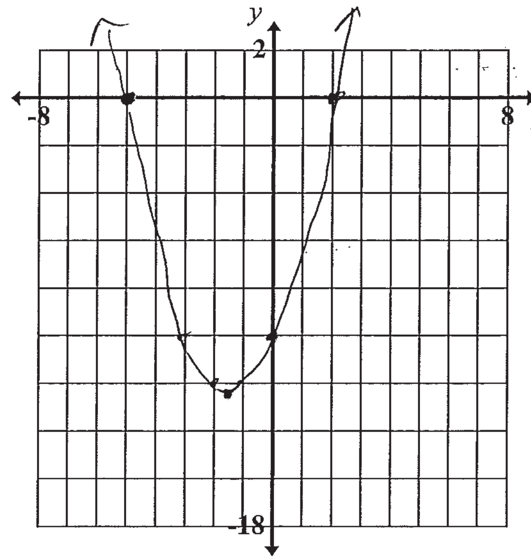
$= -4 + -8 + 12$

$f(-2) = 0$

$f(6) = -(6)^2 + 4(6) + 12$

$f(6) = -36 + 24 + 12 = 0$

9. $f(x) = x^2 + 3x - 10$



x	y
-3	-10
-2	-12
-1	-12
0	-10
1	-6
2	0
3	8

Solution(s): $\boxed{x = -5, 2}$

Verify: $f(-5) = (-5)^2 + 3(-5) - 10$

$= 25 + -15 - 10 = 0$

$f(2) = (2)^2 + 3(2) - 10$

$= 4 + 6 - 10$

$f(2) = 0$

*

#10 – 15: Use your graphing calculator to solve each equation by graphing. If needed, round your answer to the nearest hundredth. Question #13 – 15, verify that the values truly are solutions.

10. $x^2 - 7x = 11$

$x^2 - 7x - 11 = 0$

Solution(s): $x = -1.32, 8.32$

11. $6x^2 = -19x - 15$

$6x^2 + 19x + 15 = 0$

Solution(s): $x = -1.6, -1.5$

12. $5x^2 - 7x - 3 = 8$

$5x^2 - 7x - 11 = 0$

Solution(s): $x = -0.94, 2.34$

13. $\frac{1}{2}x^2 - x = 8$

$\frac{1}{2}x^2 - x - 8 = 0$

Solution(s): $x = -3.12, 5.12$

Verify: $.5(-3.12)^2 - (-3.12) - 8 \approx 0$ *

$.5(5.12)^2 - (5.12) - 8 \approx 0$

14. $x^2 + 4x = 6$

$x^2 + 4x - 6 = 0$

Solution(s): $x = -5.16, 1.16$

Verify: $(-5.16)^2 + 4(-5.16) = 5.99 \approx 6$ ✓

$(1.16)^2 + 4(1.16) = 5.99 \approx 6.$ ✓

15. $2x^2 - 2x - 5 = 0$

Solution(s): $x = -1.16, 2.16$

Verify: $2(-1.16)^2 - 2(-1.16) - 5 = .01 \approx 0$

$2(2.16)^2 - 2(2.16) - 5 = .01 \approx 0$

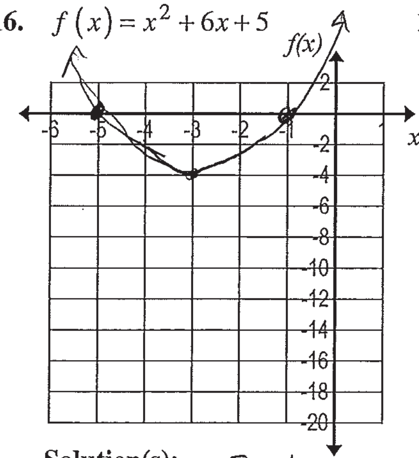
* since x values were rounded

Name ______________________ Period ________

5.1A Solving Quadratic Equations by Graphing

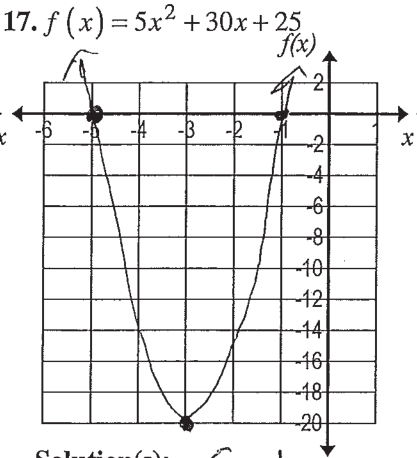
#16 – 18: Use a graphing utility to graph the following functions. Draw the graph of the function. Use the graphing utility to approximate the zeros to the nearest tenth.

16. $f(x) = x^2 + 6x + 5$



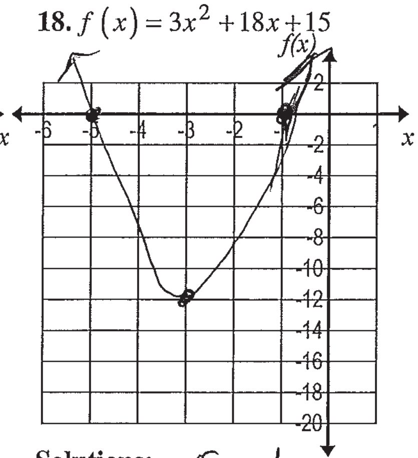
Solution(s): $-5, -1$

17. $f(x) = 5x^2 + 30x + 25$



Solution(s): $-5, -1$

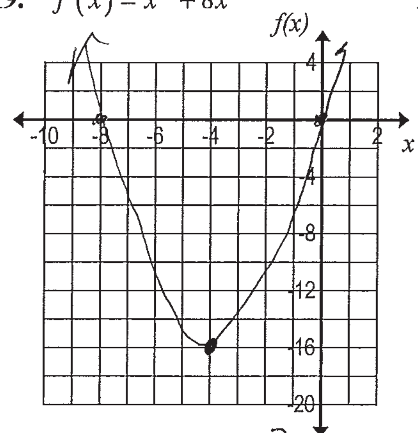
18. $f(x) = 3x^2 + 18x + 15$



Solutions: $-5, -1$

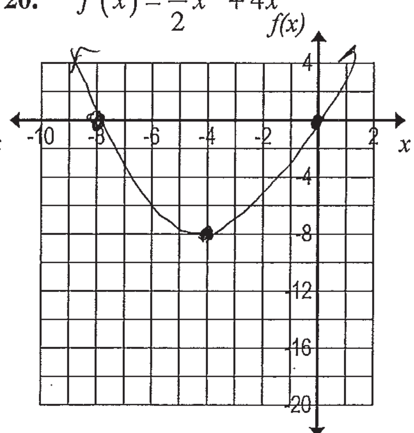
#19 – 21: Use a graphing utility to graph the following functions. Draw the graph of the function. Use the graphing utility to approximate the zeros to the nearest tenth.

19. $f(x) = x^2 + 8x$



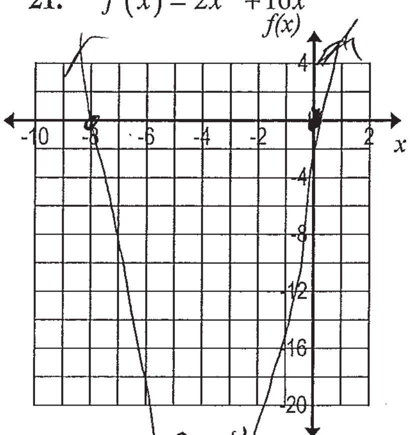
Solution(s): $0, -8$

20. $f(x) = \frac{1}{2}x^2 + 4x$



Solution(s): $0, -8$

21. $f(x) = 2x^2 + 16x$



Solutions: $0, -8$

22. Investigation:

a) Looking to Question #16 – 18, record the following:

- Function in #16 $f(x) = x^2 + 6x + 5$ Solutions in #16 $x = -5, -1$
- Function in #17 $f(x) = 5x^2 + 30x + 25$ Solutions in #17 $x = -5, -1$
- Function in #18 $f(x) = 3x^2 + 18x + 15$ Solutions in #18 $x = -5, -1$

b) Looking to Question #19 – 21, record the following:

- Function in #19 $f(x) = x^2 + 8x$ Solutions in #19 $x = 0, -8$
- Function in #20 $f(x) = \frac{1}{2}x^2 + 4x$ Solutions in #20 $x = 0, -8$
- Function in #21 $f(x) = 2x^2 + 16x$ Solutions in #21 $x = 0, -8$

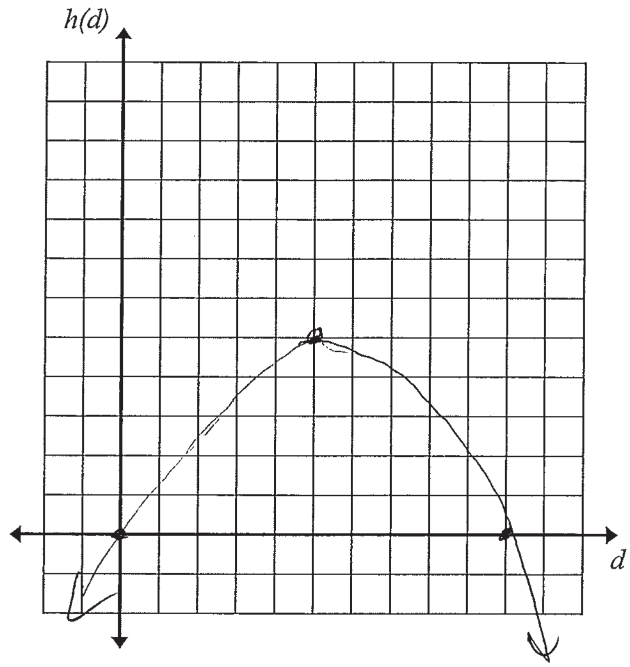
c) Comparing the functions in questions 16, 17, and 18, and then again in 19, 20, and 21, write a conjecture about the relationship of the functions within each set of questions and the solutions of those functions.

If one function is a multiple of another function, then they will have the same solutions.

5.1A Solving Quadratic Equations by Graphing

23. A bottlenose dolphin jumps out of the water. The path the dolphin travels can be modeled by the function $h(d) = -0.2d^2 + 2d$, where h represents the height, in feet, of the dolphin and d represents the horizontal distance, in feet, the dolphin traveled.

a) Sketch a graph of the quadratic equation.



b) What is the maximum height the dolphin reaches? Where is this represented on the graph of the function? 5 ft ; the vertex

c) What is the horizontal distance that the dolphin jumps? Where is this represented on the graph of the function? 10 ft ; the x intercept > 0

Section 5.1A

Name ______________________________ Period __________

5.1B Answering Real-World Questions by Graphing Quadratic Functions

#1 – 10: Use your graphing utility to solve the following problems.

1. Phillip, Peter and Pablo each throw a ball over a fence. The height of Phillip's ball with respect to time can be modeled by the equation $y=-16t^2+60t$. The height of Peter's ball with respect to time can be modeled by the equation $y=-16t^2+50t$. The height of Pablo's ball with respect to time can be modeled by the equation $y=-16t^2+40t$, where y is the height in feet and t is the time in seconds for each of the three models.

a) Phillip, Peter and Pablo want to know whose ball hit the ground first. Peter thinks that they should find the x-intercept of the graphs to determine this. Phillip thinks that they should find the vertex of each graph to find which ball hit the ground first. Which one is correct? Explain your answer.

Peter is correct; the x int gives the time (t) when y=0, which is where the ball has a height of 0 on the ground!

b) Whose ball hit the ground first? How long did it take?

Pablo ; 2.5 sec

c) Whose ball hit the ground second? How long did it take?

Peter; 3.125 sec

2. A quarterback throws a football at an initial height of 5.5 feet with an initial upward velocity of 35 feet per second. The height of a tossed ball with respect to time can be modeled by the quadratic function $h(t)=-16t^2+v_0\cdot t+h_0$ where v_0 is the initial upward velocity, h_0 is the initial height and $h(t)$ is the height of the ball after t seconds.

a) Write the function that models the height of the ball with respect to time.

$h(t)=-16t^2+35t+5.5$

b) How high will the football be after 1 second? (Consider what the 1 second represents.) (the 1 sec repr the x value)

24.5 ft

c) When will the football be 10 feet high? (Consider what the 10 feet represents.) (The 10 is the y value)

After 0.14 secs and again after 2.05 secs

d) When will the football reach its maximum height? (When graphing the function, consider what significant feature of the graph represents this concept.) (At the vertex)

After 1.09 secs.

e) What is the maximum height of the football?

24.64 ft

f) When will the football hit the ground if no one catches it? (When graphing the function, consider what significant feature of the graph represents this concept.) (the x int)

2.33 seconds

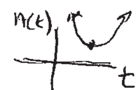
Name ______________________ Period ________

5.1B Answering Real-World Questions by Graphing Quadratic Functions

#1 – 10 (continued): Use your graphing utility to solve the following problems.

3. Suppose a batter hits a baseball, and the height of the baseball above the ground can be modeled by the function $h(t) = -16t^2 + 50t + 2$. Where is the vertex of the graph? Explain the meaning of the vertex in the context of this situation. Above the ground, the maximum y-value at (1.56, 41.06)
The vertex reveals how long it takes (1.56 sec) the ball to reach the maximum height (41.06 ft)

4. A pool is treated with a chemical to reduce the amount of algae. The amount of algae in the pool t days after the treatment begins can be approximated by the function $A(t) = 4t^2 - 88t + 500$. How many days after treatment begins will the pool have the least amount of algae?



t = 11 days (the x coordinate of the vertex)

5. The driver of a car traveling downhill on a road applied the brakes. The speed of the car, $s(t)$, in kilometers per hour t seconds after the brakes were applied is modeled by the function rule $s(t) = -4t^2 + 12t + 80$.

a) After how many seconds did the car reach its maximum speed?

1.5 secs

b) What was the maximum speed reached?

89 Km/h

c) How long will it take the car to stop?

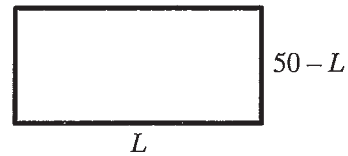
6.22 seconds

Name ______________________ Period ________

5.1B Answering Real-World Questions by Graphing Quadratic Functions

#1 – 10 (continued): Use your graphing utility to answer the following problems.

6. Andrew has 100 feet of fence to enclose a rectangular tomato patch. He wants to find the dimensions of the rectangle that encloses the most area. The width of the rectangle can be found by the expression $50 - L$ where L is the length of the rectangle.



a) In the expression representing the width of the rectangle $(50 - L)$, what does the 50 represent? Explain your thinking clearly.

50 is half the perimeter, so one length + one width = 50.

b) Write a function rule to model the area of the rectangle. $A(L)$ represents the Area of the rectangular tomato patch base on the length (L) of one side.

$A(L) = L(50-L)$

so $A(L) = -L^2 + 50L$

c) Find the coordinate representing the maximum of the graph. Explain its meaning in the context of the situation.

(25, 625) When the length is 25, width is $50-L=25$
(L, A(L)) $50-25=25$,
so Area $= l \times w$
$= 25 \times 25$
$= 625$
$A(L) = 625$

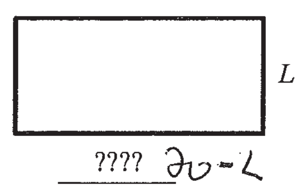
d) What size should Andrew make the tomato patch in order to enclose the most area within the fencing?

a square shape, 25 ft × 25 ft

7. Sharon needs to create a fence for her new puppy. She purchased 40 feet of fencing to enclose the four sides of a rectangular play area.

a) Determine the dimensions the enclosure play area should be to produce the **greatest** area for her puppy to play.

According to #6d, she should make a square with her 40 ft of fencing, so $40 \div 4 = 10$ ft on each side.



b) Write a function rule to model the area of the play area.

$A(L) = L(20-L)$

$A(L) = -L^2 + 20L$ Calc max shows vertex @ (10, 100)

c) What are the dimensions of the enclosure that will create the greatest area for her puppy to play?

Length $L = 10$ ft
width $20-L = 10$ ft
Area $= 100$ ft^2

5.1B Answering Real-World Questions by Graphing Quadratic Functions

#1 – 10 (continued): Use your graphing utility to answer the following problems.

8. Karen is throwing an orange to her brother Jim, who is standing on the balcony of their home. The height, h (in feet), of the orange above the ground t seconds after Karen throws the orange is given by the function $h(t) = -16t^2 + 32t + 3$. If Jim's outstretched arms are 16 feet above the ground, will the orange ever be high enough so that he can catch it? Explain your answer.

yes! The max height of the orange is 19 ft, or the y coordinate of the vertex. Jim can grab it either on the way up or the way down.

9. On wet concrete, the stopping distance, s (in feet), of a car traveling v miles per hour is given by $s(v) = 0.055v^2 + 1.1v$. At what speed could a car be traveling and still stop at a stop sign 30 feet away?

$(v, s(v))$

$(v, 30)$

use Calc intersect $v \approx 15.41$ mph, so ≈ 15 mph

10. The Buckingham Fountain in Chicago shoots water from a nozzle at the base of the fountain. The height, in feet, of the water above the ground t seconds after it leaves the nozzle is given by $h(t) = -16t^2 + 90t + 15$.

a) What is the maximum height of the water spout to the nearest tenth of a foot?

141.56 ft

b) How long does it take for the water to hit the ground?

5.79 secs

Section 5.1B

Name ______________________ Period ________

5.2A Factoring Review

#1 – 12: Factor out the greatest common factor (GCF) for each polynomial and write in *factored form.*

1. $2x+6$

$2x$	$+6$

Ans: $2(x+3)$

2. $3y-9$

$3y$	-9

Ans: $3(y-3)$

3. $7a+28$

$7a$	$+28$

Ans: $7(a+4)$

4. $36z-12$

$36z$	-12

Ans: ~~3z~~ $12(3z-1)$

5. b^2+b

b^2	$+b$

Ans: $b(b+1)$

6. $2r-r^2$

$2r$	$-r^2$

Ans: $r(2-r)$

7. $9t^2+t$

$9t^2$	$+t$

Ans: $t(9t+1)$

8. $4n^2-5n$

$4n^2$	$-5n$

Ans: $n(4n-5)$

9. $4h^2+12h$

$4h^2$	$+12h$

Ans: $4h(h+3)$

10. $9x-27x^2$

$9x$	$-27x^2$

Ans: $9x(1-3x)$

11. $2a^2+4a$

$2a^2$	$+4a$

Ans: $2a(a+2)$

12. $20d^2-24d$

$20d^2$	$-24d$

Ans: $4d(5d-6)$

#13 – 27: Factor each polynomial.

13. $x^2+13x+42$

Ans: $(x+7)(x+6)$

14. x^2+6x+9

Ans: $(x+3)(x+3)$

15. $x^2+12x+32$

Ans: $(x+8)(x+4)$

16. $x^2+3x-10$

Ans: $(x+5)(x-2)$

17. $x^2-10x+25$

Ans: $(x-5)(x-5)$

18. x^2-x-12

Ans: $(x-4)(x+3)$

19. $3x^2+x-4$

Ans: $(3x+4)(x-1)$

20. $2x^2+5x-12$

Ans: $(2x-3)(x+4)$

21. $4x^2-12x+9$

Ans: $(2x-3)(2x-3)$

Name ______________________ Period ________

5.2A Factoring Review

#13 – 27 (continued): Factor each polynomial.

22. $12x^2-8x+1$

Ans: $(6x-1)(2x-1)$

23. $2x^2-8x-10$

$2(x^2-4x-5$

Ans: $2(x+1)(x-5)$

24. $3x^2+21x+18$

$3(x^2+7x+6)$

Ans: $3(x+1)(x+6)$

25. a^2-9

Ans: $(a+3)(a-3)$

26. x^2-16

Ans: $(x+4)(x-4)$

27. $25x^2-36$

Ans: $(5x+6)(5x-6)$

#28-29: The following quadratic functions are written in *standard form*. Convert them to *factored form*.

28. $y=x^2+3x+2$

$y=(x+1)(x+2)$

29. $y=x^2-49$

$y=(x+7)(x-7)$

30. Why is *factored form* of a quadratic function also called *intercept form*?

x-intercepts have a y-coordinate of 0. Setting y=0 and easily solving for x gives the solution(s), which are the x-intercept(s).

#31 – 32: Convert the following quadratic functions *to factored form* and identify the *x*-intercepts.

31. $y=x^2-24x+80$

$0=(x-4)(x-20)$

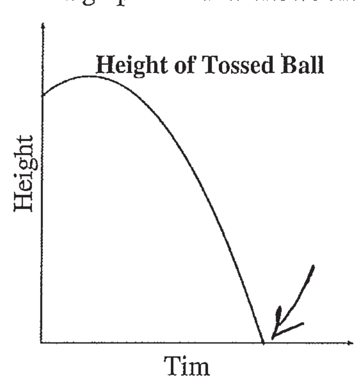
x-intercepts: 4 and 20

32. $y=x^2+9x-10$

$0=(x-1)(x+10)$

x-intercepts: 1 and -10

33. The parabola graphed below shows the height of a ball tossed into the air.



a) Draw an arrow to the location of the graph that would represent when the ball hits the ground?

b) Explain why you placed the arrow at that location.

y=0 means the ground level.

Section 5.2A

Name ________________________________ Period __________

5.2B Solve Quadratic Equations by Factoring: Part I

#1 - 3: Solve for x.

1. $(x-4)(x+9)=0$

$x=4$ or -9

2. $(x-2)(3x-6)=0$

$x=2$ (double root)

3. $(4x+3)(2x-5)=0$

$x=-\frac{3}{4}$ or $\frac{5}{2}$

#4 – 17: Factor the quadratic expression then solve the equation by factoring. Verify your solution(s).

4. $4x^2-36=0$

$4(x^2-9)=0$

$4(x+3)(x-3)=0$

$\boxed{x=-3 \text{ or } 3}$

$4(-3)^2-36=0$ ✓

$4(3)^2-36=0$ ✓

$4\cdot 9-36$

✓Verify your solution(s):

5. $5x^2-20=0$

$5(x^2-4)=0$

$5(x+2)(x-2)=0$

$\boxed{x=-2 \text{ or } 2}$

$5(-2)^2-20=0$ ✓

$5(2)^2-20=0$ ✓

✓ Verify your solution(s):

6. $3x^2-9x=0$

$3x(x-3)=0$

$\boxed{x=0 \text{ or } x=3}$

$3(0)^2-9(0)=0$ ✓

$3(3)^2-9(3)=0$ ✓

✓ Verify your solution(s):

7. $7x^2-28x=0$

$7x(x-4)=0$

$\boxed{x=0 \text{ or } 4}$

✓Verify your solution(s):

$7(0)^2-28(0)=0$ ✓

$7(4)^2-28(4)$

$7(16)-28(4)$

$112-112=0$ ✓

8. $x^2+8x-9=0$

$(x+9)(x-1)$

$\boxed{x=-9 \text{ or } x=1}$

✓ Verify your solution(s):

$(-9)^2+8(-9)-9=$

$81-72-9=0$ ✓

$(1)^2+8(1)-9$

$1+8-9=0$ ✓

9. $x^2+7x+12=0$

$(x+4)(x+3)=0$

$\boxed{x=-4 \text{ or } -3}$

✓ Verify your solution(s):

$(-4)^2+7(-4)+12$

$16-28+12=0$ ✓

$(-3)^2+7(-3)+12$

$9-21+12=0$ ✓

Name ______________________________ Period __________

5.2B Solve Quadratic Equations by Factoring: Part I

#4 – 17 (continued): Factor the quadratic expression then solve the equation by factoring. Verify your solution(s).

10. $x^2-10x+25=0$

$(x-5)(x-5)=0$

$\boxed{x=5}$

✓Verify your solution(s):

$(5)^2-10(5)+25$

$25-50+25=0$ ✓

11. $x^2-3x=4$

$x^2-3x-4=0$

$(x+1)(x-4)=0$

$\boxed{x=-1,\ 4}$

✓ Verify your solution(s):

$(-1)^2-3(-1)$

$1+3=4$ ✓

$(4)^2-3(4)$

$16-12=4$ ✓

12. $x^2-4x=5$

$x^2-4x-5=0$

$(x+1)(x-5)=0$

$\boxed{x=-1,\ 5}$

✓ Verify your solution(s):

$(-1)^2-4(-1)$

$1+4=5$ ✓

$(5)^2-4(5)$

$25-20=5$ ✓

13. $x^2-13x=-40$

$x^2-13x+40=0$

$(x-8)(x-5)=0$

$\boxed{x=8,\ 5}$

✓Verify your solution(s):

$(8)^2-13(8)$

$64-104=-40$ ✓

$(5)^2-13(5)$

$25-65=-40$ ✓

14. $3x^2+10x+8=0$

$(3x+4)(x+2)$

$\boxed{x=-4/3,\ -2}$

✓ Verify your solution(s):

$3(-\frac{4}{3})^2+10(-\frac{4}{3})+8$

$\frac{3}{1}\cdot\frac{16}{9}$

$\frac{16}{3}-\frac{40}{3}+\frac{24}{3}=0$ ✓

$3(-2)^2+10(-2)+8=0$ ✓

15. $8x^2+6x-5=0$

$(4x+5)(2x-1)=0$

$\boxed{x=-5/4,\ \frac{1}{2}}$

✓ Verify your solution(s):

$8(-\frac{5}{4})^2+6(-\frac{5}{4})-5$

$8\cdot\frac{25}{16}$

$\frac{25}{2}-\frac{15}{2}-5=0$ ✓

$8(\frac{1}{2})^2+6(\frac{1}{2})-5$

$8(\frac{1}{4})$

$2+3-5=0$ ✓

16. $5x^2+11x=-2$

$5x^2+11x+2=0$

$(5x+1)(x+2)=0$

$\boxed{x=-1/5,\ -2}$

✓Verify your solution(s):

$5(-\frac{1}{5})^2+11(-\frac{1}{5})+2$

$5\cdot\frac{1}{25}-\frac{11}{5}+\frac{10}{5}=0$ ✓

$5(-2)^2+11(-2)$

$20-22+2=0$ ✓

17. $2x^2-15x=8$

$2x^2-15x-8=0$

$(2x+1)(x-8)=0$

$\boxed{x=-1/2,\ 8}$

✓ Verify your solution(s):

$2(-\frac{1}{2})^2-15(-\frac{1}{2})$

$2(\frac{1}{4})$

$\frac{1}{2}+\frac{15}{2}=\frac{16}{2}$ ✓

$2(8)^2-15(8)$

$128-120=8$ ✓

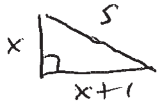
Section 5.2B

Name ______________________________ Period __________

5.2C Solve Quadratic Equations by Factoring: Part II

#1 – 9: Solve the following application problems.

1. One leg of a right triangle is 1 foot longer than the other leg. The hypotenuse is 5 feet. Find the dimensions of the right triangle.

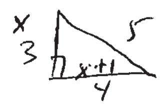


$x^2+(x+1)^2=25$

$x^2+x^2+2x+1=25$

$2x^2+2x-24=0$

$2(x^2+x-12)=0$



$2(x+4)(x-3)=0$

extraneous $\cancel{x=-4}$ $\quad x=3 \Rightarrow$ 2 legs are 3 & (3+1)

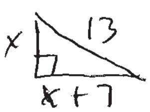
legs = 3 & 4

✓Verify your solution(s):

$3^2+4^2=5^2$

$9+16=25$ ✓

2. One leg of a right triangle is 7 feet longer than the other leg. The hypotenuse is 13. Find the dimensions of the right triangle.



$x^2+(x+7)^2=13^2$

$x^2+x^2+14x+49=169$

$2x^2+14x-120=0$

$2(x^2+7x-60)=0$

$2(x+12)(x-5)=0$

$\cancel{x=-12}$ $\quad x=5 \Rightarrow$

2 legs are 5 + 12

✓Verify your solution(s):

$5^2+12^2=13^2$

$25+144=169$ ✓

3. A rectangle has sides of $x+2$ and $x-1$. What value of x gives an area of 108?

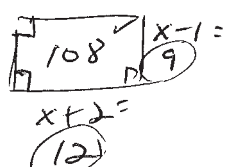
$(x+2)(x-1)=108$

$x^2+x-2-108=0$

$x^2+x-110=0$

$(x-10)(x+11)=0$

$x=10$, $\quad x=-11$ extraneous



✓Verify your solution(s): $12 \times 9 = 108$ ✓

4. A rectangle has sides of $x-1$ and $x+1$. What value of x gives an area of 120?

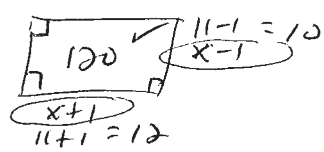
$(x-1)(x+1)=120$

$x^2-1-120=0$

$x^2-121=0$

$(x+11)(x-11)=0$

$\cancel{x=-11}$ $\quad x=11$



✓Verify your solution(s):

5. The product of two positive numbers is 120. Find the two numbers if one number is 7 more than the other.

$x(x+7)=120$

$x^2+7x-120=0$

$(x-8)(x+15)=0$

$x=8$ or $\cancel{x=-15}$

✓Verify your solution(s):

$x=8, \quad x+7=15$

$8(15)=120$ ✓

Name ______________________ Period ________

5.2C *Solve Quadratic Equations by Factoring: Part II*

6. A rectangle has a 50-foot diagonal. What are the dimensions of the rectangle if it is 34 feet longer than it is wide?

$x+34$, 50, x

$x^2 + (x+34)^2 = 50^2$

$x^2 + x^2 + 68x + 1156 = 2500$

$2x^2 + 68x - 1344 = 0$

$2(x^2 + 34x - 672) = 0$

$2(x-14)(x+48) = 0$

$x = 14$ $\quad$ $x = \cancel{-48}$

$x+34 = 48$, 14

The dimensions are 14' by 48'

✓Verify your solution(s):

$14^2 + 48^2 = 50^2$

$196 + 2304 = 2500$ ✓

7. Two positive numbers have a sum of 8, and their product is equal to the larger number plus 10. What are the numbers?

Let x = 1st #

$8-x$ = 2nd #

8 = sum

$x(8-x) = x + 10$

$8x - x^2 = x + 10$

$0 = x^2 - 7x + 10$

$0 = (x-5)(x-2)$

$x = 5$ ✓ or $x = 2$

verify each

If 1st # $x = 5$, 2nd # is $8 - 5 = 3$

Product $5(3) = 5 + 10$ ✓

So 2 #s are 5 + 3

✓Verify your solution(s):

If 1st # $x = 2$, 2nd # = 6 $\quad$ $2+6=8$

But $2(6) \neq 6 + 10$ No.

So the 2 positive #s are 5 and 3

8. The product of two negative integers is 24. The difference between the integers is 2. Find the integers.

Let x = one integer neg

$x+2$ = 2nd integer neg

Then $x(x+2) = 24$

$x^2 + 2x - 24 = 0$

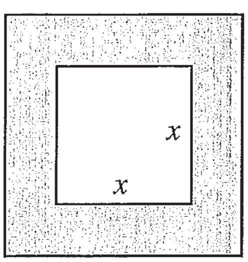
$(x-4)(x+6) = 0$ ✓Verify your solution(s):

$x = \cancel{4}$ or $x = -6$ $\Rightarrow$ $x = -6$, $x+2 = -4$ are the 2 negative ints.

extraneous

$(-6)(-4) = 24$

9. Framing Warehouse offers a picture framing service. The cost for framing a picture is made up of two parts: glass costs \$1 per square foot and the frame costs \$2 per linear foot. If the frame has to be a square, what size picture can you get framed for \$20?



linear price: $\$2(4x) = 8x$ (perimeter)

area glass: $\$1x^2$

$x^2 + 8x = 20$ \$

$x^2 + 8x - 20 = 0$

$(x+10)(x-2) = 0$

$x = -10$ $\quad$ $x = 2$

You can frame a 2 ft by 2 ft square picture

✓Verify your solution(s):

When $x = 2$, P = $8 \times 2 = 16$ \$

glass area $2 \times 2 = 4 ft^2 \times 1 = 4$ \$

$\$16 + \$4 = \$20$ ✓

Section 5.2C

Name ______________________ Period ________

5.2D *Operations with Radical Expressions*

1. Simplify each expression.

a) $\sqrt{20}$

$\sqrt{4 \cdot 5}$

$2\sqrt{5}$

b) $\sqrt{48}$

$\sqrt{16 \cdot 3}$

$4\sqrt{3}$

c) $\sqrt{200}$

$\sqrt{100 \cdot 2}$

$10\sqrt{2}$

d) $3\sqrt{20}$

$3\sqrt{4 \cdot 5}$

$3 \cdot 2\sqrt{5}$

$6\sqrt{5}$

e) $5\sqrt{24}$

$5\sqrt{4 \cdot 6}$

$5 \cdot 2\sqrt{6}$

$10\sqrt{6}$

f) $6\sqrt{98}$

$6\sqrt{49 \cdot 2}$

$6 \cdot 7\sqrt{2}$

$42\sqrt{2}$

2. Add or subtract each expression.

a) $\left(5-\sqrt{3}\right)+\left(4+\sqrt{3}\right)$

9

b) $\left(4+5\sqrt{2}\right)+\left(2+6\sqrt{2}\right)$

$6+11\sqrt{2}$

c) $\left(6-8\sqrt{7}\right)-\left(4+2\sqrt{7}\right)$

$2-10\sqrt{7}$

d) $8-\left(3+5\sqrt{2}\right)$

$5-5\sqrt{2}$

e) $-2\sqrt{5}+\left(3+\sqrt{5}\right)$

$3-\sqrt{5}$

f) $\left(6+5\sqrt{12}\right)+\left(5-\sqrt{12}\right)$

$11+4\sqrt{12}$

$11+4\sqrt{4 \cdot 3}$

$11+4 \cdot 2\sqrt{3}$

$11+8\sqrt{3}$

Name ________________________ Period ________

5.2D Operations with Radical Expressions

3. Simplify each using two different methods.

a) $\sqrt{2}\cdot\sqrt{18}$

1st method	2nd method
$\sqrt{2}\cdot\sqrt{18}$ $\sqrt{2\cdot 18}$ $\sqrt{36}$ 6	$\sqrt{2}\cdot\sqrt{18}$ $\sqrt{2}\cdot\sqrt{9\cdot 2}$ $\sqrt{2}\sqrt{9}\sqrt{2}$ $3\cdot 2=6$

b) $\sqrt{8}\cdot\sqrt{2}$

1st method	2nd method
$\sqrt{8}\cdot\sqrt{2}$ $\sqrt{8\cdot 2}$ $\sqrt{16}$ 4	$\sqrt{8}\cdot\sqrt{2}$ $\sqrt{4\cdot 2}\cdot\sqrt{2}$ $\sqrt{4}\sqrt{2}\sqrt{2}$ $2\cdot 2$ 4

4. Simplify each expression.

a) $\sqrt{5}\cdot\sqrt{5}$

5

b) $\sqrt{5}\cdot\sqrt{18}$

$\sqrt{5}\sqrt{9}\cdot\sqrt{2}$

$3\sqrt{10}$

c) $\sqrt{6}\cdot\sqrt{3}$

$\sqrt{18}$

$\sqrt{9\cdot 2}$

$3\sqrt{2}$

d) $3\sqrt{20}\cdot 2\sqrt{3}$

$6\sqrt{60}$

$6\sqrt{4\cdot 15}$

$6\cdot 2\sqrt{15}$

$12\sqrt{15}$

e) $-5\sqrt{2}\cdot 8\sqrt{2}$

$-40\cdot 2$

-80

f) $-2\sqrt{6}\cdot 3\sqrt{7}$

$-6\sqrt{42}$

Name ____________________ Period ________

5.2D Operations with Radical Expressions

5. Simplify each expression.

a) $5(3+\sqrt{7})$

$\boxed{15+5\sqrt{7}}$

b) $-3(2-\sqrt{3})$

$\boxed{-6+3\sqrt{3}}$

c) $2(1+4\sqrt{3})$

$\boxed{2+8\sqrt{3}}$

d) $\sqrt{5}(2+3\sqrt{2})$

$\boxed{2\sqrt{5}+3\sqrt{10}}$

e) $-\sqrt{2}(3+\sqrt{18})$

$-3\sqrt{2}-\sqrt{36}$

$-3\sqrt{2}-6$ or

$\boxed{-6-3\sqrt{2}}$

f) $-\sqrt{3}(5-\sqrt{8})$

$-5\sqrt{3}+\sqrt{24}$

$\sqrt{4\cdot 6}$

$\boxed{-5\sqrt{3}+2\sqrt{6}}$

6. Simplify each expression.

a) $(4+\sqrt{3})(4+\sqrt{3})$

$16+8\sqrt{3}+3$

$\boxed{19+8\sqrt{3}}$

b) $2(3-\sqrt{2})^2$

$2(3-\sqrt{2})(3-\sqrt{2})$

$2(9-6\sqrt{2}+2)$

$2(11-6\sqrt{2})$

$\boxed{22-12\sqrt{2}}$

c) $5(2-\sqrt{3})^2+6(2-\sqrt{3})$

$5(2-\sqrt{3})(2-\sqrt{3})+12-6\sqrt{3}$

$5(4-4\sqrt{3}+3)$ ↓ ↓

$5(7-4\sqrt{3})$

$35-20\sqrt{3}+12-6\sqrt{3}$

$\boxed{47-26\sqrt{3}}$

d) $4(1+3\sqrt{2})(1+3\sqrt{2})$

$4[1+6\sqrt{2}+18]$

$4(19+6\sqrt{2})$

$\boxed{76+24\sqrt{2}}$

e) $3(4-2\sqrt{7})^2$

$3(4-2\sqrt{7})(4-2\sqrt{7})$

$3(16-16\sqrt{7}+28)$

$3(44-16\sqrt{7})$

$\boxed{132-48\sqrt{7}}$

f) $3(2-\sqrt{11})^2-5(2-\sqrt{11})+7$

$3(2-\sqrt{11})(2-\sqrt{11})-10+5\sqrt{11}+7$

$3(4-4\sqrt{11}+11)$

$3(15-4\sqrt{11})$

$45-12\sqrt{11}-3+5\sqrt{11}$

$\boxed{42-7\sqrt{11}}$

Name ______________________ Period ________

5.2D Operations with Radical Expressions

7. Verify that the given answer (value of x) is a solution to the equation.

a) $x^2+7=25;\ x=3\sqrt{2}$

$(3\sqrt{2})^2+7=25$

$18+7=25$ ✓

yes

b) $x^2-14x+50=3;\ x=\left(6+\sqrt{2}\right)$

$(6+\sqrt{2})(6+\sqrt{2})-14(6+\sqrt{2})+50=3$

$36+12\sqrt{2}+2-84-14\sqrt{2}+50$

$38+12\sqrt{2}-84-14\sqrt{2}+50$

$4-2\sqrt{2} \neq 3$

$6+\sqrt{2}$ is not a solution

c) $2x^2-20x=40;\ x=\left(5-3\sqrt{5}\right)$

$x^2-10x=20$

$x^2-10x-20=0$

$(5-3\sqrt{5})(5-3\sqrt{5})-10(5-3\sqrt{5})-20=0$

$25-30\sqrt{5}+45-50+30\sqrt{5}-20=0$

$70-70-30\sqrt{5}+30\sqrt{5}$

$0=0$ ✓

yes

8. Simplify each expression.

a) $\dfrac{6a+15}{3}$

$2a+5$

b) $\dfrac{6+15\sqrt{2}}{3}$

$2+5\sqrt{2}$

c) $\dfrac{-14-10\sqrt{6}}{2}$

$-7-5\sqrt{6}$

d) $\dfrac{-25+15\sqrt{40}}{5}$

$-5+3\sqrt{40}$

$3\sqrt{4\cdot 10}$

$-5+6\sqrt{10}$

9. Verify that the given answer (value of x) is a solution to the equation.

$2x^2-3x-6=0;\ x=\dfrac{3+\sqrt{57}}{4}$

$2\left(\dfrac{3+\sqrt{57}}{4}\right)^2-3\left(\dfrac{3+\sqrt{57}}{4}\right)-6=0$

$\dfrac{9+6\sqrt{57}+57}{16}$

$\left(\dfrac{66+6\sqrt{57}}{16}\right)$

$2\left(\dfrac{33+3\sqrt{57}}{8}\right)$

$\dfrac{33+3\sqrt{57}}{4}\ \dfrac{-9-3\sqrt{57}}{4}$

$\dfrac{24}{4}-6=0$

$0=0$ ✓

Section 5.2D

Name ______________________ Period ________

5.2E Solve Quadratic Equations Using Square Roots to Find Rational Solutions

#1 – 6: Solve each equation using the square root property and check each answer.

1. $\sqrt{x^2} = \sqrt{4}$

$|x| = 2$

$x = \pm 2$

$(2)^2 = 4$ ✓
$(-2)^2 = 4$ ✓

✓Verify your solution(s):

$(2)^2 = 4$ ✓
$(-2)^2 = 4$ ✓

2. $2a^2 = 32$

$\sqrt{a^2} = \sqrt{16}$

$|a| = 4$

$a = \pm 4$

$2(4)^2 = 2(16) = 32$ ✓
$2(-4)^2 = 2(16) = 32$ ✓

✓ Verify your solution(s):

3. $3m^2 - 8 = 67$

$3m^2 = 75$

$\sqrt{m^2} = \sqrt{25}$

$|m| = 5$

$m = \pm 5$

$3(5)^2 - 8$
$75 - 8 = 67$ ✓
$3(-5)^2$
$75 - 8 = 67$ ✓

✓ Verify your solution(s):

4. $\sqrt{(x-1)^2} = \sqrt{36}$

$|x-1| = 6$

$x - 1 = 6$ or $x - 1 = -6$

$x = 7$ or $x = -5$

✓Verify your solution(s):

$(7-1)^2 = 36$
$6^2 = 36$ ✓
$(-5-1)^2 = 36$
$(-6)^2 = 36$ ✓

5. $(x+3)^2 - 16 = 0$

$\sqrt{(x+3)^2} = \sqrt{16}$

$|x+3| = 4$

$x + 3 = 4$ or $x + 3 = -4$

$x = 1$ or $x = -7$

✓ Verify your solution(s):

$(1+3)^2 - 16 = 0$
$4^2 - 16 =$
$16 - 16$
$0 = 0$ ✓

$(-7+3)^2 - 16 = 0$
$(-4)^2 - 16 = 0$
$16 - 16 = 0$ ✓

6. $2(x-2)^2 + 3 = 21$

$2(x-2)^2 = 18$

$(x-2)^2 = 9$

$\sqrt{(x-2)^2} = \sqrt{9}$

$|x-2| = 3$

$x - 2 = 3$ or $x - 2 = -3$

$x = 5$ or $x = -1$

✓ Verify your solution(s):

$2(5-2)^2 + 3 = 21$
$2(3)^2 + 3$
$18 + 3 = 21$ ✓

$2(-1-2)^2 + 3 = 21$
$2(-3)^2 + 3$
$18 + 3 = 21$ ✓

5.2E Solve Quadratic Equations Using Square Roots to Find Rational Solutions

7. A physics teacher drops an object from an initial height of 64 feet. The height of the ball (in feet) h at time t (in seconds) can be modeled by the equation $h(t) = -16t^2 + 64$. How long does it take the ball to reach the ground?

$-16t^2 + 64 = 0$
$-16t^2 = -64$
$\sqrt{t^2} = \sqrt{4}$
$|t| = 2$
$t = 2$ seconds or ~~$t = -2$~~ extraneous

$-16(2)^2 + 64 = 0$
$-16(4)$
$-64 + 64 = 0$ ✓

✓Verify your solution(s):

2 seconds

8. The stopping distance "d" (in meters) that a car needs to come to a complete stop when traveling at speed "x" (in km/h) can be modeled by the equation $d = 0.009(x+15)^2 + 3$. On a certain road, drivers cannot see a stop sign until they are approximately 20 meters away. What is the maximum speed that should be posted in order to allow cars enough room to stop in time? Round your answer to the nearest whole number and verify your solution.

$20 = 0.009(x+15)^2 + 3$
$\frac{17}{0.009} = \frac{0.009(x+15)^2}{0.009}$
$\sqrt{1888.\overline{8}} = \sqrt{(x+15)^2}$
$43.4613 = |x+15|$
$x + 15 = 43.4613$ or $x + 15 = -43.4613$
$x \approx 28$ $\quad$ $x \approx -58$ extraneous

$.009(28+15)^2 + 3$
$(43)^2$
$(.009)1849 + 3 \approx 19.6 \approx 20$ ✓

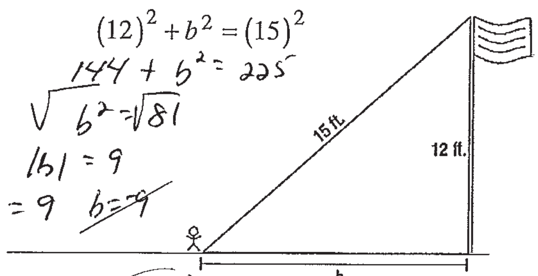
28 Kmh

9. A missing leg of a right triangle can be found using the Pythagorean Theorem: $a^2 + b^2 = c^2$, where "a" and "b" are the legs of the triangle and "c" is the hypotenuse of the triangle (the side directly across from the right angle). Andy is trying to find the missing leg of the triangle below that represents the distance that the person is from a flagpole. The flag pole is 12 feet tall and he knows that the distance from the person to the top of the flagpole is 15 feet. Andy has started the problem by putting the values into the formula. Help him find the solution.

$a^2 + b^2 = c^2$

$(12)^2 + b^2 = (15)^2$

$144 + b^2 = 225$
$\sqrt{b^2} = \sqrt{81}$
$|b| = 9$
$b = 9$ ~~$b = -9$~~



Other leg is 9 ft

Section 5.2E

Name ______________________________ Period __________

5.2F Solve Quadratic Equations Using Square Roots to Find Real Solutions

1. On a quiz, Omar solved a quadratic equation and got the answer wrong. His work is shown below. Identify his mistake and then solve the equation correctly to find the real solution.

12) $\frac{2(x-3)^2}{2} = \frac{18}{2}$

$(x-3)^2 = 9$

error here → $+3 \quad +3$

$\sqrt{x^2} = \sqrt{12}$

$x = 3.464, -3.464$

$\sqrt{(x-3)^2} = \sqrt{9}$

$|x-3| = 3$

$x-3=3$ or $x-3=-3$

$x=6$ or $x=0$

#2 – 5: Verify that each of the following values are solutions to the given equation. Show ALL of your work.

2. $2x^2+3=21;\ x=3, x=-3$

$2(3)^2+3=21$

$2(9)+3$

$18+3=21$ ✓

$2(-3)^2+3=21$

$2(9)+3$

$18+3=21$ ✓

3. $(x-5)^2+1=17;\ x=9, x=1$

$(9-5)^2+1=17$

$(4)^2+1$

$16+1=17$ ✓

$(1-5)^2+1=17$

$(-4)^2+1$

$16+1=17$ ✓

4. $x^2+7=35;\ x=2\sqrt{7}, x=-2\sqrt{7}$

$(2\sqrt{7})^2+7=35$

$4\cdot 7$

$28+7=35$ ✓

$(-2\sqrt{7})^2+7=35$

$4\cdot 7$

$28+7=35$ ✓

5. $(x+3)^2-5=70;\ x=-3+\sqrt{5}, x=-3-\sqrt{5}$

$((-3+\sqrt{5})+3)^2-5=70$

$(\sqrt{5})^2$

$5-5\neq 70$

$(-3+\sqrt{5})$ is NOT a soln.

$((-3-\sqrt{5})+3)^2-5=70$

$(-\sqrt{5})^2$

$5-5\neq 70$

$(-3-\sqrt{5})$ is NOT a soln.

Name ______________________________ Period __________

5.2F *Solve Quadratic Equations Using Square Roots to Find Real Solutions*

6. Samantha solved the following problem on a test and got the right answer. Unfortunately, she doesn't know which answer is the actual solution. Explain to her which solution is correct and why.

> 5) The height "h" of a water balloon (in feet) at time "x" (in seconds) is given by the equation. $h(x) = -16(x-0.65)^2 + 10$. If a student throws the balloon and it hits a student who is 6 feet tall in the head, how long was the balloon in the air?

$6 = -16(x - 0.65)^2 + 10$

$-10 \qquad\qquad -10$

$-4 = -16(x - 0.65)^2$

$-16 \qquad -16$

$\sqrt{0.25} = \sqrt{(x - 0.65)^2}$

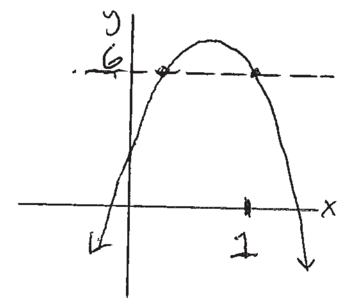
$0.5 = x - 0.65 \qquad\qquad -0.5 = x - 0.65$

$+0.65 \quad +0.65 \qquad\qquad +0.65 \quad +0.65$

$x = 1.15$ seconds or $x = 0.15$ seconds

Not sure which is right??? HELP!

Both answers are correct. The balloon could have hit a 6 ft tall student on the way up after 0.15 seconds, OR it could have hit the student on its descending path after 1.15 seconds. See graph at right.



#7 – 10: Solve each equation for real solutions and simplify your answers. Verify your solutions!

7. $x^2 + 3 = 21$

$\sqrt{x^2} = \sqrt{18}$

$|x| = \sqrt{9 \cdot 2}$

$x = \pm 3\sqrt{2}$

$(3\sqrt{2})^2 + 3$

$9 \cdot 2 + 3 = 21$ ✓

$(-3\sqrt{2})^2 + 3$

$9 \cdot 2 + 3 = 21$ ✓

✓Verify your solution(s):

8. $\sqrt{(x-1)^2} = \sqrt{32}$

$|x-1| = \sqrt{16 \cdot 2}$

$x - 1 = \pm 4\sqrt{2}$

$x = 1 \pm 4\sqrt{2}$

✓ Verify your solution(s):

$((1+4\sqrt{2})-1)^2 = 32$

$(4\sqrt{2})^2 = 32$

$16 \cdot 2 = 32$ ✓

$((1-4\sqrt{2})-1)^2 = 32$

$(-4\sqrt{2})^2 = 32$

$16 \cdot 2 = 32$ ✓

Name ______________________________ Period __________

5.2F Solve Quadratic Equations Using Square Roots to Find Real Solutions

#7 – 10 (continued): Solve each equation for real solutions and simplify your answers. Verify your solutions!

9. $2x^2 - 8 = 0$

$2x^2 = 8$

$\sqrt{x^2} = \sqrt{4}$

$|x| = 2$

$\boxed{x = \pm 2}$

✓Verify your solution(s):

$2(2)^2 - 8 = 0$

$2 \cdot 4 - 8 = 0$ ✓

$2(-2)^2 - 8$

$2(4) - 8 = 0$ ✓

10. $5(x-1)^2 - 3 = 42$

$5(x-1)^2 = 45$

$\sqrt{(x-1)^2} = \sqrt{9}$

$|x-1| = 3$

$x - 1 = 3$ or $x - 1 = -3$

$+1 \quad +1 \qquad +1 \quad +1$

$\boxed{x = 4 \text{ or } x = -2}$

✓ Verify your solution(s):

$5(4-1)^2 - 3$

$5(3)^2 - 3$

$5 \cdot 9 - 3 = 42$ ✓

$5(-2-1)^2 - 3 = 42$

$5(-3)^2 - 3$

$5 \cdot 9 - 3 = 42$ ✓

#11 – 14: Find the roots of each function and simplify your answers. Verify your solutions!

11. $f(x) = x^2 - 75$

$x^2 - 75 = 0$

$\sqrt{x^2} = \sqrt{75}$

$|x| = \sqrt{25 \cdot 3}$

$\boxed{x = \pm 5\sqrt{3}}$

✓Verify your solution(s):

$(5\sqrt{3})^2 - 75 = 0$

$25 \cdot 3 - 75 = 0$ ✓

$(-5\sqrt{3})^2$

$25 \cdot 3 - 75 = 0$ ✓

12. $f(x) = (x+2)^2$

$\sqrt{(x+2)^2} = \sqrt{0}$

$|x+2| = 0$

$x + 2 = 0$

$-2 \quad -2$

$\boxed{x = -2}$

✓ Verify your solution(s):

$(-2+2)^2 = 0$

$0^2 = 0$ ✓

5.2F Solve Quadratic Equations Using Square Roots to Find Real Solutions

#11 – 14 (continued): Find the roots of each function and simplify your answers. Verify your solutions!

13. $f(x)=2(x-1)^2-18$

$2(x-1)^2-18=0$
$2(x-1)^2=18$
$(x-1)^2=9$
$\sqrt{(x-1)^2}=\sqrt{9}$
$|x-1|=3$
$x-1=3$ or $x-1=-3$
$\boxed{x=4 \text{ or } x=-2}$

✓Verify your solution(s):

$2(4-1)^2-18$
$2(3)^2$
$2\cdot 9-18=0$ ✓

$2(-2-1)^2-18$
$2(-3)^2$
$2\cdot 9-18=0$ ✓

14. $f(x)=3x^2-24$

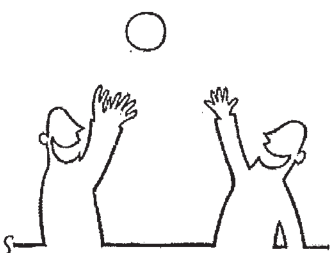
$3x^2-24=0$
$3x^2=24$
$\sqrt{x^2}=\sqrt{8}$
$|x|=2\sqrt{2}$
$\boxed{x=\pm 2\sqrt{2}}$

✓ Verify your solution(s):

$3(2\sqrt{2})^2-24$
$3(4\cdot 2)-24$
$24-24=0$ ✓

$3(-2\sqrt{2})^2-24$
$3(4\cdot 2)-24$
$3\cdot 8-24=0$ ✓

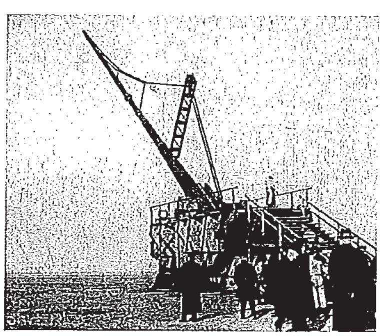
15. The height of a ball in the air, h, at time t can be modeled by the equation $h(t)=-16(t-1)^2+32$. How long does it take for the ball to reach the ground? (round answers to the nearest hundredth)



$-16(t-1)^2+32=0$
$-16(t-1)^2=-32$
$\sqrt{(t-1)^2}=\sqrt{2}$
$|t-1|=\sqrt{2}$

$t-1=\sqrt{2}$ or $t-1=-\sqrt{2}$
$t=1+\sqrt{2}$ $\quad$ $t=1-\sqrt{2}$
$t\approx 2.41$ sec $\quad$ ~~$t\approx -.41$~~ extraneous

16. Big Bertha, a cannon used in WW1, could fire shells incredibly long distances. The page of a shell could be modeled by $y=-0.0196(x-25)^2+12$ where x was the horizontal distance traveled (in miles), and y was the height (in miles). How far could Big Bertha fire a shell? (round answers to the nearest mile)



Big Bertha (Paris Gun), courtesy http://www.militaryimages.net

$-0.0196(x-25)^2+12=0$
$-0.0196(x-25)^2=-12$
$\sqrt{(x-25)^2}=\sqrt{612.245}$
$|x-25|=24.74$

$x-25=24.74$ or $x-25=-24.74$
$+25 \quad 25$ $\quad$ $+25 \quad 25$
$x=49.74$ $\quad$ $x=0.26\approx 0$ miles extraneous
$x\approx 49.74$

Section 5.2F

Name ______________________________ Period __________

5.2G Operations with Complex Expressions

1. Simplify each expression.

a) i^2 $= -1$

b) i^4 $= 1$

c) i^{17} $= i$

d) $-i^{10}$ $= -(-1) = 1$

e) i^{101} $= i$

f) i^{87} $= -i$

g) $\sqrt{-25}$ $\boxed{5i}$

h) $4\sqrt{-9}$
$4(3i) = \boxed{12i}$

i) $5\sqrt{-28}$
$5i\sqrt{4\cdot 7}$
$5i2\cdot\sqrt{7} = \boxed{10i\sqrt{7}}$

j) $7\sqrt{-8}$
$7i\sqrt{8}$
$7i\sqrt{4\cdot 2}$
$7i2\sqrt{2} = \boxed{14i\sqrt{2}}$

k) $-4\sqrt{-10}$
$-4i\sqrt{10} = \boxed{-4i\sqrt{10}}$

l) $5\sqrt{-100}$
$5i\sqrt{100}$
$5i\cdot 10 = \boxed{50i}$

2. Add or subtract each expression.

a) $(5-3i)+(4+7i)$
$\boxed{9+4i}$

b) $(4-8i)+(9+2i)$
$\boxed{13-6i}$

c) $(3-2i)-(5-4i)$
$\boxed{-2+2i}$

d) $6-\left(-5-\sqrt{-9}\right)$
$\boxed{11+3i}$

e) $8i+(9-11i)$
$\boxed{9-3i}$

f) $\left(7-\sqrt{-81}\right)+\left(5-\sqrt{-100}\right)$
$7-9i+5-10i$
$\boxed{12-19i}$

3. Simplify each expression.

a) $3i\cdot 2i$
$6i^2$
$6(-1) = \boxed{-6}$

b) $-6i\cdot 2i$
$-12i^2$
$-12(-1) = \boxed{12}$

c) $4\sqrt{-6}\cdot 2\sqrt{3}$
$4i\sqrt{6}\cdot 2\sqrt{3}$
$8i\sqrt{18}$
$8i\sqrt{9\cdot 2}$
$8i3\cdot\sqrt{2} = \boxed{24i\sqrt{2}}$

d) $-3\sqrt{-20}\cdot 2\sqrt{5}$
$-6i\sqrt{4\cdot 5}\cdot\sqrt{5}$
$-6i\cdot 2\cdot 5 = \boxed{-60i}$

e) $8\sqrt{-2}\cdot 3\sqrt{2}$
$24i\sqrt{2}\sqrt{2} = \boxed{48i}$

f) $-2\sqrt{-6}\cdot 3\sqrt{-3}$
$-6i\sqrt{6}\,i\sqrt{3}$
$-6i^2\sqrt{18}$
$-6(-1)\sqrt{9\cdot 2}$
$6\cdot 3\sqrt{2} = \boxed{18\sqrt{2}}$

Name ______________________ Period ________

5.2G Operations with Complex Expressions

4. Kasem simplified the expression $\sqrt{-4}\cdot\sqrt{-9}$ using various methods.

1 WRONG		2 CORRECT		3 CORRECT	
Line 1	$\sqrt{-4}\cdot\sqrt{-9}$	Line 1	$\sqrt{-4}\cdot\sqrt{-9}$	Line 1	$\sqrt{-4}\cdot\sqrt{-9}$
Line 2	$\sqrt{-4\cdot -9}$	Line 2	$\sqrt{-1\cdot 4}\cdot\sqrt{-1\cdot 9}$	Line 2	$\sqrt{-1\cdot 4}\cdot\sqrt{-1\cdot 9}$
Line 3	$\sqrt{36}$	Line 3	$\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{-1}\cdot\sqrt{9}$	Line 3	$\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{-1}\cdot\sqrt{9}$
Line 4	6	Line 4	$\sqrt{-1}\cdot\sqrt{-1}\cdot\sqrt{4}\cdot\sqrt{9}$	Line 4	$i\cdot 2\cdot i\cdot 3$
		Line 5	$i\cdot i\cdot\sqrt{4\cdot 9}$	Line 5	$2\cdot 3\cdot i\cdot i$
		Line 6	$i^2\cdot\sqrt{36}$	Line 6	$6\cdot i^2$
		Line 7	$(-1)6$	Line 7	$6\cdot(-1)$
		Line 8	-6	Line 8	-6

a) Other than simplifying to the different numeric values, identify 2 or more differences in the process shown used in methods 1 and 2.

1) In method 1, Kasem never used the definition of $i=\sqrt{-1}$ to rewrite either radical
2) Method 1 used the product property of radicals incorrectly; only if a and b are BOTH nonnegative with an even index, is $\sqrt{a}\sqrt{b}=\sqrt{ab}$
Method 2 used the product property of radicals correctly.

b) Although the methods have created different numeric values when simplifying, identify 2 or more similarities in the process shown in methods 1 and 2.

1) Both methods combined $\sqrt{4\cdot 9}=\sqrt{36}$ instead of simplifying each separately.
2) Both methods used fact that $\sqrt{36}=6$

c) Although the methods have created the same numeric value, identify 2 or more differences in the process shown through methods 2 and 3.

1) From line 3 to line 4, multiplication was done in a different order by commutative property for multiplication.
2) Method 3 simplified the radicals $\sqrt{4}$ and $\sqrt{9}$; method 2 multiplied them together.

d) Other than simplifying to the same numeric values, identify 2 or more similarities in the process shown in methods 2 and 3.

1) Both used definition of $i=\sqrt{-1}$ wherever there was a $\sqrt{-1}$.
2) Both used fact that $i^2=-1$

5. Simplify each expression.

a) $\sqrt{-6}\cdot\sqrt{-2}$
$i\sqrt{6}\cdot i\sqrt{2}$
$i^2\sqrt{12}$
$-1\sqrt{4\cdot 3}$
$-1\cdot 2\sqrt{3}$ $= -2\sqrt{3}$

b) $3\sqrt{-5}\cdot 2\sqrt{-8}$
$6i\sqrt{5}\,i\sqrt{4\cdot 2}$
$6i^2\sqrt{5}\cdot 2\cdot\sqrt{2}$
$12(-1)\sqrt{10}$ $= -12\sqrt{10}$

c) $-3\sqrt{-2}\cdot 7\sqrt{-9}$
$-21i\sqrt{2}\cdot 3i$
$-63i^2\sqrt{2}$
$-63(-1)\sqrt{2}$ $= 63\sqrt{2}$

Name ______________________________ Period __________

5.2G *Operations with Complex Expressions*

6. Simplify each expression.

a) $5(1-3i)$

$\boxed{5-15i}$

b) $3(3+i)$

$\boxed{9+3i}$

c) $3\left(2-4\sqrt{-25}\right)$

$6-12\sqrt{-25}$

$6-12\sqrt{-1\cdot 25}$

$6-12i\cdot 5$

$\boxed{6-60i}$

d) $-3i(2+4i)$

$-6i-12i^2$

$-6i-12(-1)$

$-6i+12$

$\boxed{12-6i}$

e) $-5i(-8+2i)$

$40i-10(i^2)$

$40i-10(-1)$

$\boxed{10+40i}$

f) $\sqrt{-10}\left(2-\sqrt{-2}\right)$

$i\sqrt{10}(2-i\sqrt{2})$

$2i\sqrt{10}-i^2\sqrt{20}$

$2i\sqrt{10}-(-1)2\sqrt{5}$

$2i\sqrt{10}+2\sqrt{5}$

$\boxed{2\sqrt{5}+2i\sqrt{10}}$

7. Simplify each expression.

a) $(3+5i)(3+5i)$

$9+30i+25i^2$

$9+30i-25$

$\boxed{-16+30i}$

b) $3(4-2i)^2$

$3(16-16i+4i^2$

$3(16-16i-4)$

$3(12-16i)$

$\boxed{36-48i}$

c) $\left(5-3i\sqrt{2}\right)^2$

$25-30i\sqrt{2}+9\cdot 2i^2$

$25-30i\sqrt{2}-18$

$\boxed{7-30i\sqrt{2}}$

8. Simplify each expression.

a) $4\left(1+3i\sqrt{2}\right)\left(1+3i\sqrt{2}\right)$

$4(1+6i\sqrt{2}+9i^2\cdot 2)$

$4(1+6i\sqrt{2}-18)$

$4(-17+6i\sqrt{2})$

$\boxed{-68+24i\sqrt{2}}$

b) $3\left(4-2i\sqrt{7}\right)^2$

$3(16-16i\sqrt{7}+4i^2\cdot 7)$

$3(16-16i\sqrt{7}-28)$

$3(-12-16i\sqrt{7})$

$\boxed{-36-48i\sqrt{7}}$

c) $3\left(2-i\sqrt{11}\right)^2-5\left(2-i\sqrt{11}\right)+7$

$3(4-4i\sqrt{11}+i^2\cdot 11)$

$3(4-4i\sqrt{11}-11)$

$3(-7-4i\sqrt{11})$

$-21-12i\sqrt{11}-10+5i\sqrt{11}+7$

$\boxed{-24-7i\sqrt{11}}$

Name ______________________ Period ________

5.2G Operations with Complex Expressions

9. Verify that the given answer (value of x) is a solution to the equation.

a) $-2x^2+6=56;\ x=5i$

$-2(5i)^2+6=56$

$-2(25i^2)$

$-2(-25)$

$50+6=56$ ✓

b) $x^2-8x+30=5;\ x=(4+3i)$

$(4+3i)^2-8(4+3i)+30=5$

$16+24i+9i^2$

$16+24i-9$

$7+24i-32-24i+30=5$

$5=5$ ✓

c) $x^2-2x+48=2;\ x=(1-3i\sqrt{5})$

$(1-3i\sqrt{5})^2-2(1-3i\sqrt{5})+48=2$

$1-6i\sqrt{5}+9i^2 5$

$1-6i\sqrt{5}-45$

$-44-6i\sqrt{5}-2+6i\sqrt{5}+48=$

$-46+48$

$2=2$ ✓

10. Simplify each expression (no decimal values allowed).

a) $\dfrac{6+8i}{2}$

$\boxed{3+4i}$

b) $\dfrac{6+10i\sqrt{2}}{2}$

$\boxed{3+5i\sqrt{2}}$

c) $\dfrac{-15-21i\sqrt{6}}{3}$

$\boxed{-5-7i\sqrt{6}}$

d) $\dfrac{-30+\sqrt{-75}}{5}$

$\sqrt{-1\cdot 25\cdot 3}$

$\dfrac{-30+5i\sqrt{3}}{5}=$

$\boxed{-6+i\sqrt{3}}$

11. Verify that the given answer (value of x) is a solution to the equation.

$2x^2-6x+8=3;\ x=\dfrac{3+i}{2}$

$2\left(\frac{3+i}{2}\right)^2-6\left(\frac{3+i}{2}\right)+8=3$

$2\left(\frac{9+6i+i^2}{4}\right)-3(3+i)+8$

$2\left(\frac{8+6i}{4}\right)-9-3i+8$

$\frac{8+6i}{2}$

$4+3i-1-3i$

$3=3$ ✓

Section 5.2G

Name ______________________ Period ________

5.2H Solve Quadratic Equations Using Square Roots to Find Real or Complex Solutions

#1 – 6: Simplify the following radicals.

1. $\sqrt{-9}$

$\boxed{3i}$

2. $7\sqrt{-50}$

$7\sqrt{-1\cdot 25\cdot 2}$

$7i5\sqrt{2}$

$\boxed{35i\sqrt{2}}$

3. $2\sqrt{-3}$

$\boxed{2i\sqrt{3}}$

4. $3\sqrt{-100}$

$\boxed{30i}$

5. $11\sqrt{-18}$

$11\sqrt{-1\cdot 9\cdot 2}$

$\boxed{33i\sqrt{2}}$

6. $\sqrt{-121}$

$\boxed{11i}$

7. What is the value of i^2? $\boxed{-1}$

#8 – 13: Simplify the following complex expressions.

8. $(3i)^2$

$9i^2$

$\boxed{-9}$

9. $(-5i)^2$

$25i^2$

$\boxed{-25}$

10. $(-i)^2$

$i^2 =$

$\boxed{-1}$

11. $(3-5i)^2$

$9-30i+25i^2$

$9-30i-25$

$\boxed{-16-30i}$

12. $2(3+4i)^2$

$2(9+24i+16i^2)$

$2(-7+24i)$

$\boxed{-14+48i}$

13. $(-2-7i)^2+5$

$4+28i+49i^2+5$

$4+28i-49+5$

$\boxed{-40+28i}$

#14 – 17: Verify that each of the following values are solutions to the given equation. Show all of your work.

14. $-2x^2+3=21;\ x=3i, x=-3i$

$-2(3i)^2+3 = 21$

$-2(9i^2)$

$-2(9)(-1)$

$18+3 = 21$ ✓

$-2(-3i)^2+3 = 21$

$-2(9i^2)$

$-18i^2$

$18+3 = 21$ ✓

15. $(x-5)^2-1=-17;\ x=5+4i, x=5-4i$

$((5+4i)-5)^2-1 = -17$

$(4i)^2$

$16i^2$

$-16 - 1 = -17$ ✓

$((5-4i)-5)^2-1 = -17$

$(-4i)^2-1$

$16i^2$

$-16 - 1 = -17$ ✓

Name ______________________________ Period __________

5.2H Solve Quadratic Equations Using Square Roots to Find Real or Complex Solutions

#14 – 17 (continued): Verify that each of the following values are solutions to the given equation. Show all of your work.

16. $x^2+11=7;\ x=2i, x=-2i$

$(2i)^2+11=7$

$4i^2+11$

$4(-1)+11=7$ ✓

$(-2i)^2+11=7$

$4i^2$

$-4+11=7$ ✓

17. $(x+2)^2=-25;\ x=-2+5i, x=-2-5i$

$((-2+5i)+2)^2=-25$

$(5i)^2$

$25i^2$

$25(-1)=-25$ ✓

$((-2-5i)+2)^2=-25$

$(-5i)^2$

$25i^2$

$25(-1)=-25$ ✓

#18 – 21: Solve each equation for real or complex solutions. Verify your solutions.

18. $x^2+3=51$

$\sqrt{x^2}=\sqrt{48}$

$|x|=\sqrt{16\cdot 3}$

$\boxed{x=\pm 4\sqrt{3}}$

✓Verify your solution(s):

$(4\sqrt{3})^2+3=51$

$16\cdot 3+3=51$ ✓

$(-4\sqrt{3})^2+3$

$16\cdot 3+3=51$ ✓

19. $\sqrt{(x-1)^2}=\sqrt{-24}$

$|x-1|=\sqrt{-1\cdot 4\cdot 6}$

$x-1=\pm 2i\sqrt{6}$

$+1\quad +1$

$\boxed{x=1+2i\sqrt{6},\ x=1-2i\sqrt{6}}$

✓ Verify your solution(s):

$((1+2i\sqrt{6})-1)^2=-24$

$(2i\sqrt{6})^2$

$4i^2\cdot 6$

$24(-1)=-24$ ✓

$((1-2i\sqrt{6})-1)^2=-24$

$(-2i\sqrt{6})^2$

$4i^2\cdot 6$

$24(-1)=-24$ ✓

Name ______________________ Period ________.

5.2H Solve Quadratic Equations Using Square Roots to Find Real or Complex Solutions

#18 – 21 (continued): Solve each equation for real or complex solutions. Verify your solutions.

20. $3x^2 - 27 = 0$

$3x^2 = 27$

$\sqrt{x^2} = \sqrt{9}$

$|x| = 3$

$\boxed{x = 3 \text{ or } x = -3}$

✓Verify your solution(s):

$3(3)^2 - 27 = 0$

$3 \cdot 9 - 27 = 0$ ✓

$3(-3)^2 - 27$

$3 \cdot 9 - 27 = 0$ ✓

21. $5(x+1)^2 - 3 = -48$

$+3 \quad +3$

$5(x+1)^2 = -45$

$\sqrt{(x+1)^2} = \sqrt{-9}$

$|x+1| = 3i$

$x+1 = \pm 3i$

$\boxed{x = -1 + 3i,\ x = -1 - 3i}$

✓ Verify your solution(s):

$5((-1+3i)+1)^2 - 3 = -48$

$5(3i)^2$

$5 \cdot 9i^2$

$45i^2$

$-45 - 3 = -48$ ✓

$5((-1-3i)+1)^2 - 3 = -48$

$5(-3i)^2$

$5 \cdot 9i^2$

$45i^2$

$-45 - 3 = -48$ ✓

22. The height, h, of a water balloon (in feet) at time t (in seconds) is given by the equation $h(t) = -16(t - 0.45)^2 + 32$. If a student throws the balloon and it lands on the ground, how long is the balloon in the air? Verify your solution(s).

$0 = -16(t - .45)^2 + 32$

$-32 = -16(t - 0.45)^2$

$\sqrt{2} = \sqrt{(t - 0.45)^2}$

$\pm\sqrt{2} = |t - 0.45|$

* $\boxed{t = 0.45 + \sqrt{2}}$, $t = 0.45 - \sqrt{2} \approx -.96$

$t \approx 1.86$ sec

extraneous

$-16(1.86 - .45)^2 + 32 = 0$

$(1.41)^2$

$-16(1.9881) + 32$

* $0.1904 \approx 0$ ✓

*Using $t \approx 1.8642$ to 4 decimal places yields $0 = 0$ ✓

#23 – 26: Find the real and/or complex roots of each function. Verify your solutions!

23. $f(x) = x^2 - 125$

$x^2 - 125 = 0$

$\sqrt{x^2} = \sqrt{125}$

$|x| = \sqrt{25 \cdot 5}$

$\boxed{x = 5\sqrt{5} \text{ or } x = -5\sqrt{5}}$

✓Verify your solution(s):

$(5\sqrt{5})^2 - 125 = 0$

$25 \cdot 5 - 125 = 0$ ✓

$(-5\sqrt{5})^2 - 125 = 0$

$25 \cdot 5 - 125 = 0$ ✓

24. $f(x) = (x+7)^2$

$\sqrt{(x+7)^2} = \sqrt{0}$

$|x+7| = 0$

$x+7 = 0$

$\boxed{x = -7}$

✓ Verify your solution(s):

$(-7+7)^2 = 0$

$0^2 = 0$ ✓

Name ______________________________ Period __________

5.2H *Solve Quadratic Equations Using Square Roots to Find Real or Complex Solutions*

#23 – 26 (continued): Find the real and/or complex roots of each function. Verify your solutions!

25. $f(x) = -2(x-2)^2 - 18$

$-2(x-2)^2 - 18 = 0$

$-2(x-2)^2 = 18$

$\sqrt{(x-2)^2} = \sqrt{-9}$

$|x-2| = 3i$

$x-2 = \pm 3i$

$\boxed{x = 2+3i,\ x = 2-3i}$

✓Verify your solution(s):

$-2(2+3i-2)^2 - 18 = 0$

$-2(3i)^2$

$9i^2$

$-2(-9) - 18 = 0$ ✓

$-2(2-3i-2)^2 - 18 = 0$

$-2(-3i)^2$

$9i^2$

$-2(-9) - 18 = 0$ ✓

26. $f(x) = 4x^2 + 24$

$4x^2 + 24 = 0$

$4x^2 = -24$

$\sqrt{x^2} = \sqrt{-6}$

$|x| = i\sqrt{6}$

$\boxed{x = i\sqrt{6} \text{ or } -i\sqrt{6}}$

✓ Verify your solution(s):

$4(i\sqrt{6})^2 + 24 = 0$

$4 \cdot i^2 \cdot 6$

$-24 + 24 = 0$ ✓

$4(-i\sqrt{6})^2 + 24 = 0$

$4(i^2 \cdot 6)$

$-24 + 24 = 0$ ✓

27. The area of a square can be found using the formula $A = s^2$, where "*A*" is the area and "*s*" is the length of one side. If the area of a square is 50 square inches, what is the length of one side? Round your answer to the nearest thousandth and verify your solution(s).

$A = s^2$

$\sqrt{50} = \sqrt{s^2}$

$\sqrt{25 \cdot 2} = |s|$

$\pm 5\sqrt{2} = s$

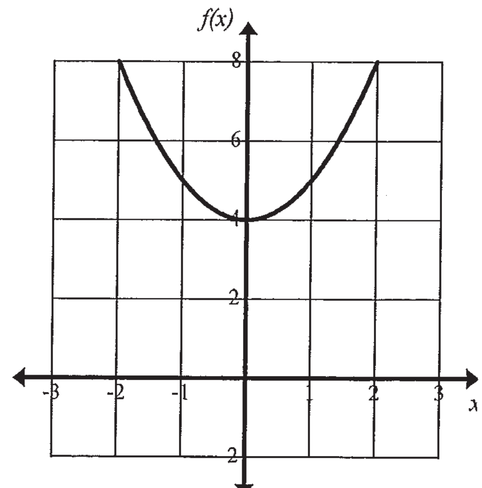
But length is positive

$\boxed{s \approx 7.071 \text{ in}}$

$(7.071)^2 = 50$

$49.999 \approx 50$ ✓

28. The function $f(x) = x^2 + 4$ has no *x*-intercepts, as shown in the graph to the right. Use algebra to show that no *real roots* exist for this function.



$x^2 + 4 = 0$

$\sqrt{x^2} = \sqrt{-4}$

$|x| = 2i$

$\boxed{x = 2i \text{ or } x = -2i}$

Both roots are imaginary

Section 5.2H

Name ______________________ Period __________

5.2I Solving Quadratic Equations by Completing the Square to Find Rational Solutions

#1 – 3: Solve using square roots.

1. $\sqrt{(x-2)^2} = \sqrt{25}$

$|x-2| = 5$

$x-2 = 5$ or $x-2 = -5$

$\boxed{x = 7 \text{ or } x = -3}$

2. $\sqrt{(x+4)^2} = \sqrt{16}$

$|x+4| = 4$

$x+4 = 4$ or $x+4 = -4$

$\boxed{x = 0 \text{ or } x = -8}$

3. $\sqrt{121} = \sqrt{(x-7)^2}$

$|x-7| = 11$

$x-7 = 11$ or $x-7 = -11$

$\boxed{x = 18 \text{ or } x = -4}$

4. Find and explain the error made when solving the following equation. $x^2 + 10x = 24$

Line 1	$x^2 + 10x = 24$
Line 2	$x^2 + 10x + 25 = 24$
Line 3	$(x+5)^2 = 24$
Line 4	$\sqrt{(x+5)^2} = \sqrt{24}$
Line 5	$\lvert x+5\rvert = \sqrt{24}$
Line 6	$x+5 = \pm 2\sqrt{6}$
Line 7	$x+5 = 2\sqrt{6}$ and $x+5 = -2\sqrt{6}$
Line 8	$x = -5 + 2\sqrt{6}$ $\quad x = -5 - 2\sqrt{6}$
Line 9	$x = -5 \pm 2\sqrt{6}$

* Forgot to add 25 to Both Sides of the Equation.

#5 – 6: Fill in the missing value to create a perfect square trinomial. Then solve by *Completing the Square*.

5. $40 + \underline{9} = x^2 + 6x + \underline{9}$

$\sqrt{49} = \sqrt{(x+3)^2}$

$7 = |x+3|$

$x+3 = 7$ or $x+3 = -7$

$x = 4$ or $x = -10$

6. $x^2 - 18x + \underline{81} = 88 + \underline{81}$

$\sqrt{(x-9)^2} = \sqrt{169}$

$|x-9| = 13$

$x-9 = 13$ or $x-9 = -13$

$x = 22$ or $x = -4$

#7 – 10: Solve by *Completing the Square*. Then verify your solutions.

7. $29 = x^2 + 28x$

$+196 \qquad +196$

$\sqrt{225} = \sqrt{(x+14)^2}$

$15 = |x+14|$

$x+14 = 15$ or $x+14 = -15$

$x = 1$ or $x = -29$

checks

$(1)^2 + 28(1) = 29$ ✓

$(-29)^2 + 28(-29) =$

$841 - 812 = 29$ ✓

8. $x^2 - 10x - 56 = 0$

$x^2 - 10x + 25 = 56 + 25$

$\sqrt{(x-5)^2} = \sqrt{81}$

$x-5 = 9$ or $x-5 = -9$

$x = 14$ or $x = -4$

Checks:

$(14)^2 - 10(14) - 56$

$196 - 140 - 56 = 0$ ✓

$(-4)^2 - 10(-4) - 56$

$16 + 40 - 56 = 0$ ✓

9. $-23 = x^2 - 12x + 13$

$-13 \qquad -13$

$-36 = x^2 - 12x$

$+36 \qquad +36$

$\sqrt{0} = \sqrt{(x-6)^2}$

$|x-6| = 0$

$x-6 = 0$, $x = 6$

Ch:

$(6)^2 - 12(6) + 13 =$

$36 - 72 + 13 = -23$ ✓

10. $x^2 + 7 = 30x - 74$

$x^2 - 30x = -81$

$x^2 - 30x + 225 = -81 + 225$

$\sqrt{(x-15)^2} = \sqrt{144}$

$|x-15| = 12$

$x-15 = 12$ or $x-15 = -12$

$x = 27$ or $x = 3$

Checks:

$(27)^2 + 7 = 30(27) - 74$

$736 = 736$ ✓

$(3)^2 + 7 = 30(3) - 74$

$16 = 16$ ✓

Name ______________________________ Period __________

5.2I Solving Quadratic Equations by Completing the Square to Find Rational Solutions

11. The product of two consecutive positive even integers is 528. What are the numbers?

(Solve by Completing the Square)

Let n = smaller of 2 pos. ints

$n+2$ = next consecutive pos int.

$n(n+2) = 528$

$n^2+2n+1 = 528+1$

$\sqrt{(n+1)^2} = \sqrt{529}$

$|n+1| = 23$

$n+1 = 23$ or $n+1 = -23$

$-1 \quad -1$

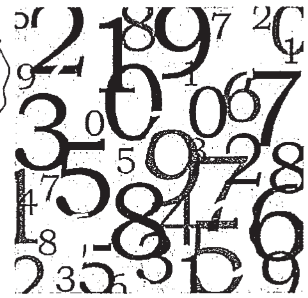
$n = -24$ extraneous

$\boxed{n = 22 \quad n+2 = 24}$

The 2 consecutive positive even integers are 22 & 24.

Check

$22(24) = 528$ ✓



12. A square garden is altered so that one dimension is decreased by 3 yards, while the other dimension is increased by 5 yards. The area of the resulting rectangle is 20 square yards. Find the length of each side of the original garden and its area. (Solve by Completing the Square)

x = length of side on square

$(x-3)(x+5) = 20$

$x^2+2x-15 = 20$

$x^2+2x+1 = 35+1$

$\sqrt{(x+1)^2} = \sqrt{36}$

$|x+1| = 6$

$x+1 = 6$ or $\cancel{x+1 = -6}$ extraneous $\cancel{x = -7}$

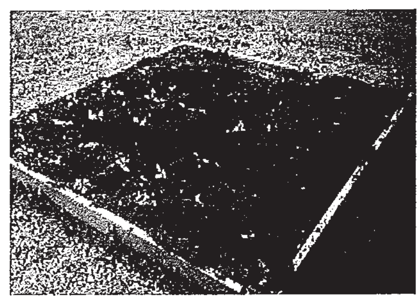
$\boxed{x = 5 \text{ yds}}$

$\boxed{\text{Area} = 5^2 = 25 \text{ yd}^2}$

Check

$(5-3)(5+5) = 20$

$2(10) = 20$ ✓



13. A foul ball leaves the end of a baseball bat and travels according to the formula $h(t) = -16t^2 + 64t$ where h is the height of the ball in feet and t is the time in seconds. How long will it take for the ball to reach a height of 64 feet in the air? (Solve by Completing the Square)

$\frac{-16t^2+64t}{-16} = \frac{64}{-16}$

$t^2-4t = -4$

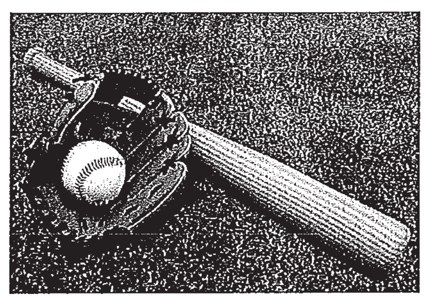
$t^2-4t+4 = -4+4$

$\sqrt{(t-2)^2} = \sqrt{0}$

$|t-2| = 0$

$t-2 = 0$

$\boxed{t = 2 \text{ secs}}$



Section 5.2I

Name ______________________________ Period __________

5.2J *Solving Quadratic Equations by Completing the Square to Find Real Solutions*

#1 – 3: Solve using square roots.

1. $\sqrt{(x+7)^2} = \sqrt{15}$

$|x+7| = \sqrt{15}$

$x+7 = 15$ or $x+7 = -15$

$x = -7+\sqrt{15}$ $\quad x = -7-\sqrt{15}$

$\boxed{x = -7 \pm \sqrt{15}}$

2. $\sqrt{(x-9)^2} = \sqrt{12}$

$|x-9| = 2\sqrt{3}$

$x-9 = \pm 2\sqrt{3}$

$\boxed{x = 9 \pm 2\sqrt{3}}$

3. $\sqrt{32} = \sqrt{(x-5)^2}$

$|x-5| = 4\sqrt{2}$

$x-5 = \pm 4\sqrt{2}$

$\boxed{x = 5 \pm 4\sqrt{2}}$

#4 – 5: Fill in the missing value to create a perfect square trinomial. Then solve by *Completing the Square*.

4. $7 + \underline{\frac{25}{4}} = x^2 + 5x + \underline{\frac{25}{4}}$

$\frac{28}{4} + \frac{25}{4} = (x + \frac{5}{2})^2$

$\sqrt{\frac{53}{4}} = \sqrt{(x+\frac{5}{2})^2}$

$\frac{\sqrt{53}}{2} = |x + \frac{5}{2}|$

$\pm \frac{\sqrt{53}}{2} = x + \frac{5}{2}$

$\boxed{x = \frac{-5 \pm \sqrt{53}}{2}}$

5. $x^2 - 7x + \underline{\frac{49}{4}} = -3 + \underline{\frac{49}{4}}$

$(x - \frac{7}{2})^2 = \frac{-12}{4} + \frac{49}{4}$

$\sqrt{(x-\frac{7}{2})^2} = \sqrt{\frac{37}{4}}$

$|x - \frac{7}{2}| = \frac{\sqrt{37}}{2}$

$x - \frac{7}{2} = \pm \frac{\sqrt{37}}{2}$

$\boxed{x = \frac{7 \pm \sqrt{37}}{2}}$

#6 – 9: Solve by *Completing the Square*. Then verify your solutions.

(see next page)

6. $-88 = x^2 + 28x$

$+196 \qquad +196$

$\sqrt{108} = \sqrt{(x+14)^2}$

$6\sqrt{3} = |x+14|$

$\pm 6\sqrt{3} = x+14$

$\boxed{x = -14 \pm 6\sqrt{3}}$

7. $2x^2 - 7x - 13 = -10$

$2x^2 - 7x = 3$

$x^2 - \frac{7}{2}x + \frac{49}{16} = \frac{3}{2} + \frac{49}{16}$

$\sqrt{(x-\frac{7}{4})^2} = \sqrt{\frac{73}{16}}$

$|x - \frac{7}{4}| = \frac{\sqrt{73}}{4}$

$x - \frac{7}{4} = \pm \frac{\sqrt{73}}{4}$

$\boxed{x = \frac{7 \pm \sqrt{73}}{4}}$

8. $8 = 4x^2 + 4x - 13$

$21 = 4x^2 + 4x$

$\frac{21}{4} = x^2 + x$

$\frac{21}{4} + \frac{1}{4} = x^2 + x + \frac{1}{4}$

$\sqrt{\frac{22}{4}} = \sqrt{(x+\frac{1}{2})^2}$

$\frac{\sqrt{22}}{2} = |x + \frac{1}{2}|$

$\pm \frac{\sqrt{22}}{2} = x + \frac{1}{2}$

$\boxed{x = \frac{-1 \pm \sqrt{22}}{2}}$

9. $9x^2 - 1 = 6x$

$9x^2 - 6x = 1$

$x^2 - \frac{2}{3}x = \frac{1}{9}$

$x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{1}{9} + \frac{1}{9}$

$\sqrt{(x-\frac{1}{3})^2} = \sqrt{\frac{2}{9}}$

$|x - \frac{1}{3}| = \frac{\sqrt{2}}{3}$

$x - \frac{1}{3} = \pm \frac{\sqrt{2}}{3}$

$\boxed{x = \frac{1 \pm \sqrt{2}}{3}}$

10. For which values of y, given $y = x^2 + bx + \left(\frac{b}{2}\right)^2$, will you find 2 real solutions? 1 real solution?

when $y > 0$, there will be 2 real solutions.

when $y = 0$, there will be 1 real solution.

P-35 Checks

#6 $\boxed{-88 = x^2 + 28x}$ Verifying $x = -14 \pm 6\sqrt{3}$

$$-88 = (-14+6\sqrt{3})^2 + 28(-14+6\sqrt{3})$$
$$= (196 - 168\sqrt{3} + 108)$$
$$= 304 - 168\sqrt{3} \quad - 392 + 168\sqrt{3}$$
$$-88 = -88 \checkmark$$

Also

$$-88 = (-14-6\sqrt{3})^2 + 28(-14-6\sqrt{3})$$
$$= 196 + 168\sqrt{3} + 108$$
$$= 304 + 168\sqrt{3} \quad - 392 - 168\sqrt{3}$$
$$-88 = -88 \checkmark$$

#7 $\boxed{2x^2 - 7x - 13 = -10}$ Verifying $x = \frac{7 \pm \sqrt{73}}{4}$

$$2\left(\frac{7+\sqrt{73}}{4}\right)^2 - 7\left(\frac{7+\sqrt{73}}{4}\right) - 13 = -10$$
$$2\left(\frac{49+14\sqrt{73}+73}{16}\right)$$
$$\frac{122+14\sqrt{73}}{8}$$
$$\frac{61+7\sqrt{73}}{4} \quad \frac{-49-7\sqrt{73}}{4}$$
$$\frac{12}{4} \quad - 13$$
$$3 \quad - 13 = -10 \checkmark$$

Also

$$2\left(\frac{7-\sqrt{73}}{4}\right)^2 - 7\left(\frac{7-\sqrt{73}}{4}\right) - 13 = -10$$
$$2\left(\frac{49-14\sqrt{73}+73}{16}\right)$$
$$\frac{122-14\sqrt{73}}{8}$$
$$\frac{61-7\sqrt{73}}{4} \quad \frac{-49+7\sqrt{73}}{4}$$
$$\frac{12}{4} \quad - 13$$
$$3 \quad - 13 = -10 \checkmark$$

P-35 checks

#8) $\boxed{4x^2+4x-13 = 8}$ Verifying $x = \frac{-1 \pm \sqrt{22}}{2}$

$4\left(\frac{-1+\sqrt{22}}{2}\right)^2 + 4\left(\frac{-1+\sqrt{22}}{2}\right) - 13 = 8$

$4\left(\frac{1-2\sqrt{22}+22}{4}\right)$

$23 - 2\sqrt{22} \quad - 2 + 2\sqrt{22} \quad - 13$

$21 \qquad -13 = 8$ ✓

Also

$4\left(\frac{-1-\sqrt{22}}{2}\right)^2 + 4\left(\frac{-1-\sqrt{22}}{2}\right) - 13 = 8$

$4\left(\frac{1+2\sqrt{22}+22}{4}\right) + 2(-1-\sqrt{22}) \quad -13$

$23 + 2\sqrt{22} \quad - 2 - 2\sqrt{22} \quad -13$

$21 \qquad -13$

$8 = 8$ ✓

#9) $\boxed{9x^2 - 1 = 6x}$ verifying $x = \frac{1 \pm \sqrt{2}}{3}$

$9\left(\frac{1+\sqrt{2}}{3}\right)^2 - 1 = 6\left(\frac{1+\sqrt{2}}{3}\right)$

$9\left(\frac{1+2\sqrt{2}+2}{9}\right) = 2(1+\sqrt{2})$

$3 + 2\sqrt{2} \quad -1 =$

$2 + 2\sqrt{2} = 2 + 2\sqrt{2}$ ✓

Also

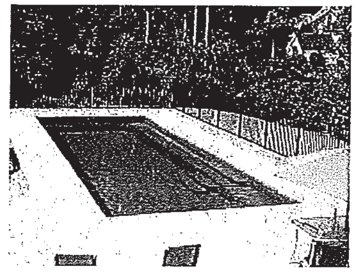
$9\left(\frac{1-\sqrt{2}}{3}\right)^2 - 1 = 6\left(\frac{1-\sqrt{2}}{3}\right)$

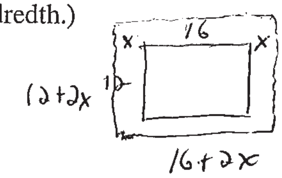
$9\left(\frac{1-2\sqrt{2}+2}{9}\right) - 1 = 2(1-\sqrt{2})$

$3 - 2\sqrt{2} \quad -1$

$2 - 2\sqrt{2} = 2 - 2\sqrt{2}$ ✓

11. A pool measuring 12 meters by 16 meters is to have a sidewalk installed all around it, increasing the total area to 285 square meters. What will be the width of the sidewalk? (Solve by Completing the Square and round your solution to the nearest hundredth.)





$w \cdot l = 285$

$(12+2x)(16+2x) = 285$

$192 + 24x + 32x + 4x^2 = 285$

$\dfrac{4x^2 + 56x}{4} = \dfrac{93}{4}$

$x^2 + 14x + 49 = \frac{93}{4} + 49$

$\sqrt{(x+7)^2} = \sqrt{72.25}$

$|x+7| = 8.5$

$x+7 = 8.5$ or $x+7 = -8.5$

$\boxed{x = 1.5}$ $\quad x = -15.5$ extraneous

width $12+2(1.5) = 15$ meters
length $16+2(1.5) = 19$ meters

$15 \cdot 19 = 285$ ✓

12. The height h in feet of an arrow shot upward from the top of a 96-foot tall tower when time $t = 0$ is given by $h(t) = -16t^2 + 80t + 96$. How long will it take the arrow to strike the ground? (Solve by Completing the Square and round your solution to the nearest hundredth.)

$-16t^2 + 80t + 96 = 0$

$t^2 - 5t - 6 = 0$

$t^2 - 5t + \frac{25}{4} = 6 + \frac{25}{4}$

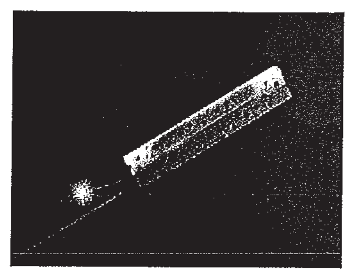
$\sqrt{(t - \frac{5}{2})^2} = \sqrt{\frac{49}{4}}$

$|t - \frac{5}{2}| = \frac{7}{2}$

$t - \frac{5}{2} = \frac{7}{2}$ or $t - \frac{5}{2} = \frac{-7}{2}$

$t = \frac{12}{2} = 6$ seconds $\quad t = -1$ extraneous

13. The height h in feet of a bottle rocket launched from a deck 8 feet above the ground is given by $h(t) = -16t^2 + 240t + 8$, where t is the time in seconds.



a) What is the height after 2 seconds?

$h(2) = -16(2)^2 + 240(2) + 8$

$\boxed{h(2) = 424 \text{ ft}}$

b) At what times will the rocket be at a height of 400 feet? (Solve by Completing the Square and round your solution to the nearest hundredth.)

$-16t^2 + 240t + 8 = 400$

$\dfrac{-16t^2 + 240t}{-16} = \dfrac{392}{-16}$

$t^2 - 15t + \frac{225}{4} = -24.5 + \frac{225}{4}$

$(t - \frac{15}{2})^2 = 31.75$

$\sqrt{(t - \frac{15}{2})^2} = \sqrt{31.75}$

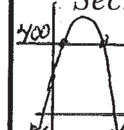
$|t - 7.5| = 5.6347$

$t - 7.5 = 5.6347$ or $t - 7.5 = -5.6347$

$\boxed{t \approx 13.13 \text{ secs OR } t = 1.87 \text{ secs}}$

Both solns are correct

Section 5.2J



Name ____________________ Period __________

5.2K *Solving Quadratic Equations by Completing the Square to Find Real or Complex Solutions*

#1 – 3: Solve using square roots.

1. $\sqrt{(x-5)^2} = \sqrt{-144}$

$|x-5| = 12i$

$\boxed{x = 5 \pm 12i}$

2. $\sqrt{(x-7)^2} = \sqrt{-24}$

$|x-7| = 2i\sqrt{6}$

$\boxed{x = 7 \pm 2i\sqrt{6}}$

3. $\sqrt{-128} = \sqrt{(x+13)^2}$

$8i\sqrt{2} = |x+13|$

$\boxed{x = -13 \pm 8i\sqrt{2}}$

#4 – 5: Fill in the missing value to create a perfect square trinomial. Then solve by *Completing the Square*.

4. $-40 + \underline{36} = x^2 + 12x + \underline{36}$

$\sqrt{-4} = \sqrt{(x+6)^2}$

$2i = |x+6|$

$\boxed{x = -6 \pm 2i}$

5. $x^2 - 24x + \underline{144} = -216 + \underline{144}$

$\sqrt{(x-12)^2} = \sqrt{-72}$

$|x-12| = 6i\sqrt{2}$

$\boxed{x = 12 \pm 6i\sqrt{2}}$

#6 – 9: Solve by *Completing the Square*. Then verify your solutions. (see next page)

6. $4x = x^2 + 5x + 4$

$\frac{1}{4} + -4 = x^2 + x + \frac{1}{4}$

$\sqrt{\frac{-15}{4}} = \sqrt{(x+\frac{1}{2})^2}$

$\frac{i\sqrt{15}}{2} = |x + \frac{1}{2}|$

$\boxed{x = \frac{-1 \pm i\sqrt{15}}{2}}$

7. $x^2 - 4x + 20 = 0$

$x^2 - 4x + 4 = -20 + 4$

$\sqrt{(x-2)^2} = \sqrt{-16}$

$|x-2| = 4i$

$\boxed{x = 2 \pm 4i}$

8. $6x + 23 = 10x^2 + 26$

$10x^2 - 6x = -3$

$x^2 - \frac{3}{5}x = \frac{-3}{10}$

$x^2 - \frac{3}{5}x + \frac{9}{100} = -\frac{3}{10} + \frac{9}{100}$

$\sqrt{(x-\frac{3}{10})^2} = \sqrt{\frac{-21}{100}}$

$|x - \frac{3}{10}| = \frac{i\sqrt{21}}{10}$

$\boxed{x = \frac{3 \pm i\sqrt{21}}{10}}$

9. $-5 = 8x^2 + 6x$

$x^2 + \frac{3}{4}x + \frac{9}{64} = -\frac{5}{8} + \frac{9}{64}$

$\sqrt{(x+\frac{3}{8})^2} = \sqrt{\frac{-31}{64}}$

$|x + \frac{3}{8}| = \frac{i\sqrt{31}}{8}$

$\boxed{x = \frac{-3 \pm i\sqrt{31}}{8}}$

p-37 checks

#6 $\boxed{4x = x^2 + 5x + 4}$ Verifying $x = \frac{-1 \pm i\sqrt{15}}{2}$

$4\left(\frac{-1+i\sqrt{15}}{2}\right) = \left(\frac{-1+i\sqrt{15}}{2}\right)^2 + 5\left(\frac{-1+i\sqrt{15}}{2}\right) + 4$

$2(-1+i\sqrt{15}) \qquad \frac{1-2i\sqrt{15}-15}{4}$

$-2 + 2i\sqrt{15} = \frac{-14-2i\sqrt{15}}{4}$

$= \frac{-7-i\sqrt{15}}{2} + \frac{-5+5i\sqrt{15}}{2} + \frac{8}{2}$

$= \frac{-4+4i\sqrt{15}}{2}$

$= -2 + 2i\sqrt{15}$ ✓

Also

$4\left(\frac{-1-i\sqrt{15}}{2}\right) = \left(\frac{-1-i\sqrt{15}}{2}\right)^2 + 5\left(\frac{-1-i\sqrt{15}}{2}\right) + 4$

$2(-1-i\sqrt{15}) = \frac{1+2i\sqrt{15}-15}{4}$

$-2-2i\sqrt{15}$

$= \frac{-14+2i\sqrt{15}}{4}$

$\frac{-7+i\sqrt{15}}{2} \quad \frac{-5-5i\sqrt{15}}{2} + \frac{8}{2}$

$= \frac{-4-4i\sqrt{15}}{2}$

$-2-2i\sqrt{15} = -2-2i\sqrt{15}$ ✓

#7 $\boxed{x^2 - 4x + 20 = 0}$ Verifying $x = 2 \pm 4i$

$(2+4i)^2 - 4(2+4i) + 20 = 0$

$4 + 16i - 16 - 8 - 16i + 20$

$0 = 0$ ✓

Also

$(2-4i)^2 - 4(2-4i) + 20 =$

$4 - 16i - 16 - 8 + 16i + 20 =$

$0 = 0$ ✓

#8) $\boxed{6x + 23 = 10x^2 + 26}$ Verifying $x = \dfrac{3 \pm i\sqrt{21}}{10}$

$$6\left(\frac{3+i\sqrt{21}}{10}\right) + 23 = 10\left(\frac{3+i\sqrt{21}}{10}\right)^2 + 26$$

$$-23 \qquad\qquad -23$$

$$3\left(\frac{3+i\sqrt{21}}{5}\right)$$

$$\frac{9+3i\sqrt{21}}{5} = 10\left(\frac{9+6i\sqrt{21}-21}{100}\right) + 3$$

$$= \frac{-12+6i\sqrt{21}}{10}$$

$$= \frac{-6+3i\sqrt{21}}{5} + \frac{15}{5}$$

$$= \frac{9+3i\sqrt{21}}{5} \checkmark$$

Also

$$6\left(\frac{3-i\sqrt{21}}{10}\right) + 23 = 10\left(\frac{3-i\sqrt{21}}{10}\right)^2 + 26$$

$$-23 \qquad\qquad -23$$

$$3\left(\frac{3-i\sqrt{21}}{5}\right) = 10\left(\frac{9-6i\sqrt{21}-21}{100}\right) + 3$$

$$\frac{9-3i\sqrt{21}}{5} = \frac{-12-6i\sqrt{21}}{10} + 3$$

$$= \frac{-6-3i\sqrt{21}}{5} + \frac{15}{5}$$

$$= \frac{9-3i\sqrt{21}}{5} \checkmark$$

#9 $\boxed{-5 = 8x^2 + 6x}$ Verifying $x = \frac{-3 \pm i\sqrt{31}}{8}$

$$8\left(\frac{-3+i\sqrt{31}}{8}\right)^2 + 6\left(\frac{-3+i\sqrt{31}}{8}\right) = -5$$

$$8\left(\frac{9-6i\sqrt{31}-31}{64}\right) + \frac{-18+6i\sqrt{31}}{8} =$$

$$\frac{-22-6i\sqrt{31}}{8}$$

$$\frac{-11-3i\sqrt{31}}{4} + \frac{-9+3i\sqrt{31}}{4} =$$

$$\frac{-20}{4} = -5 \checkmark$$

Also

$$8\left(\frac{-3-i\sqrt{31}}{8}\right)^2 + 6\left(\frac{-3-i\sqrt{31}}{8}\right) = -5$$

$$8\left(\frac{9+6i\sqrt{31}-31}{64}\right) + \frac{-18-6i\sqrt{31}}{8}$$

$$\frac{-22+6i\sqrt{31}}{8}$$

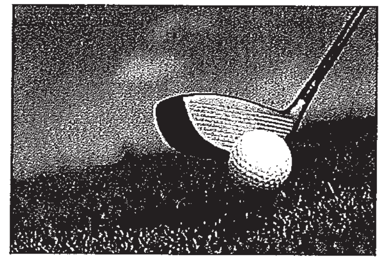
$$\frac{-11+3i\sqrt{31}}{4} + \frac{-9-3i\sqrt{31}}{4}$$

$$-\frac{20}{4} = -5 \checkmark$$

Name ______________________________ Period __________

5.2K *Solving Quadratic Equations by Completing the Square to Find Real or Complex Solutions*

10. Emma hits a golf ball off the tee. The height of the ball is given by $h(x) = -16x^2 + 4000x + 3248$ where h is the height in yards above the ground and x is the horizontal distance from the tee in yards. How far does Emma hit the ball? (Solve by Completing the Square and round your solution to the nearest hundredth.)



$-16x^2 + 4000x = -3248$

$x^2 - 250x + 15625 = 203 + 15625$

$\sqrt{(x-125)^2} = \sqrt{15828}$

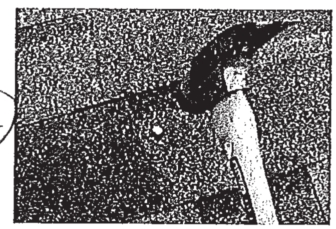
$|x-125| = 125.81$

$x - 125 = \pm 125.81$

$x \approx 250.81$ yds

$x \approx 0.81$ extraneous

11. Gail and Veronica are fixing a leak in a roof. Gail is working on the roof and Veronica is tossing up supplies to Gail. When Gail tosses up a tape measure, the height h, in feet, of the object above the ground t seconds after Veronica tosses it is $h(t) = -16t^2 + 32t + 5$. Gail can catch the object any time it is above 17 feet. How much time does Gail have to try to catch the tape measure? (Solve by Completing the Square and round your solution to the nearest hundredth.)



$-16t^2 + 32t + 5 = 17$

$-16t^2 + 32t = 12$

$t^2 - 2t + 1 = -\frac{3}{4} + 1$

$\sqrt{(t-1)^2} = \sqrt{\frac{1}{4}}$

$|t-1| = \frac{1}{2}$

$t - 1 = \frac{1}{2}$ or $t - 1 = -\frac{1}{2}$

$t = 1\frac{1}{2}$ sec or $t = \frac{1}{2}$ sec

So Gail can catch tape measure at $\frac{1}{2} < t < 1\frac{1}{2}$ seconds

12. For which values of y, given $y = x^2 + bx + \left(\frac{b}{2}\right)^2$, will you find 2 complex solutions?

when $y < 0$

13. Solve the following quadratic for x by ***Completing the Square***.

What is the result? $\frac{ax^2 + bx + c}{a} = \frac{0}{a}$

$x^2 + \frac{bx}{a} = -\frac{c}{a}$

$x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{-\frac{4ac}{4a^2} + \frac{b^2}{4a^2}}$

$\left|x + \frac{b}{2a}\right| = \sqrt{\frac{b^2 - 4ac}{4a^2}}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

The Quadratic Formula!

Section 5.2K