Will the boats’ paths ever cross?

PERPENDICULAR AND PARALLEL LINES
APPLICATION: Sailing

When you float in an inner tube on a windy day, you get blown in the direction of the wind. Sailboats are designed to sail against the wind. Most sailboats can sail at an angle of 45° to the direction from which the wind is blowing, as shown below. If a sailboat heads directly into the wind, the sail flaps and is useless.

You’ll learn how to analyze lines such as the paths of sailboats in Chapter 3.

Think & Discuss

1. What do you think the measure of $\angle 1$ is? Use a protractor to check your answer.
2. If the boats always sail at a 45° angle to the wind, and the wind doesn’t change direction, do you think the boats’ paths will ever cross?

Learn More About It

You will learn more about the paths of sailboats in Example 4 on p. 152.

APPLICATION LINK Visit www.mcdougallittell.com for more information about sailing.
What’s the chapter about?

Chapter 3 is about **lines** and **angles**. In Chapter 3, you’ll learn

- properties of parallel and perpendicular lines.
- six ways to prove that lines are parallel.
- how to write an equation of a line with given characteristics.

### KEY VOCABULARY

**Review**
- linear pair, p. 44
- vertical angles, p. 44
- perpendicular lines, p. 79

**New**
- parallel lines, p. 129
- skew lines, p. 129
- parallel planes, p. 129
- transversal, p. 131
- alternate interior angles, p. 131
- alternate exterior angles, p. 131
- consecutive interior angles, p. 131
- flow proof, p. 136

Are you ready for the chapter?

**SKILL REVIEW** Do these exercises to review key skills that you’ll apply in this chapter. See the given reference page if there is something you don’t understand.

**USING ALGEBRA** Solve each equation. (Skills Review, p. 789 and 790)

1. $47 + x = 180$
2. $135 = 3x - 6$
3. $m = \frac{5 - 7}{2 - (-6)}$
4. $\frac{1}{2} = -5\left(\frac{7}{2}\right) + b$
5. $5x + 9 = 6x - 11$
6. $2(x - 1) + 15 = 90$

Use the diagram. Write the reason that supports the statement. (Review pp. 44–46)

7. $m \angle 1 = 90^\circ$
8. $\angle 2 \equiv \angle 4$
9. $\angle 2$ and $\angle 3$ are supplementary.

Write the reason that supports the statement. (Review pp. 96–98)

10. If $m \angle A = 30^\circ$ and $m \angle B = 30^\circ$, then $\angle A \equiv \angle B$.
11. If $x + 4 = 9$, then $x = 5$.
12. $3(x + 5) = 3x + 15$

Here’s a study strategy!

**Write Sample Questions**

Write at least six questions about topics in the chapter. Focus on the concepts that you found difficult. Include both short-answer questions and more involved ones. Then answer your questions.
3.1 Lines and Angles

### GOAL 1 RELATIONSHIPS BETWEEN LINES

Two lines are **parallel lines** if they are coplanar and do not intersect. Lines that do not intersect and are not coplanar are called **skew lines**. Similarly, two planes that do not intersect are called **parallel planes**.

To write “\( AB \parallel CD \),” you write \( \overrightarrow{AB} \parallel \overrightarrow{CD} \). Triangles like those on \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) are used on diagrams to indicate that lines are parallel. Segments and rays are parallel if they lie on parallel lines. For example, \( \overrightarrow{AB} \parallel \overrightarrow{CD} \).

#### EXAMPLE 1 Identifying Relationships in Space

Think of each segment in the diagram as part of a line. Which of the lines appear to fit the description?

- **a.** parallel to \( \overrightarrow{AB} \) and contains \( D \)
- **b.** perpendicular to \( \overrightarrow{AB} \) and contains \( D \)
- **c.** skew to \( \overrightarrow{AB} \) and contains \( D \)
- **d.** Name the plane(s) that contain \( D \) and appear to be parallel to plane \( ABE \).

#### SOLUTION

- **a.** \( \overrightarrow{CD} \), \( \overrightarrow{GH} \), and \( \overrightarrow{EF} \) are all parallel to \( \overrightarrow{AB} \), but only \( \overrightarrow{CD} \) passes through \( D \) and is parallel to \( \overrightarrow{AB} \).
- **b.** \( \overrightarrow{BC} \), \( \overrightarrow{AD} \), \( \overrightarrow{AE} \), and \( \overrightarrow{BF} \) are all perpendicular to \( \overrightarrow{AB} \), but only \( \overrightarrow{AD} \) passes through \( D \) and is perpendicular to \( \overrightarrow{AB} \).
- **c.** \( \overrightarrow{DG} \), \( \overrightarrow{DH} \), and \( \overrightarrow{DE} \) all pass through \( D \) and are skew to \( \overrightarrow{AB} \).
- **d.** Only plane \( DCH \) contains \( D \) and is parallel to plane \( ABE \).
Notice in Example 1 that, although there are many lines through $D$ that are skew to $\overline{AB}$, there is only one line through $D$ that is parallel to $\overline{AB}$ and there is only one line through $D$ that is perpendicular to $\overline{AB}$.

**PARALLEL AND PERPENDICULAR POSTULATES**

**POSTULATE 13 Parallel Postulate**
If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

**POSTULATE 14 Perpendicular Postulate**
If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

You can use a compass and a straightedge to construct the line that passes through a given point and is perpendicular to a given line. In Lesson 6.6, you will learn why this construction works.

You will learn how to construct a parallel line in Lesson 3.5.

**ACTIVITY**

**A Perpendicular to a Line**

Use the following steps to construct a line that passes through a given point $P$ and is perpendicular to a given line $l$.

1. Place the compass point at $P$ and draw an arc that intersects line $l$ twice. Label the intersections $A$ and $B$.
2. Draw an arc with center $A$. Using the same radius, draw an arc with center $B$. Label the intersection of the arcs $Q$.
3. Use a straightedge to draw $\overrightarrow{PQ}$.

$\overrightarrow{PQ} \perp l$. 
3.1 Lines and Angles

### GOAL 2 Identifying Angles Formed by Transversals

A **transversal** is a line that intersects two or more coplanar lines at different points. For instance, in the diagrams below, line $t$ is a transversal. The angles formed by two lines and a transversal are given special names.

![Diagram](image)

Two angles are **corresponding angles** if they occupy corresponding positions. For example, angles 1 and 5 are corresponding angles.

Two angles are **alternate exterior angles** if they lie outside the two lines on opposite sides of the transversal. Angles 1 and 8 are alternate exterior angles.

Two angles are **alternate interior angles** if they lie between the two lines on opposite sides of the transversal. Angles 3 and 6 are alternate interior angles.

Two angles are **consecutive interior angles** if they lie between the two lines on the same side of the transversal. Angles 3 and 5 are consecutive interior angles.

Consecutive interior angles are sometimes called **same side interior angles**.

### Example 2 Identifying Angle Relationships

List all pairs of angles that fit the description.

- **a. corresponding**
  - $\angle 1$ and $\angle 5$
  - $\angle 2$ and $\angle 6$
  - $\angle 3$ and $\angle 7$
  - $\angle 4$ and $\angle 8$

- **b. alternate exterior**
  - $\angle 1$ and $\angle 8$
  - $\angle 2$ and $\angle 7$

- **c. alternate interior**
  - $\angle 3$ and $\angle 6$
  - $\angle 4$ and $\angle 5$

- **d. consecutive interior**
  - $\angle 3$ and $\angle 5$
  - $\angle 4$ and $\angle 6$

![Diagram](image)
1. Draw two lines and a transversal. Identify a pair of alternate interior angles.

2. How are skew lines and parallel lines alike? How are they different?

Match the photo with the corresponding description of the chopsticks.

A. skew  
B. parallel  
C. intersecting

3. In Exercises 6–9, use the diagram at the right.

6. Name a pair of corresponding angles.

7. Name a pair of alternate interior angles.

8. Name a pair of alternate exterior angles.

9. Name a pair of consecutive interior angles.

10. \( \overrightarrow{DE}, \overrightarrow{AB}, \) and \( \overrightarrow{GC} \) are \( \text{?} \).

11. \( \overrightarrow{DE} \) and \( \overrightarrow{BE} \) are \( \text{?} \).

12. \( \overrightarrow{BE} \) and \( \overrightarrow{GC} \) are \( \text{?} \).

13. Plane \( \overline{GAD} \) and plane \( \overline{CBE} \) are \( \text{?} \).

LINE RELATIONSHIPS  Think of each segment in the diagram as part of a line. Fill in the blank with parallel, skew, or perpendicular.

14. Name a line parallel to \( \overrightarrow{QR} \).

15. Name a line perpendicular to \( \overrightarrow{QR} \).

16. Name a line skew to \( \overrightarrow{QR} \).

17. Name a line parallel to plane \( \overline{QRS} \).

IDENTIFYING RELATIONSHIPS  Think of each segment in the diagram as part of a line. There may be more than one right answer.

APPLYING POSTULATES  How many lines can be drawn that fit the description?

18. through \( L \) parallel to \( \overrightarrow{JK} \)

19. through \( L \) perpendicular to \( \overrightarrow{JK} \)
20. **Tightrope Walking** Philippe Petit sometimes uses a long pole to help him balance on the tightrope. Are the rope and the pole at the left intersecting, perpendicular, parallel, or skew?

**Angle Relationships** Complete the statement with corresponding, alternate interior, alternate exterior, or consecutive interior.

21. \( \angle 8 \) and \( \angle 12 \) are ______ angles.
22. \( \angle 9 \) and \( \angle 14 \) are ______ angles.
23. \( \angle 10 \) and \( \angle 12 \) are ______ angles.
24. \( \angle 11 \) and \( \angle 12 \) are ______ angles.
25. \( \angle 8 \) and \( \angle 15 \) are ______ angles.
26. \( \angle 10 \) and \( \angle 14 \) are ______ angles.

**Roman Numerals** Write the Roman numeral that consists of the indicated segments. Then write the base ten value of the Roman numeral. For example, the base ten value of XII is 10 + 1 + 1 = 12.

<table>
<thead>
<tr>
<th>Roman numeral</th>
<th>I</th>
<th>V</th>
<th>X</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base ten value</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>50</td>
<td>1000</td>
</tr>
</tbody>
</table>

27. Three parallel segments
28. Two non-congruent perpendicular segments
29. Two congruent segments that intersect to form only one angle
30. Two intersecting segments that form vertical angles
31. Four segments, two of which are parallel

**Escalators** In Exercises 32–36, use the following information. The steps of an escalator are connected to a chain that runs around a drive wheel, which moves continuously. When a step on an up-escalator reaches the top, it flips over and goes back down to the bottom. Each step is shaped like a wedge, as shown at the right. On each step, let plane \( A \) be the plane you stand on.

32. As each step moves around the escalator, is plane \( A \) always parallel to ground level?
33. When a person is standing on plane \( A \), is it parallel to ground level?
34. Is line \( \ell \) on any step always parallel to \( \ell \) on any other step?
35. Is plane \( A \) on any step always parallel to plane \( A \) on any other step?
36. As each step moves around the escalator, how many positions are there at which plane \( A \) is perpendicular to ground level?
37. **Logical Reasoning** If two parallel planes are cut by a third plane, explain why the lines of intersection are parallel.

38. **Writing** What does “two lines intersect” mean?

39. **Construction** Draw a horizontal line \( l \) and a point \( P \) above \( l \). Construct a line through \( P \) perpendicular to \( l \).

40. **Construction** Draw a diagonal line \( m \) and a point \( Q \) below \( m \). Construct a line through \( Q \) perpendicular to \( m \).

41. **Multiple Choice** In the diagram at the right, how many lines can be drawn through point \( P \) that are perpendicular to line \( l \)?

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) More than 3

42. **Multiple Choice** If two lines intersect, then they must be

- (A) perpendicular
- (B) parallel
- (C) coplanar
- (D) skew
- (E) None of these

**Angle Relationships** Complete each statement. List all possible correct answers.

43. \( \angle 1 \) and \( \angle ? \) are corresponding angles.

44. \( \angle 1 \) and \( \angle ? \) are consecutive interior angles.

45. \( \angle 1 \) and \( \angle ? \) are alternate interior angles.

46. \( \angle 1 \) and \( \angle ? \) are alternate exterior angles.

**Mixed Review**

47. **Angle Bisector** The ray \( \overline{BD} \) bisects \( \angle ABC \), as shown at the right. Find \( m\angle ABD \) and \( m\angle ABC \). *(Review 1.5 for 3.2)*

**Complements and Supplements** Find the measures of a complement and a supplement of the angle. *(Review 1.6 for 3.2)*

48. \( 71^\circ \)  
49. \( 13^\circ \)  
50. \( 56^\circ \)

51. \( 88^\circ \)  
52. \( 27^\circ \)  
53. \( 68^\circ \)

54. \( 1^\circ \)  
55. \( 60^\circ \)  
56. \( 45^\circ \)

**Writing Reasons** Solve the equation and state a reason for each step. *(Review 2.4 for 3.2)*

57. \( x + 13 = 23 \)  
58. \( x - 8 = 17 \)  
59. \( 4x + 11 = 31 \)

60. \( 2x + 9 = 4x - 29 \)  
61. \( 2(x - 1) + 3 = 17 \)  
62. \( 5x + 7(x - 10) = -94 \)
### Comparing Types of Proofs

There is more than one way to write a proof. The two-column proof below is from Lesson 2.6. It can also be written as a paragraph proof or as a flow proof. A flow proof uses arrows to show the flow of the logical argument. Each reason in a flow proof is written below the statement it justifies.

**Example 1: Comparing Types of Proof**

**Given**
\[ \angle 5 \text{ and } \angle 6 \text{ are a linear pair.} \]
\[ \angle 6 \text{ and } \angle 7 \text{ are a linear pair.} \]

**Prove**
\[ \angle 5 \equiv \angle 7 \]

**Method 1: Two-column Proof**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 5 ) and ( \angle 6 ) are a linear pair. ( \angle 6 ) and ( \angle 7 ) are a linear pair.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 5 ) and ( \angle 6 ) are supplementary. ( \angle 6 ) and ( \angle 7 ) are supplementary.</td>
<td>2. Linear Pair Postulate</td>
</tr>
<tr>
<td>3. ( \angle 5 \equiv \angle 7 )</td>
<td>3. Congruent Supplements Theorem</td>
</tr>
</tbody>
</table>

**Method 2: Paragraph Proof**

Because \( \angle 5 \) and \( \angle 6 \) are a linear pair, the Linear Pair Postulate says that \( \angle 5 \) and \( \angle 6 \) are supplementary. The same reasoning shows that \( \angle 6 \) and \( \angle 7 \) are supplementary. Because \( \angle 5 \) and \( \angle 7 \) are both supplementary to \( \angle 6 \), the Congruent Supplements Theorem says that \( \angle 5 \equiv \angle 7 \).

**Method 3: Flow Proof**

- \( \angle 5 \) and \( \angle 6 \) are a linear pair. [Given]
- \( \angle 5 \) and \( \angle 6 \) are supplementary. [Linear Pair Postulate]
- \( \angle 6 \) and \( \angle 7 \) are a linear pair. [Given]
- \( \angle 6 \) and \( \angle 7 \) are supplementary. [Linear Pair Postulate]
- \( \angle 5 \equiv \angle 7 \) [Congruent Supplements Theorem]
PROOFING RESULTS ABOUT PERPENDICULAR LINES

THEOREMS

THEOREM 3.1
If two lines intersect to form a linear pair of congruent angles, then the lines are perpendicular.

THEOREM 3.2
If two sides of two adjacent acute angles are perpendicular, then the angles are complementary.

THEOREM 3.3
If two lines are perpendicular, then they intersect to form four right angles.

You will prove Theorem 3.2 and Theorem 3.3 in Exercises 17–19.

EXAMPLE 2 Proof of Theorem 3.1

Write a proof of Theorem 3.1.

**SOLUTION**

**GIVEN** \( \angle 1 \equiv \angle 2, \angle 1 \) and \( \angle 2 \) are a linear pair.

**PROVE** \( g \perp h \)

**Plan for Proof** Use \( m\angle 1 + m\angle 2 = 180^\circ \) and \( m\angle 1 = m\angle 2 \) to show \( m\angle 1 = 90^\circ \).

\[ m\angle 1 + m\angle 2 = 180^\circ \]

Def. of supplementary \( \angle \)

\[ m\angle 1 + m\angle 1 = 180^\circ \]

Substitution prop. of equality

\[ 2 \cdot (m\angle 1) = 180^\circ \]

Distributive prop.

\[ m\angle 1 = 90^\circ \]

Div. prop. of equality

\[ \angle 1 \] is a right \( \angle \).

Def. of right angle

\[ g \perp h \]

Def. of \( \perp \) lines

**STUDENT HELP**

Study Tip
When you write a complicated proof, it may help to write a plan first. The plan will also help others to understand your proof.
1. **Define** perpendicular lines.

2. Which postulate or theorem guarantees that there is only one line that can be constructed perpendicular to a given line from a given point not on the line? Write the postulate or theorem that justifies the statement about the diagram.

3. \( \angle 1 \equiv \angle 2 \)

4. \( j \perp k \)

Write the postulate or theorem that justifies the statement, given that \( g \perp h \).

5. \( m\angle 5 + m\angle 6 = 90^\circ \)

6. \( \angle 3 \) and \( \angle 4 \) are right angles.

Find the value of \( x \).

7. 

8. 

9. 

10. **ERROR ANALYSIS** It is given that \( \angle ABC \equiv \angle CBD \). A student concludes that because \( \angle ABC \) and \( \angle CBD \) are congruent adjacent angles, \( \overline{AB} \perp \overline{CB} \). What is wrong with this reasoning? Draw a diagram to support your answer.
**Practice and Applications**

**Using Algebra** Find the value of $x$.

11. 

![Diagram of a right angle with $x^\circ$]

12. 

![Diagram of a right angle with $80^\circ$ and $x^\circ$]

13. 

![Diagram of a right angle with $55^\circ$]

**Logical Reasoning** What can you conclude about the labeled angles?

14. $AB \perp CB$

15. $n \perp m$

16. $h \perp k$

![Diagrams of labeled angles]

17. **Developing Paragraph Proof** Fill in the lettered blanks to complete the proof of Theorem 3.2.

**Given** $\overrightarrow{BA} \perp \overrightarrow{BC}$

**Prove** $\angle 3$ and $\angle 4$ are complementary.

Because $\overrightarrow{BA} \perp \overrightarrow{BC}$, $\angle ABC$ is a ______ a.____ and $m\angle ABC$ = ______ b.____.

According to the ______ c.____ Postulate, $m\angle 3 + m\angle 4 = m\angle ABC$. So, by the substitution property of equality, ______ d.____ + ______ e.____ = ______ f.____.

By definition, $\angle 3$ and $\angle 4$ are complementary.

18. **Developing Flow Proof** Fill in the lettered blanks to complete the proof of part of Theorem 3.3. Because the lines are perpendicular, they intersect to form a right angle. Call that $\angle 1$.

**Given** $j \perp k$, $\angle 1$ and $\angle 2$ are a linear pair.

**Prove** $\angle 2$ is a right $\angle$.

![Diagram of perpendicular lines]

**Student Help**

- Extra Practice to help you master skills is on p. 807.

**Homework Help**

Example 1: Exs. 17–23
Example 2: Exs. 11–19, 24, 25
19. **DEVELOPING TWO-COLUMN PROOF** Fill in the blanks to complete the proof of part of Theorem 3.3.

**GIVEN** $\angle 1$ is a right angle.

**PROVE** $\angle 3$ is a right angle.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1$ and $\angle 3$ are vertical angles.</td>
<td>1. Definition of vertical angles</td>
</tr>
<tr>
<td>2. $\quad ?$</td>
<td>2. Vertical Angles Theorem</td>
</tr>
<tr>
<td>3. $m\angle 1 = m\angle 3$</td>
<td>3. $\quad ?$</td>
</tr>
<tr>
<td>4. $\angle 1$ is a right angle.</td>
<td>4. $\quad ?$</td>
</tr>
<tr>
<td>5. $\quad ?$</td>
<td>5. Definition of right angle</td>
</tr>
<tr>
<td>6. $\quad ?$</td>
<td>6. Substitution prop. of equality</td>
</tr>
<tr>
<td>7. $\quad ?$</td>
<td>7. Definition of right angle</td>
</tr>
</tbody>
</table>

**DEVELOPING PROOF** In Exercises 20–23, use the following information.

Dan is trying to figure out how to prove that $\angle 5 \equiv \angle 6$ below. First he wrote everything that he knew about the diagram, as shown below in blue.

**GIVEN** $m \perp n$, $\angle 3$ and $\angle 4$ are complementary.

**PROVE** $\angle 5 \equiv \angle 6$

$m \perp n \rightarrow \angle 3$ and $\angle 6$ are complementary.

$\angle 3$ and $\angle 4$ are complementary.

$\angle 4$ and $\angle 5$ are vertical angles. $\rightarrow \angle 4 \equiv \angle 5$

20. Write a justification for each statement Dan wrote in blue.

21. After writing all he knew, Dan wrote what he was supposed to prove in red.
He also wrote $\angle 4 \equiv \angle 6$ because he knew that if $\angle 4 \equiv \angle 6$ and $\angle 4 \equiv \angle 5$, then $\angle 5 \equiv \angle 6$. Write a justification for this step.

22. How can you use Dan’s blue statements to prove that $\angle 4 \equiv \angle 6$?

23. Copy and complete Dan’s flow proof.

24. **CIRCUIT BOARDS** The diagram shows part of a circuit board. Write any type of proof.

**GIVEN** $AB \perp BC$, $BC \perp CD$

**PROVE** $\angle 7 \equiv \angle 8$

**Plan for Proof** Show that $\angle 7$ and $\angle 8$ are both right angles.
25. **WINDOW REPAIR** Cathy is fixing a window frame. She fits two strips of wood together to make the crosspieces. For the glass panes to fit, each angle of the crosspieces must be a right angle. Must Cathy measure all four angles to be sure they are all right angles? Explain.

26. **MULTIPLE CHOICE** Which of the following is true if \( g \perp h \)?

- A. \( m \angle 1 + m \angle 2 > 180° \)
- B. \( m \angle 1 + m \angle 2 < 180° \)
- C. \( m \angle 1 + m \angle 2 = 180° \)
- D. Cannot be determined

27. **MULTIPLE CHOICE** Which of the following must be true if \( m \angle ACD = 90° \)?

I. \( \angle BCE \) is a right angle.
II. \( \overline{AE} \perp \overline{BD} \)
III. \( \angle BCA \) and \( \angle BCE \) are complementary.

- A. I only
- B. I and II only
- C. III only
- D. I, II, and III
- E. None of these

28. **REFLECTIONS** Ann has a full-length mirror resting against the wall of her room. Ann notices that the floor and its reflection do not form a straight angle. She concludes that the mirror is not perpendicular to the floor. Explain her reasoning.

**EXTRA CHALLENGE**

- www.mcdougallittell.com

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**MIXED REVIEW**

**ANGLE MEASURES** Complete the statement given that \( s \perp t \). (Review 2.6 for 3.3)

29. If \( m \angle 1 = 38° \), then \( m \angle 4 = \_\_\_\_. \)

30. \( m \angle 2 = \_\_\_\_. \)

31. If \( m \angle 6 = 51° \), then \( m \angle 1 = \_\_\_\_. \)

32. If \( m \angle 3 = 42° \), then \( m \angle 1 = \_\_\_\_. \)

**ANGLES** List all pairs of angles that fit the description. (Review 3.1)

33. Corresponding angles
34. Alternate interior angles
35. Alternate exterior angles
36. Consecutive interior angles
3.3 Parallel Lines and Transversals

**GOAL 1** Properties of Parallel Lines

In the activity on page 142, you may have discovered the following results.

**POSTULATE**

**POSTULATE 15** Corresponding Angles Postulate

If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

\[ \angle 1 \cong \angle 2 \]

You are asked to prove Theorems 3.5, 3.6, and 3.7 in Exercises 27–29.

**THEOREMS ABOUT PARALLEL LINES**

**THEOREM 3.4** Alternate Interior Angles

If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

\[ \angle 3 \cong \angle 4 \]

**THEOREM 3.5** Consecutive Interior Angles

If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

\[ m\angle 5 + m\angle 6 = 180^\circ \]

**THEOREM 3.6** Alternate Exterior Angles

If two parallel lines are cut by a transversal, then the pairs of alternate exterior angles are congruent.

\[ \angle 7 \cong \angle 8 \]

**THEOREM 3.7** Perpendicular Transversal

If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other.
**EXAMPLE 1** Proving the Alternate Interior Angles Theorem

Prove the Alternate Interior Angles Theorem.

**SOLUTION**

**GIVEN** \( p \parallel q \)

**PROVE** \( \angle 1 \equiv \angle 2 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( p \parallel q )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \equiv \angle 3 )</td>
<td>2. Corresponding Angles Postulate</td>
</tr>
<tr>
<td>3. ( \angle 3 \equiv \angle 2 )</td>
<td>3. Vertical Angles Theorem</td>
</tr>
<tr>
<td>4. ( \angle 1 \equiv \angle 2 )</td>
<td>4. Transitive Property of Congruence</td>
</tr>
</tbody>
</table>

**EXAMPLE 2** Using Properties of Parallel Lines

Given that \( m\angle 5 = 65^\circ \), find each measure. Tell which postulate or theorem you use.

- a. \( m\angle 6 \)
- b. \( m\angle 7 \)
- c. \( m\angle 8 \)
- d. \( m\angle 9 \)

**SOLUTION**

- a. \( m\angle 6 = m\angle 5 = 65^\circ \) Vertical Angles Theorem
- b. \( m\angle 7 = 180^\circ - m\angle 5 = 115^\circ \) Linear Pair Postulate
- c. \( m\angle 8 = m\angle 5 = 65^\circ \) Corresponding Angles Postulate
- d. \( m\angle 9 = m\angle 7 = 115^\circ \) Alternate Exterior Angles Theorem

**EXAMPLE 3** Classifying Leaves

**BOTANY** Some plants are classified by the arrangement of the veins in their leaves. In the diagram of the leaf, \( j \parallel k \). What is \( m\angle 1 \)?

**SOLUTION**

\[
m\angle 1 + 120^\circ = 180^\circ \quad \text{Consecutive Interior Angles Theorem}
\]
\[
m\angle 1 = 60^\circ \quad \text{Subtract.}
\]
**EXAMPLE 4** Using Properties of Parallel Lines

Use properties of parallel lines to find the value of \( x \).

**Solution**

\[ m\angle 4 = 125^\circ \quad \text{Corresponding Angles Postulate} \]
\[ m\angle 4 + (x + 15)^\circ = 180^\circ \quad \text{Linear Pair Postulate} \]
\[ 125^\circ + (x + 15)^\circ = 180^\circ \quad \text{Substitute} \]
\[ x = 40 \quad \text{Subtract} \]

**EXAMPLE 5** Estimating Earth’s Circumference

**History Connection** Eratosthenes was a Greek scholar. Over 2000 years ago, he estimated Earth’s circumference by using the fact that the Sun’s rays are parallel.

Eratosthenes chose a day when the Sun shone exactly down a vertical well in Syene at noon. On that day, he measured the angle the Sun’s rays made with a vertical stick in Alexandria at noon. He discovered that

\[ m\angle 2 \approx \frac{1}{50} \text{ of a circle}. \]

By using properties of parallel lines, he knew that \( m\angle 1 = m\angle 2 \). So he reasoned that

\[ m\angle 1 \approx \frac{1}{50} \text{ of a circle}. \]

At the time, the distance from Syene to Alexandria was believed to be 575 miles.

\[ \frac{1}{50} \text{ of a circle} \approx \frac{575 \text{ miles}}{\text{Earth’s circumference}} \]

Earth’s circumference \( \approx 50(575 \text{ miles}) \) \( \approx 29,000 \) miles

How did Eratosthenes know that \( m\angle 1 = m\angle 2 \)?

**Solution**

Because the Sun’s rays are parallel, \( \ell_1 \parallel \ell_2 \). Angles 1 and 2 are alternate interior angles, so \( \angle 1 \cong \angle 2 \). By the definition of congruent angles, \( m\angle 1 = m\angle 2 \).
Guided Practice

Vocabulary Check ✓ 1. Sketch two parallel lines cut by a transversal. Label a pair of consecutive interior angles.

Concept Check ✓ 2. In the figure at the right, \( j \parallel k \). How many angle measures must be given in order to find the measure of every angle? Explain your reasoning.

Skill Check ✓ State the postulate or theorem that justifies the statement.

3. \( \angle 2 \equiv \angle 7 \)
4. \( \angle 4 \equiv \angle 5 \)
5. \( m\angle 3 + m\angle 5 = 180^\circ \)
6. \( \angle 2 \equiv \angle 6 \)
7. In the diagram of the feather below, lines \( p \) and \( q \) are parallel. What is the value of \( x \)?

Practice and Applications

Using Parallel Lines Find \( m\angle 1 \) and \( m\angle 2 \). Explain your reasoning.

8. 

9. 

10. 

Using Parallel Lines Find the values of \( x \) and \( y \). Explain your reasoning.

11. 

12.

13. 

14. 

15.

16.
17. **Using Properties of Parallel Lines**
Use the given information to find the measures of the other seven angles in the figure at the right.

**GIVEN** \( j \parallel k, m\angle 1 = 107^\circ \)

\[ \begin{array}{c}
\text{1. Using Algebra} \quad \text{Find the value of \( y \).} \\
\begin{array}{c}
\text{18.} \\
\text{19.} \\
\text{20.}
\end{array}
\end{array} \]

\[ \begin{array}{c}
\text{21.} \\
\text{22.} \\
\text{23.}
\end{array} \]

\[ \begin{array}{c}
\text{24.} \\
\text{25.} \\
\text{26.}
\end{array} \]

27. **Developing Proof**
Complete the proof of the Consecutive Interior Angles Theorem.

**GIVEN** \( \rho \parallel q \)

**PROVE** \( \angle 1 \) and \( \angle 2 \) are supplementary.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 \equiv \angle 3 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 3 \equiv \angle 5 )</td>
<td>2. ( \angle 3 \equiv \angle 5 )</td>
</tr>
<tr>
<td>3. ( \angle 2 \equiv \angle 6 )</td>
<td>3. Definition of congruent angles</td>
</tr>
<tr>
<td>4. ( \angle 4 \equiv \angle 7 )</td>
<td>4. Definition of linear pair</td>
</tr>
<tr>
<td>5. ( m\angle 3 + m\angle 2 = 180^\circ )</td>
<td>5. ( m\angle 3 + m\angle 2 = 180^\circ )</td>
</tr>
<tr>
<td>6. ( m\angle 1 + m\angle 2 = 180^\circ )</td>
<td>6. Substitution prop. of equality</td>
</tr>
<tr>
<td>7. ( \angle 1 ) and ( \angle 2 ) are supplementary.</td>
<td>7. ( \angle 1 ) and ( \angle 2 ) are supplementary.</td>
</tr>
</tbody>
</table>

**Student Help**
**Proving Theorems 3.6 and 3.7** In Exercises 28 and 29, complete the proof.

28. To prove the Alternate Exterior Angles Theorem, first show that \( \angle 1 \equiv \angle 3 \). Then show that \( \angle 3 \equiv \angle 2 \). Finally, show that \( \angle 1 \equiv \angle 2 \).

\[ \text{GIVEN} \quad j \parallel k \]
\[ \text{PROVE} \quad \angle 1 \equiv \angle 2 \]

29. To prove the Perpendicular Transversal Theorem, show that \( \angle 1 \) is a right angle, \( \angle 1 \equiv \angle 2 \), \( \angle 2 \) is a right angle, and finally that \( p \perp r \).

\[ \text{GIVEN} \quad p \perp q, q \parallel r \]
\[ \text{PROVE} \quad p \perp r \]

**30. Forming Rainbows**

When sunlight enters a drop of rain, different colors leave the drop at different angles. That’s what makes a rainbow. For red light, \( m \angle 2 = 42^\circ \). What is \( m \angle 1 \)? How do you know?

**31. Multi-Step Problem** You are designing a lunch box like the one below.

a. The measure of \( \angle 1 \) is 70°. What is the measure of \( \angle 2 \)? What is the measure of \( \angle 3 \)?

b. Writing Explain why \( \angle ABC \) is a straight angle.

**32. Using Properties of Parallel Lines**

Use the given information to find the measures of the other labeled angles in the figure. For each angle, tell which postulate or theorem you used.

\[ \text{GIVEN} \quad PQ \parallel RS, \quad LM \perp NK, \quad m \angle 1 = 48^\circ \]
**Mixed Review**

**Angle Measures** \( \angle 1 \) and \( \angle 2 \) are supplementary. Find \( m\angle 2 \). (Review 1.6)

33. \( m\angle 1 = 50^\circ \)  
34. \( m\angle 1 = 73^\circ \)  
35. \( m\angle 1 = 101^\circ \)  
36. \( m\angle 1 = 107^\circ \)  
37. \( m\angle 1 = 111^\circ \)  
38. \( m\angle 1 = 118^\circ \)

**Converses** Write the converse of the statement. (Review 2.1 for 3.4)

39. If the measure of an angle is 19°, then the angle is acute.
40. I will go to the park if you go with me.
41. I will go fishing if I do not have to work.

**Finding Angles** Complete the statement, given that \( \overline{DE} \parallel \overline{DG} \) and \( \overline{AB} \parallel \overline{DC} \). (Review 2.6)

42. If \( m\angle 1 = 23^\circ \), then \( m\angle 2 = ? \).
43. If \( m\angle 4 = 69^\circ \), then \( m\angle 3 = ? \).
44. If \( m\angle 2 = 70^\circ \), then \( m\angle 4 = ? \).

**Quiz 1**

Complete the statement. (Lesson 3.1)

1. \( \angle 2 \) and \( ? \) are corresponding angles.
2. \( \angle 3 \) and \( ? \) are consecutive interior angles.
3. \( \angle 3 \) and \( ? \) are alternate interior angles.
4. \( \angle 2 \) and \( ? \) are alternate exterior angles.

5. **Proof** Write a plan for a proof. (Lesson 3.2)

   **Given** \( \angle 1 \equiv \angle 2 \)

   **Prove** \( \angle 3 \) and \( \angle 4 \) are right angles.

Find the value of \( x \). (Lesson 3.3)

6. \[
   \begin{align*}
   138^\circ & \quad 2x^\circ \\
   \end{align*}
   \]

7. \[
   \begin{align*}
   151^\circ & \quad (2x + 1)^\circ \\
   \end{align*}
   \]

8. \[
   \begin{align*}
   81^\circ & \quad (7x + 15)^\circ \\
   \end{align*}
   \]

9. **Flag of Puerto Rico** Sketch the flag of Puerto Rico shown at the right. Given that \( m\angle 3 = 55^\circ \), determine the measure of \( \angle 1 \). Justify each step in your argument. (Lesson 3.3)
The following theorems are converses of those in Lesson 3.3. Remember that the converse of a true conditional statement is not necessarily true. Thus, each of the following must be proved to be true. Theorems 3.8 and 3.9 are proved in Examples 1 and 2. You are asked to prove Theorem 3.10 in Exercise 30.

### THEOREMS ABOUT TRANSVERSALS

**THEOREM 3.8 Alternate Interior Angles Converse**

If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.

**THEOREM 3.9 Consecutive Interior Angles Converse**

If two lines are cut by a transversal so that consecutive interior angles are supplementary, then the lines are parallel.

**THEOREM 3.10 Alternate Exterior Angles Converse**

If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel.
3.4 Proving Lines are Parallel

**EXAMPLE 1  Proof of the Alternate Interior Angles Converse**

Prove the Alternate Interior Angles Converse.

**SOLUTION**

**GIVEN** $\angle 1 \cong \angle 2$

**PROVE** $m \parallel n$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1 \cong \angle 2$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 2 \cong \angle 3$</td>
<td>2. Vertical Angles Theorem</td>
</tr>
<tr>
<td>3. $\angle 1 \cong \angle 3$</td>
<td>3. Transitive Property of Congruence</td>
</tr>
<tr>
<td>4. $m \parallel n$</td>
<td>4. Corresponding Angles Converse</td>
</tr>
</tbody>
</table>

When you prove a theorem you may use only earlier results. For example, to prove Theorem 3.9, you may use Theorem 3.8 and Postulate 16, but you may not use Theorem 3.9 itself or Theorem 3.10.

**EXAMPLE 2  Proof of the Consecutive Interior Angles Converse**

Prove the Consecutive Interior Angles Converse.

**SOLUTION**

**GIVEN** $\angle 4$ and $\angle 5$ are supplementary.

**PROVE** $g \parallel h$

**Paragraph Proof** You are given that $\angle 4$ and $\angle 5$ are supplementary. By the Linear Pair Postulate, $\angle 5$ and $\angle 6$ are also supplementary because they form a linear pair. By the Congruent Supplements Theorem, it follows that $\angle 4 \cong \angle 6$. Therefore, by the Alternate Interior Angles Converse, $g$ and $h$ are parallel.

**EXAMPLE 3  Applying the Consecutive Interior Angles Converse**

Find the value of $x$ that makes $j \parallel k$.

**SOLUTION**

Lines $j$ and $k$ will be parallel if the marked angles are supplementary.

$$x^\circ + 4x^\circ = 180^\circ$$

$$5x = 180$$

$$x = 36$$

So, if $x = 36$, then $j \parallel k$. 

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GOAL 2 USING THE PARALLEL CONVERSES

EXAMPLE 4 Using the Corresponding Angles Converse

SAILING If two boats sail at a 45° angle to the wind as shown, and the wind is constant, will their paths ever cross? Explain.

SOLUTION
Because corresponding angles are congruent, the boats’ paths are parallel. Parallel lines do not intersect, so the boats’ paths will not cross.

EXAMPLE 5 Identifying Parallel Lines

Decide which rays are parallel.

a. Is \( \overrightarrow{EB} \parallel \overrightarrow{HD} \)?

b. Is \( \overrightarrow{EA} \parallel \overrightarrow{HC} \)?

SOLUTION

a. Decide whether \( \overrightarrow{EB} \parallel \overrightarrow{HD} \).

\[ m\angle BEH = 58^\circ \]
\[ m\angle DHG = 61^\circ \]

\( \angle BEH \) and \( \angle DHG \) are corresponding angles, but they are not congruent, so \( \overrightarrow{EB} \) and \( \overrightarrow{HD} \) are not parallel.

b. Decide whether \( \overrightarrow{EA} \parallel \overrightarrow{HC} \).

\[ m\angle AEH = 62^\circ + 58^\circ = 120^\circ \]
\[ m\angle CHG = 59^\circ + 61^\circ = 120^\circ \]

\( \angle AEH \) and \( \angle CHG \) are congruent corresponding angles, so \( \overrightarrow{EA} \parallel \overrightarrow{HC} \).
3.4 Proving Lines are Parallel

**GUIDED PRACTICE**

1. What are parallel lines?

2. Write the converse of Theorem 3.8. Is the converse true?

3. Can you prove that lines \( p \) and \( q \) are parallel? If so, describe how.

4. Find the value of \( x \) that makes \( j \parallel k \). Which postulate or theorem about parallel lines supports your answer?

**PRACTICE AND APPLICATIONS**

**LOGICAL REASONING** Is it possible to prove that lines \( m \) and \( n \) are parallel? If so, state the postulate or theorem you would use.

**USING ALGEBRA** Find the value of \( x \) that makes \( r \parallel s \).

---

**Extra Practice**

to help you master skills is on p. 808.

**STUDENT HELP**

- **Homework Help**
  - Example 1: Exs. 28, 30
  - Example 2: Exs. 28, 30
  - Example 3: Exs. 10–18
  - Example 4: Exs. 19, 29, 31
  - Example 5: Exs. 20–27
19. **ARCHAEOLOGY** A farm lane in Ohio crosses two long, straight earthen mounds that may have been built about 2000 years ago. The mounds are about 200 feet apart, and both form a 63° angle with the lane, as shown. Are the mounds parallel? How do you know?

**LOGICAL REASONING** Is it possible to prove that lines \( a \) and \( b \) are parallel? If so, explain how.

20. \[ \angle 1 \text{ and } \angle 2 \text{ are supplementary.} \]

21. \[ \angle 1 \text{ and } \angle 3 \text{ are a linear pair.} \]

22. \[ \angle 1 \parallel \angle 2 \]

23. \[ \angle 1 \text{ and } \angle 2 \text{ are supplementary.} \]

24. \[ \angle 1 \text{ and } \angle 2 \text{ are a linear pair.} \]

25. \[ \angle 1 \parallel \angle 2 \]

26. \[ \angle 1 \text{ and } \angle 2 \text{ are supplementary.} \]

27. \[ \angle 1 \text{ and } \angle 2 \text{ are a linear pair.} \]

28. **PROOF** Complete the proof.

**GIVEN** \( \angle 1 \text{ and } \angle 2 \text{ are supplementary.} \)

**PROVE** \( \ell_1 \parallel \ell_2 \)

**Statements** | **Reasons**
---|---
1. \( \angle 1 \text{ and } \angle 2 \text{ are supplementary.} \) | 1. \( ? \)
2. \( \angle 1 \text{ and } \angle 3 \text{ are a linear pair.} \) | 2. Definition of linear pair
3. \( ? \) | 3. Linear Pair Postulate
4. \( ? \) | 4. Congruent Supplements Theorem
5. \( \ell_1 \parallel \ell_2 \) | 5. \( ? \)
29. **Building Stairs** One way to build stairs is to attach triangular blocks to an angled support, as shown at the right. If the support makes a 32° angle with the floor, what must \( m \angle 1 \) be so the step will be parallel to the floor? The sides of the angled support are parallel.

30. **Proving Theorem 3.10** Write a two-column proof for the Alternate Exterior Angles Converse: If two lines are cut by a transversal so that alternate exterior angles are congruent, then the lines are parallel.

- **GIVEN** \( \angle 4 \cong \angle 5 \)
- **PROVE** \( g \parallel h \)

**Plan for Proof** Show that \( \angle 4 \) is congruent to \( \angle 6 \), show that \( \angle 6 \) is congruent to \( \angle 5 \), and then use the Corresponding Angles Converse.

31. **Writing** In the diagram at the right, \( m \angle 5 = 110^\circ \) and \( m \angle 6 = 110^\circ \). Explain why \( p \parallel q \).

32. **Logical Reasoning** Use the information given in the diagram.

33. What can you prove about \( \angle 1, \angle 2, \angle 3, \) and \( \angle 4? \) Explain.

34. **Proof** Write a proof.

- **GIVEN** \( m \angle 7 = 125^\circ, m \angle 8 = 55^\circ \)
- **PROVE** \( j \parallel k \)

35. **Given** \( a \parallel b, \angle 1 \cong \angle 2 \)

- **PROVE** \( c \parallel d \)

36. **Technology** Use geometry software to construct a line \( l \), a point \( P \) not on \( l \), and a line \( n \) through \( P \) parallel to \( l \). Construct a point \( Q \) on \( l \) and construct \( PQ \). Choose a pair of alternate interior angles and construct their angle bisectors. Are the bisectors parallel? Make a conjecture. Write a plan for a proof of your conjecture.
37. **MULTIPLE CHOICE** What is the converse of the following statement?

If \( \angle 1 \cong \angle 2 \), then \( n \parallel m \).

- A: \( \angle 1 \cong \angle 2 \) if and only if \( n \parallel m \).
- B: \( \angle 2 \cong \angle 1 \), then \( m \parallel n \).
- C: \( \angle 1 \cong \angle 2 \) if \( n \parallel m \).
- D: \( \angle 1 \cong \angle 2 \) only if \( n \parallel m \).

38. **MULTIPLE CHOICE** What value of \( x \) would make lines \( l_1 \) and \( l_2 \) parallel?

- A: 13
- B: 35
- C: 37
- D: 78
- E: 102

39. **Challenge** 

**Snow Making** To shoot the snow as far as possible, each snowmaker below is set at a 45° angle. The axles of the snowmakers are all parallel. It is possible to prove that the barrels of the snowmakers are also parallel, but the proof is difficult in 3 dimensions. To simplify the problem, think of the illustration as a flat image on a piece of paper. The axles and barrels are represented in the diagram on the right. Lines \( j \) and \( l_2 \) intersect at \( C \).

**GIVEN** \( l_1 \parallel l_2 \), \( m\angle A = m\angle B = 45° \)

**PROVE** \( j \parallel k \)

40. **FINDING THE MIDPOINT** Use a ruler to draw a line segment with the given length. Then use a compass and straightedge to construct the midpoint of the line segment. *(Review 1.5 for 3.5)*

- 3 inches
- 8 centimeters
- 5 centimeters
- 1 inch

41. **CONGRUENT SEGMENTS** Find the value of \( x \) if \( AB \equiv AD \) and \( CD \equiv AD \). Explain your steps. *(Review 2.5)*

42. **IDENTIFYING ANGLES** Use the diagram to complete the statement. *(Review 3.1)*

- \( \angle 12 \) and \( \_\_\_\_ \) are alternate exterior angles.
- \( \angle 10 \) and \( \_\_\_\_ \) are corresponding angles.
- \( \angle 10 \) and \( \_\_\_\_ \) are alternate interior angles.
- \( \angle 9 \) and \( \_\_\_\_ \) are consecutive interior angles.
3.5 Using Properties of Parallel Lines

**GOAL 1** Using Parallel Lines in Real Life

When a team of rowers competes, each rower keeps his or her oars parallel to the adjacent rower’s oars. If any two adjacent oars on the same side of the boat are parallel, does this imply that any two oars on that side are parallel? This question is examined below.

Example 1 justifies Theorem 3.11, and you will prove Theorem 3.12 in Exercise 38.

**EXAMPLE 1** Proving Two Lines are Parallel

Lines $m$, $n$, and $k$ represent three of the oars above. $m \parallel n$ and $n \parallel k$. Prove that $m \parallel k$.

**SOLUTION**

**GIVEN** $m \parallel n$, $n \parallel k$

**PROVE** $m \parallel k$

<table>
<thead>
<tr>
<th>Statements</th>
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<tbody>
<tr>
<td>1. $m \parallel n$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1 \equiv \angle 2$</td>
<td>2. Corresponding Angles Postulate</td>
</tr>
<tr>
<td>3. $n \parallel k$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $\angle 2 \equiv \angle 3$</td>
<td>4. Corresponding Angles Postulate</td>
</tr>
<tr>
<td>5. $\angle 1 \equiv \angle 3$</td>
<td>5. Transitive Property of Congruence</td>
</tr>
<tr>
<td>6. $m \parallel k$</td>
<td>6. Corresponding Angles Converse</td>
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</tbody>
</table>

**THEOREMS ABOUT PARALLEL AND PERPENDICULAR LINES**

**THEOREM 3.11**
If two lines are parallel to the same line, then they are parallel to each other.

**THEOREM 3.12**
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.
**EXAMPLE 2**

**Explaining Why Steps are Parallel**

In the diagram at the right, each step is parallel to the step immediately below it and the bottom step is parallel to the floor. Explain why the top step is parallel to the floor.

**SOLUTION**

You are given that \( k_1 \parallel k_2 \) and \( k_2 \parallel k_3 \). By transitivity of parallel lines, \( k_1 \parallel k_3 \). Since \( k_1 \parallel k_3 \) and \( k_3 \parallel k_4 \), it follows that \( k_1 \parallel k_4 \). So, the top step is parallel to the floor.

**EXAMPLE 3**

**Building a CD Rack**

You are building a CD rack. You cut the sides, bottom, and top so that each corner is composed of two 45° angles. Prove that the top and bottom front edges of the CD rack are parallel.

**SOLUTION**

**GIVEN**  
\( m\angle 1 = 45^\circ, m\angle 2 = 45^\circ \)  
\( m\angle 3 = 45^\circ, m\angle 4 = 45^\circ \)

**PROVE**  
\( BA \parallel CD \)

\[
\begin{align*}
m\angle ABC &= m\angle 1 + m\angle 2 \\
&= 45^\circ + 45^\circ \\
&= 90^\circ \\
\angle ABC &\text{ is a right angle.} \\
\angle BCD &= m\angle 3 + m\angle 4 \\
&= 45^\circ + 45^\circ \\
&= 90^\circ \\
\angle BCD &\text{ is a right angle.} \\
BA &\perp BC \\
BC &\perp CD \\
BA &\parallel CD
\end{align*}
\]

In a plane, 2 lines \( \perp \) to the same line are \( \parallel \).
To construct parallel lines, you first need to know how to copy an angle.

**Copying an Angle**

Use these steps to construct an angle that is congruent to a given $\angle A$.

1. Draw a line. Label a point on the line $D$.
2. Draw an arc with center $A$. Label $B$ and $C$. With the same radius, draw an arc with center $D$. Label $E$.
3. Draw $\overrightarrow{DE}$.
4. $\angle EDF \cong \angle BAC$.

In Chapter 4, you will learn why the *Copying an Angle* construction works. You can use the *Copying an Angle* construction to construct two congruent corresponding angles. If you do, the sides of the angles will be parallel.

**Parallel Lines**

Use these steps to construct a line that passes through a given point $P$ and is parallel to a given line $m$.

1. Draw points $Q$ and $R$ on $m$. Draw $\overrightarrow{PQ}$.
2. Draw an arc with the compass point at $Q$ so that it crosses $\overrightarrow{QP}$ and $\overrightarrow{QR}$.
3. Copy $\angle PQR$ on $\overrightarrow{QP}$ as shown. Be sure the two angles are corresponding. Label the new angle $\angle TPS$ as shown.
4. Draw $\overrightarrow{PS}$. Because $\angle TPS$ and $\angle PQR$ are congruent corresponding angles, $\overrightarrow{PS} \parallel \overrightarrow{QR}$.
**GUIDED PRACTICE**

1. Name two ways, from this lesson, to prove that two lines are parallel. If they are parallel, if they are perpendicular to the same line. State the theorem that you can use to prove that \( r \) is parallel to \( s \).

2. **GIVEN** \( r \parallel t, t \parallel s \)

3. **GIVEN** \( r \perp t, t \perp s \)

Determine which lines, if any, must be parallel. Explain your reasoning.

4. \( \ell_1 \parallel \ell_2 \)

5. \( \ell_1 \parallel \ell_2 \)

6. Draw any angle \( \angle A \). Then construct \( \angle B \) congruent to \( \angle A \).

7. Given a line \( l \) and a point \( P \) not on \( l \), describe how to construct a line through \( P \) parallel to \( l \).

**PRACTICE AND APPLICATIONS**

**Logical Reasoning** State the postulate or theorem that allows you to conclude that \( j \parallel k \).

8. **GIVEN** \( j \parallel n, k \parallel n \)

9. **GIVEN** \( j \perp n, k \perp n \)

10. **GIVEN** \( \angle 1 \equiv \angle 2 \)

**Showing Lines are Parallel** Explain how you would show that \( k \parallel j \). State any theorems or postulates that you would use.

11. \( 112^\circ \)

12. \( 99^\circ \)

13. \( 52^\circ \)

14. **Writing** Make a list of all the ways you know to prove that two lines are parallel.
3.5 Using Properties of Parallel Lines

**SHOWING LINES ARE PARALLEL** Explain how you would show that $k \parallel j$.

15. 

16. 

17. 

**USING ALGEBRA** Explain how you would show that $g \parallel h$.

18. 

19. 

20. 

**NAMING PARALLEL LINES** Determine which lines, if any, must be parallel. Explain your reasoning.

21. 

22. 

23. 

24. 

**CONSTRUCTIONS** Use a straightedge to draw an angle that fits the description. Then use the *Copying an Angle* construction on page 159 to copy the angle.

25. An acute angle

26. An obtuse angle

27. **CONSTRUCTING PARALLEL LINES** Draw a horizontal line and construct a line parallel to it through a point above the line.

28. **CONSTRUCTING PARALLEL LINES** Draw a diagonal line and construct a line parallel to it through a point to the right of the line.

29. **JUSTIFYING A CONSTRUCTION** Explain why the lines in Exercise 28 are parallel. Use a postulate or theorem from Lesson 3.4 to support your answer.

30. **Football Field** The white lines along the long edges of a football field are called *sidelines*. *Yard lines* are perpendicular to the sidelines and cross the field every five yards. Explain why you can conclude that the yard lines are parallel.

31. **Hanging Wallpaper** When you hang wallpaper, you use a tool called a *plumb line* to make sure one edge of the first strip of wallpaper is vertical. If the edges of each strip of wallpaper are parallel and there are no gaps between the strips, how do you know that the rest of the strips of wallpaper will be parallel to the first?

32. **Error Analysis** It is given that $j \perp k$ and $k \perp l$. A student reasons that lines $j$ and $l$ must be parallel. What is wrong with this reasoning? Sketch a counterexample to support your answer.

**Categorizing** Tell whether the statement is *sometimes*, *always*, or *never* true.

33. Two lines that are parallel to the same line are parallel to each other.

34. In a plane, two lines that are perpendicular to the same line are parallel to each other.

35. Two noncoplanar lines that are perpendicular to the same line are parallel to each other.

36. Through a point not on a line you can construct a parallel line.

37. **Latticework** You are making a lattice fence out of pieces of wood called slats. You want the top of each slat to be parallel to the bottom. At what angle should you cut $\angle 1$?

38. **Proving Theorem 3.12** Rearrange the statements to write a flow proof of Theorem 3.12. Remember to include a reason for each statement.

**Given** $m \perp p$, $n \perp p$

**Prove** $m \parallel n$

$\angle 1 \equiv \angle 2$

$n \perp p$

$\angle 1$ is a right $\angle$.

$m \parallel n$

$m \perp p$

$\angle 2$ is a right $\angle$. 
39. **Optical Illusion** The radiating lines make it hard to tell if the red lines are straight. Explain how you can answer the question using only a straightedge and a protractor.

   a. Are the red lines straight?
   b. Are the red lines parallel?

40. **Constructing with Perpendiculars** Draw a horizontal line \( l \) and a point \( P \) not on \( l \). Construct a line \( m \) through \( P \) perpendicular to \( l \). Draw a point \( Q \) not on \( m \) or \( l \). Construct a line \( n \) through \( Q \) perpendicular to \( m \). What postulate or theorem guarantees that the lines \( l \) and \( n \) are parallel?

41. **Multi-Step Problem** Use the information given in the diagram at the right.

   a. Explain why \( AB \parallel CD \).
   b. Explain why \( CD \parallel EF \).
   c. **Writing** What is \( m \angle 1 \)? How do you know?

42. **Science Connection** When light enters glass, the light bends. When it leaves glass, it bends again. If both sides of a pane of glass are parallel, light leaves the pane at the same angle at which it entered. Prove that the path of the exiting light is parallel to the path of the entering light.

   **Given** \( \angle 1 \equiv \angle 2 \), \( j \parallel k \)
   **Prove** \( r \parallel s \)

### Mixed Review

**Using the Distance Formula** Find the distance between the two points. (*Review 1.3 for 3.6*)

43. \( A(0, -6), B(14, 0) \) 
44. \( A(-3, -8), B(2, -1) \) 
45. \( A(0, -7), B(6, 3) \) 
46. \( A(-9, -5), B(-1, 11) \) 
47. \( A(5, -7), B(-11, 6) \) 
48. \( A(4, 4), B(-3, -3) \)

**Finding Counterexamples** Give a counterexample that demonstrates that the converse of the statement is false. (*Review 2.2*)

49. If an angle measures 42°, then it is acute.
50. If two angles measure 150° and 30°, then they are supplementary.
51. If a polygon is a rectangle, then it contains four right angles.

52. **Using Properties of Parallel Lines** Use the given information to find the measures of the other seven angles in the figure shown at the right. (*Review 3.3*)

   **Given** \( j \parallel k \), \( m \angle 1 = 33° \)
1. In the diagram shown at the right, determine whether you can prove that lines \( j \) and \( k \) are parallel. If you can, state the postulate or theorem that you would use. \( \text{(Lesson 3.4)} \)

Use the given information and the diagram to determine which lines must be parallel. \( \text{(Lesson 3.5)} \)

2. \( \angle 1 \) and \( \angle 2 \) are right angles.

3. \( \angle 4 \cong \angle 3 \)

4. \( \angle 2 \cong \angle 3, \angle 3 \cong \angle 4 \).

5. FIREPLACE CHIMNEY In the illustration at the right, \( \angle ABC \) and \( \angle DEF \) are supplementary. Explain how you know that the left and right edges of the chimney are parallel. \( \text{(Lesson 3.4)} \)

**Math & History**

Measuring Earth’s Circumference

AROUND 230 B.C., the Greek scholar Eratosthenes estimated Earth’s circumference. In the late 15th century, Christopher Columbus used a smaller estimate to convince the king and queen of Spain that his proposed voyage to India would take only 30 days.

TODAY, satellites and other tools are used to determine Earth’s circumference with great accuracy.

1. The actual distance from Syene to Alexandria is about 500 miles. Use this value and the information on page 145 to estimate Earth’s circumference. How close is your value to the modern day measurement in the table at the right?

<table>
<thead>
<tr>
<th>Measuring Earth’s Circumference</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference estimated by Eratosthenes (230 B.C.)</td>
<td>About 29,000 mi</td>
</tr>
<tr>
<td>Circumference assumed by Columbus (about 1492)</td>
<td>About 17,600 mi</td>
</tr>
<tr>
<td>Modern day measurement</td>
<td>24,902 mi</td>
</tr>
</tbody>
</table>
Parallel Lines in the Coordinate Plane

### Goal 1: Slope of Parallel Lines

In algebra, you learned that the slope of a nonvertical line is the ratio of the vertical change (the rise) to the horizontal change (the run). If the line passes through the points \((x_1, y_1)\) and \((x_2, y_2)\), then the slope is given by

\[
\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}.
\]

Slope is usually represented by the variable \(m\).

#### Example 1: Finding the Slope of Train Tracks

**Cog Railway** A cog railway goes up the side of Mount Washington, the tallest mountain in New England. At the steepest section, the train goes up about 4 feet for each 10 feet it goes forward. What is the slope of this section?

**Solution**

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4 \text{ feet}}{10 \text{ feet}} = 0.4
\]

#### Example 2: Finding the Slope of a Line

Find the slope of the line that passes through the points \((0, 6)\) and \((5, 2)\).

**Solution**

Let \((x_1, y_1) = (0, 6)\) and \((x_2, y_2) = (5, 2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 6}{5 - 0} = \frac{-4}{5}
\]

The slope of the line is \(-\frac{4}{5}\).
You can use the slopes of two lines to tell whether the lines are parallel.

**POSTULATE 17** \( \text{Slopes of Parallel Lines} \)

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.

**EXAMPLE 3** \( \text{Deciding Whether Lines are Parallel} \)

Find the slope of each line. Is \( j_1 \parallel j_2 \)?

**SOLUTION**

Line \( j_1 \) has a slope of

\[
m_1 = \frac{4}{2} = 2
\]

Line \( j_2 \) has a slope of

\[
m_2 = \frac{2}{1} = 2
\]

Because the lines have the same slope, \( j_1 \parallel j_2 \).

**EXAMPLE 4** \( \text{Identifying Parallel Lines} \)

Find the slope of each line. Which lines are parallel?

**SOLUTION**

Find the slope of \( k_1 \). Line \( k_1 \) passes through \((0, 6)\) and \((2, 0)\).

\[
m_1 = \frac{0 - 6}{2 - 0} = \frac{-6}{2} = -3
\]

Find the slope of \( k_2 \). Line \( k_2 \) passes through \((-2, 6)\) and \((0, 1)\).

\[
m_2 = \frac{1 - 6}{0 - (-2)} = \frac{-5}{0 + 2} = \frac{-5}{2}
\]

Find the slope of \( k_3 \). Line \( k_3 \) passes through \((-6, 5)\) and \((-4, 0)\).

\[
m_3 = \frac{0 - 5}{-4 - (-6)} = \frac{-5}{-4 + 6} = \frac{-5}{2}
\]

Compare the slopes. Because \( k_2 \) and \( k_3 \) have the same slope, they are parallel. Line \( k_1 \) has a different slope, so it is not parallel to either of the other lines.
**GOAL 2** WRITING EQUATIONS OF PARALLEL LINES

In algebra, you learned that you can use the slope $m$ of a nonvertical line to write an equation of the line in *slope-intercept form.*

\[
y = mx + b
\]

The $y$-intercept is the $y$-coordinate of the point where the line crosses the $y$-axis.

**EXAMPLE 5** Writing an Equation of a Line

Write an equation of the line through the point $(2, 3)$ that has a slope of 5.

**SOLUTION**

Solve for $b$. Use $(x, y) = (2, 3)$ and $m = 5$.

\[
y = mx + b
\]

\[
3 = 5(2) + b
\]

\[
3 = 10 + b
\]

\[
b = -7
\]

Write an equation. Since $m = 5$ and $b = -7$, an equation of the line is $y = 5x - 7$.

**EXAMPLE 6** Writing an Equation of a Parallel Line

Line $n_1$ has the equation $y = -\frac{1}{3}x - 1$.

Line $n_2$ is parallel to $n_1$ and passes through the point $(3, 2)$. Write an equation of $n_2$.

**SOLUTION**

Find the slope.

The slope of $n_1$ is $-\frac{1}{3}$. Because parallel lines have the same slope, the slope of $n_2$ is also $-\frac{1}{3}$.

Solve for $b$. Use $(x, y) = (3, 2)$ and $m = -\frac{1}{3}$.

\[
y = mx + b
\]

\[
2 = -\frac{1}{3}(3) + b
\]

\[
2 = -1 + b
\]

\[
b = 3
\]

Write an equation.

Because $m = -\frac{1}{3}$ and $b = 3$, an equation of $n_2$ is $y = -\frac{1}{3}x + 3$. 
1. What does intercept mean in the expression slope-intercept form?

2. The slope of line \( j \) is 2 and \( j \parallel k \). What is the slope of line \( k \)?

3. What is the slope of a horizontal line? What is the slope of a vertical line?

Find the slope of the line that passes through the labeled points.

4. 5. 6.

Determine whether the two lines shown in the graph are parallel. If they are parallel, explain how you know.

7. 8. 9.

10. Write an equation of the line that passes through the point \((2, -3)\) and has a slope of \(-1\).

### Practice and Applications

**Calculating Slope**

What is the slope of the line?


Find the slope of the line that passes through the labeled points on the graph.

14. 15. 16.

---

**Student Help**

Extra Practice to help you master skills is on p. 808.

**Homework Help**

Example 1: Exs. 11–16, 23, 46, 49–52
Example 2: Exs. 11–16
IDENTIFYING PARALLELS  Find the slope of each line. Are the lines parallel?

17. 18.

19. 20.

21. 22.

23. UNDERGROUND RAILROAD  The photo at the right shows a monument in Oberlin, Ohio, that is dedicated to the Underground Railroad. The slope of each of the rails is about \( \frac{3}{5} \) and the sculpture is about 12 feet long. What is the height of the ends of the rails? Explain how you found your answer.

IDENTIFYING PARALLELS  Find the slopes of \( \overrightarrow{AB} \), \( \overrightarrow{CD} \), and \( \overrightarrow{EF} \). Which lines are parallel, if any?

24. \( A(0, -6), B(4, -4) \)  
   \( C(0, 2), D(2, 3) \)  
   \( E(0, -4), F(1, -7) \)

25. \( A(2, 6), B(4, 7) \)  
   \( C(0, -1), D(6, 2) \)  
   \( E(4, -5), F(-8, -2) \)

26. \( A(-4, 10), B(-8, 7) \)  
   \( C(-5, 7), D(-2, 4) \)  
   \( E(2, -3), F(6, -7) \)

WRITING EQUATIONS  Write an equation of the line.

27. slope = 3  
   y-intercept = 2

28. slope = \( \frac{1}{3} \)  
   y-intercept = -4

29. slope = \( -\frac{2}{9} \)  
   y-intercept = 0

30. slope = \( \frac{1}{2} \)  
   y-intercept = 6

31. slope = 0  
   y-intercept = -3

32. slope = \( -\frac{2}{9} \)  
   y-intercept = -\( \frac{3}{5} \)
**WRITING EQUATIONS** Write an equation of the line that has a \( y \)-intercept of 3 and is parallel to the line whose equation is given.

33. \( y = -6x + 2 \)  
34. \( y = x - 8 \)  
35. \( y = \frac{4}{3}x \)

**WRITING EQUATIONS** Write an equation of the line that passes through the given point \( P \) and has the given slope.

36. \( P(0, -6), m = -2 \)  
37. \( P(-3, 9), m = -1 \)  
38. \( P\left(\frac{3}{2}, 4\right), m = \frac{1}{2} \)  
39. \( P(2, -4), m = 0 \)  
40. \( P(-7, -5), m = \frac{3}{4} \)  
41. \( P(6, 1), \) undefined slope

**USING ALGEBRA** Write an equation of the line that passes through point \( P \) and is parallel to the line with the given equation.

42. \( P(-3, 6), y = -x - 5 \)  
43. \( P(1, -2), y = \frac{5}{4}x - 8 \)  
44. \( P(8, 7), y = 3 \)  
45. **USING ALGEBRA** Write an equation of a line parallel to \( y = \frac{3}{2}x - 16 \).

**ZIP LINE** A zip line is a taut rope or cable that you can ride down on a pulley. The zip line at the right goes from a 9 foot tall tower to a 6 foot tall tower. The towers are 20 feet apart. What is the slope of the zip line?

**COORDINATE GEOMETRY** In Exercises 47 and 48, use the five points: \( P(0, 0), Q(1, 3), R(4, 0), S(8, 2), \) and \( T(9, 5) \).

47. Plot and label the points. Connect every pair of points with a segment.
48. Which segments are parallel? How can you verify this?

**CIVIL ENGINEERING** In Exercises 49–52, use the following information. The slope of a road is called the road’s grade. Grades are measured in percents. For example, if the slope of a road is \( \frac{1}{20} \), the grade is 5%. A warning sign is needed before any hill that fits one of the following descriptions.

- 5% grade and more than 3000 feet long
- 6% grade and more than 2000 feet long
- 7% grade and more than 1000 feet long
- 8% grade and more than 750 feet long
- 9% grade and more than 500 feet long

Source: U.S. Department of Transportation

What is the grade of the hill to the nearest percent? Is a sign needed?

49. The hill is 1400 feet long and drops 70 feet.
50. The hill is 2200 feet long and drops 140 feet.
51. The hill is 600 feet long and drops 55 feet.
52. The hill is 450 feet long and drops 40 feet.
**TECHNOLOGY** Using a square viewing screen on a graphing calculator, graph a line that passes through the origin and has a slope of 1.

53. Write an equation of the line you graphed. Approximately what angle does the line form with the x-axis?

54. Graph a line that passes through the origin and has a slope of 2. Write an equation of the line. When you doubled the slope, did the measure of the angle formed with the x-axis double?

55. **MULTIPLE CHOICE** If two different lines with equations \( y = m_1x + b_1 \) and \( y = m_2x + b_2 \) are parallel, which of the following must be true?

- (A) \( b_1 = b_2 \) and \( m_1 \neq m_2 \)
- (B) \( b_1 \neq b_2 \) and \( m_1 \neq m_2 \)
- (C) \( b_1 \neq b_2 \) and \( m_1 = m_2 \)
- (D) \( b_1 = b_2 \) and \( m_1 = m_2 \)
- (E) None of these

56. **MULTIPLE CHOICE** Which of the following is an equation of a line parallel to \( y = -2x + \frac{3}{2} \)?

- (A) \( y = -2x + 3 \)
- (B) \( y = 2x + 1 \)
- (C) \( y = \frac{1}{2}x - 6 \)
- (D) \( y = 7x - 1 \)
- (E) \( y = -\frac{1}{2}x - 8 \)

57. **CHALLENGE** USING ALGEBRA Find a value for \( k \) so that the line through \((4, k)\) and \((-2, -1)\) is parallel to \( y = -2x + \frac{3}{2} \).

58. **CHALLENGE** USING ALGEBRA Find a value for \( k \) so that the line through \((k, -10)\) and \((5, -6)\) is parallel to \( y = -\frac{1}{4}x + 3 \).

---

**MIXED REVIEW**

**RECIPIRALS** Find the reciprocal of the number. (Skills Review, p. 788)

- 59. \( 20 \)
- 60. \( -3 \)
- 61. \( -11 \)
- 62. \( 340 \)
- 63. \( \frac{3}{7} \)
- 64. \( -\frac{13}{3} \)
- 65. \( -\frac{1}{2} \)
- 66. \( 0.25 \)

**MULTIPLYING NUMBERS** Evaluate the expression. (Skills Review, p. 785)

- 67. \( \frac{3}{4} \cdot (-12) \)
- 68. \( -\frac{3}{2} \cdot \left( -\frac{8}{3} \right) \)
- 69. \( -10 \cdot \frac{7}{6} \)
- 70. \( -\frac{2}{9} \cdot (-33) \)

**PROVING LINES PARALLEL** Can you prove that lines \( m \) and \( n \) are parallel? If so, state the postulate or theorem you would use. (Review 3.4)

- 71.
- 72.
- 73.
Perpendicular Lines in the Coordinate Plane

**SLOPE OF PERPENDICULAR LINES**

In the activity below, you will trace a piece of paper to draw perpendicular lines on a coordinate grid. Points where grid lines cross are called *lattice points*.

**ACTIVITY**

*Investigating Slopes of Perpendicular Lines*

1. Put the corner of a piece of paper on a lattice point. Rotate the corner so each edge passes through another lattice point but neither edge is vertical. Trace the edges.
2. Find the slope of each line.
3. Multiply the slopes.
4. Repeat Steps 1–3 with the paper at a different angle.

In the activity, you may have discovered the following.

**POSTULATE**

**POSTULATE 18 Slopes of Perpendicular Lines**

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is $-1$.

Vertical and horizontal lines are perpendicular.

product of slopes $= 2 \left(-\frac{1}{2}\right) = -1$

**EXAMPLE 1**  

*Deciding Whether Lines are Perpendicular*

Find each slope.

Slope of $j_1 = \frac{3 - 1}{0 - 3} = \frac{2}{3}$

Slope of $j_2 = \frac{3 - (-3)}{0 - (-4)} = \frac{6}{4} = \frac{3}{2}$

Multiply the slopes.

The product is $\left(-\frac{2}{3}\right)\left(\frac{3}{2}\right) = -1$, so $j_1 \perp j_2$. 

---

**REAL LIFE**

Investigating Slopes of Perpendicular Lines

Put the corner of a piece of paper on a lattice point. Rotate the corner so each edge passes through another lattice point but neither edge is vertical. Trace the edges.

Find the slope of each line.

Multiply the slopes.

Repeat Steps 1–3 with the paper at a different angle.

**POSTULATE**

**POSTULATE 18 Slopes of Perpendicular Lines**

In a coordinate plane, two nonvertical lines are perpendicular if and only if the product of their slopes is $-1$.

Vertical and horizontal lines are perpendicular.

product of slopes $= 2 \left(-\frac{1}{2}\right) = -1$

**EXAMPLE 1**  

*Deciding Whether Lines are Perpendicular*

Find each slope.

Slope of $j_1 = \frac{3 - 1}{0 - 3} = \frac{2}{3}$

Slope of $j_2 = \frac{3 - (-3)}{0 - (-4)} = \frac{6}{4} = \frac{3}{2}$

Multiply the slopes.

The product is $\left(-\frac{2}{3}\right)\left(\frac{3}{2}\right) = -1$, so $j_1 \perp j_2$. 

---

**REAL LIFE**

Investigating Slopes of Perpendicular Lines

Put the corner of a piece of paper on a lattice point. Rotate the corner so each edge passes through another lattice point but neither edge is vertical. Trace the edges.

Find the slope of each line.

Multiply the slopes.

Repeat Steps 1–3 with the paper at a different angle.
**EXAMPLE 2** **Deciding Whether Lines are Perpendicular**

Decide whether \( \overrightarrow{AC} \) and \( \overrightarrow{DB} \) are perpendicular.

**SOLUTION**

Slope of \( \overrightarrow{AC} \) = \( \frac{2 - (-4)}{4 - 1} = \frac{6}{3} = 2 \)

Slope of \( \overrightarrow{DB} \) = \( \frac{2 - (-1)}{-1 - 5} = \frac{3}{-6} = -\frac{1}{2} \)

The product is \( 2 \cdot \left(-\frac{1}{2}\right) = -1 \), so \( \overrightarrow{AC} \perp \overrightarrow{DB} \).

**EXAMPLE 3** **Deciding Whether Lines are Perpendicular**

Decide whether the lines are perpendicular.

**SOLUTION**

The slope of line \( h \) is \( \frac{3}{4} \). 

The slope of line \( j \) is \( -\frac{4}{3} \).

The product is \( \left(\frac{3}{4}\right)\left(-\frac{4}{3}\right) = -1 \), so the lines are perpendicular.

**EXAMPLE 4** **Deciding Whether Lines are Perpendicular**

Decide whether the lines are perpendicular.

**SOLUTION**

**Rewrite** each equation in slope-intercept form to find the slope.

**line** \( r \): \( 4x + 5y = 2 \)

**line** \( s \): \( 5x + 4y = 3 \)

\( 4x + 5y = 2 \)

\( 5y = -4x + 2 \)

\( y = -\frac{4}{5}x + \frac{2}{5} \)

Slope = \( -\frac{4}{5} \)

\( 5x + 4y = 3 \)

\( 4y = -5x + 3 \)

\( y = -\frac{5}{4}x + \frac{3}{4} \)

Slope = \( -\frac{5}{4} \)

Multiply the slopes to see if the lines are perpendicular.

\( \left(-\frac{4}{5}\right)\left(-\frac{5}{4}\right) = 1 \)

The product of the slopes is \( -1 \). So, \( r \) and \( s \) are not perpendicular.
**GOAL 2**

**Writing Equations of Perpendicular Lines**

**EXAMPLE 5**

### Writing the Equation of a Perpendicular Line

Line \( l_1 \) has equation \( y = -2x + 1 \). Find an equation of the line \( l_2 \) that passes through \( P(4, 0) \) and is perpendicular to \( l_1 \). First you must find the slope, \( m_2 \).

\[
\begin{align*}
  m_1 \cdot m_2 &= -1 \\
  -2 \cdot m_2 &= -1 \\
  m_2 &= \frac{1}{2}
\end{align*}
\]

The product of the slopes of \( \perp \) lines is \( -1 \).

Divide both sides by \( -2 \).

Then use \( m = \frac{1}{2} \) and \( (x, y) = (4, 0) \) to find \( b \).

\[
\begin{align*}
  y &= mx + b \\
  0 &= \frac{1}{2}(4) + b \\
  -2 &= b
\end{align*}
\]

Simplify.

So, an equation of \( l_2 \) is \( y = \frac{1}{2}x - 2 \).

**RAY TRACING**

Computer illustrators use ray tracing to make accurate reflections. To figure out what to show in the mirror, the computer traces a ray of light as it reflects off the mirror. This calculation has many steps. One of the first steps is to find the equation of a line perpendicular to the mirror.

**EXAMPLE 6**

### Writing the Equation of a Perpendicular Line

The equation \( y = \frac{3}{2}x + 3 \) represents a mirror. A ray of light hits the mirror at \((-2, 0)\). What is the equation of the line \( p \) that is perpendicular to the mirror at this point?

**SOLUTION**

The mirror’s slope is \( \frac{3}{2} \), so the slope of \( p \) is \( -\frac{2}{3} \).

Use \( m = -\frac{2}{3} \) and \( (x, y) = (-2, 0) \) to find \( b \).

\[
\begin{align*}
  0 &= -\frac{2}{3}(-2) + b \\
  -\frac{4}{3} &= b
\end{align*}
\]

So, an equation for \( p \) is \( y = -\frac{2}{3}x - \frac{4}{3} \).
3.7 Perpendicular Lines in the Coordinate Plane

**Guided Practice**

1. Define slope of a line.
2. The slope of line \( m \) is \(-\frac{1}{5}\). What is the slope of a line perpendicular to \( m \)?
3. In the coordinate plane shown at the right, is \( \overrightarrow{AC} \) perpendicular to \( \overrightarrow{BD} \)? Explain.
4. Decide whether the lines with the equations \( y = 2x - 1 \) and \( y = -2x + 1 \) are perpendicular.
5. Decide whether the lines with the equations \( 5y - x = 15 \) and \( y + 5x = 2 \) are perpendicular.
6. The line \( l_1 \) has the equation \( y = 3x \). The line \( l_2 \) is perpendicular to \( l_1 \) and passes through the point \( P(0, 0) \). Write an equation of \( l_2 \).

**Practice and Applications**

**Slopes of Perpendicular Lines** The slopes of two lines are given. Are the lines perpendicular?

- **Example 1:** Exs. 7–20
- **Example 2:** Exs. 21–24, 33–37
- **Example 3:** Exs. 25–28, 47–50
- **Example 4:** Exs. 29–32
- **Example 5:** Exs. 38–41
- **Example 6:** Exs. 42–46

- 7. \( m_1 = 2 \), \( m_2 = \frac{1}{2} \)
- 8. \( m_1 = \frac{2}{3} \), \( m_2 = \frac{3}{2} \)
- 9. \( m_1 = \frac{1}{4} \), \( m_2 = -4 \)
- 10. \( m_1 = \frac{5}{7} \), \( m_2 = -\frac{7}{5} \)
- 11. \( m_1 = \frac{1}{2} \), \( m_2 = -\frac{1}{2} \)
- 12. \( m_1 = -1 \), \( m_2 = 1 \)

**Slopes of Perpendicular Lines** Lines \( j \) and \( n \) are perpendicular. The slope of line \( j \) is given. What is the slope of line \( n \)? Check your answer.

- 13. 2
- 14. 5
- 15. -3
- 16. -7
- 17. \( \frac{2}{3} \)
- 18. \( \frac{1}{5} \)
- 19. \( -\frac{1}{3} \)
- 20. \( -\frac{4}{3} \)

**Identifying Perpendicular Lines** Find the slope of \( \overrightarrow{AC} \) and \( \overrightarrow{BD} \). Decide whether \( \overrightarrow{AC} \) is perpendicular to \( \overrightarrow{BD} \).

- 21.
- 22.
- 23.
- 24.
USING ALGEBRA Decide whether lines \( k_1 \) and \( k_2 \) are perpendicular. Then graph the lines to check your answer.

25. \[ \text{line } k_1: y = 3x \]
   \[ \text{line } k_2: y = -\frac{1}{3}x - 2 \]

26. \[ \text{line } k_1: y = -\frac{4}{5}x - 2 \]
   \[ \text{line } k_2: y = \frac{1}{5}x + 4 \]

27. \[ \text{line } k_1: y = -\frac{3}{4}x + 2 \]
   \[ \text{line } k_2: y = \frac{4}{3}x + 5 \]

28. \[ \text{line } k_1: y = \frac{1}{3}x - 10 \]
   \[ \text{line } k_2: y = 3x \]

USING ALGEBRA Decide whether lines \( p_1 \) and \( p_2 \) are perpendicular.

29. \[ \text{line } p_1: 3y - 4x = 3 \]
   \[ \text{line } p_2: 4y + 3x = -12 \]

30. \[ \text{line } p_1: y - 6x = 2 \]
   \[ \text{line } p_2: 6y - x = 12 \]

31. \[ \text{line } p_1: 3y + 2x = -36 \]
   \[ \text{line } p_2: 4y - 3x = 16 \]

32. \[ \text{line } p_1: 5y + 3x = -15 \]
   \[ \text{line } p_2: 3y - 5x = -33 \]

LINE RELATIONSHIPS Find the slope of each line. Identify any parallel or perpendicular lines.

33.

34.

35.

36.

37. NEEDLEPOINT To check whether two stitched lines make a right angle, you can count the squares. For example, the lines at the right are perpendicular because one goes up 8 as it goes over 4, and the other goes over 8 as it goes down 4. Why does this mean the lines are perpendicular?

WRITING EQUATIONS Line \( j \) is perpendicular to the line with the given equation and line \( j \) passes through \( P \). Write an equation of line \( j \).

38. \[ y = \frac{1}{2}x - 1, P(0, 3) \]
39. \[ y = \frac{5}{3}x + 2, P(5, 1) \]
40. \[ y = -4x - 3, P(-2, 2) \]
41. \[ 3y + 4x = 12, P(-3, -4) \]
**Writing Equations** The line with the given equation is perpendicular to line \( j \) at point \( R \). Write an equation of line \( j \).

42. \( y = -\frac{3}{4}x + 6, R(8, 0) \)
43. \( y = \frac{1}{7}x - 11, R(7, -10) \)
44. \( y = 3x + 5, R(-3, -4) \)
45. \( y = -\frac{2}{5}x - 3, R(5, -5) \)

46. **Sculpture** Helaman Ferguson designs sculptures on a computer. The computer is connected to his stone drill and tells how far he should drill at any given point. The distance from the drill tip to the desired surface of the sculpture is calculated along a line perpendicular to the sculpture.

Suppose the drill tip is at \((-1, -1)\) and the equation \( y = \frac{1}{4}x + 3 \) represents the surface of the sculpture. Write an equation of the line that passes through the drill tip and is perpendicular to the sculpture.

**Line Relationships** Decide whether the lines with the given equations are parallel, perpendicular, or neither.

47. \( y = -2x - 1 \)  
48. \( y = -\frac{1}{2}x + 3 \)  
49. \( y = -3x + 1 \)  
50. \( y = 4x + 10 \)  

51. **Multi-Step Problem** Use the diagram at the right.

a. Is \( l_1 \parallel l_2 \)? How do you know?

b. Is \( l_2 \perp n \)? How do you know?

c. **Writing** Describe two ways to prove that \( l_1 \perp n \).

**Distance to a Line** In Exercises 52–54, use the following information.

The distance from a point to a line is defined to be the length of the perpendicular segment from the point to the line. In the diagram at the right, the distance \( d \) between point \( P \) and line \( l \) is given by \( QP \).

52. Find an equation of \( \overrightarrow{QP} \).

53. Solve a system of equations to find the coordinates of point \( Q \), the intersection of the two lines.

54. Use the Distance Formula to find \( QP \).
**Mixed Review**

**Angle Measures** Use the diagram to complete the statement.
(Review 2.6 for 4.1)

55. If \( m\angle 5 = 38^\circ \), then \( m\angle 8 = \_\_\_\_. \)
56. If \( m\angle 3 = 36^\circ \), then \( m\angle 4 = \_\_\_\_. \)
57. If \( \angle 8 \cong \angle 4 \) and \( m\angle 2 = 145^\circ \), then \( m\angle 7 = \_\_\_\_. \)
58. If \( m\angle 1 = 38^\circ \) and \( \angle 3 \cong \angle 5 \), then \( m\angle 6 = \_\_\_. \)

**Identifying Angles** Use the diagram to complete the statement.
(Review 3.1 for 4.1)

59. \( \angle 3 \) and \( \_\_\_\_\_\_ \) are consecutive interior angles.
60. \( \angle 1 \) and \( \_\_\_\_\_\_ \) are alternate exterior angles.
61. \( \angle 4 \) and \( \_\_\_\_\_\_ \) are alternate interior angles.
62. \( \angle 1 \) and \( \_\_\_\_\_\_ \) are corresponding angles.
63. Writing Describe the three types of proofs you have learned so far.
(Review 3.2)

**Quiz 3**

Self-Test for Lessons 3.6 and 3.7

Find the slope of \( \overrightarrow{AB} \). (Lesson 3.6)

1. \( A(1, 2), B(5, 8) \) 
2. \( A(2, -3), B(-1, 5) \)

Write an equation of line \( j_2 \) that passes through point \( P \) and is parallel to line \( j_1 \). (Lesson 3.6)

3. \( \text{line } j_1: y = 3x - 2 \) 
   \( P(0, 2) \)
4. \( \text{line } j_1: y = \frac{1}{2}x + 1 \) 
   \( P(2, -4) \)

Decide whether \( k_1 \) and \( k_2 \) are perpendicular. (Lesson 3.7)

5. \( \text{line } k_1: y = 2x - 1 \) 
   \( \text{line } k_2: y = \frac{1}{2}x + 2 \)
6. \( \text{line } k_1: y = -3x = -2 \) 
   \( \text{line } k_2: 3y = x = 12 \)

7. **Angle of Repose** When a granular substance is poured into a pile, the slope of the pile depends only on the substance. For example, when barley is poured into piles, every pile has the same slope. A pile of barley that is 5 feet tall would be about 10 feet wide. What is the slope of a pile of barley? (Lesson 3.6)
Chapter Summary

**WHAT did you learn?**

- Identify relationships between lines. (3.1)
- Identify angles formed by coplanar lines intersected by a transversal. (3.1)
- Prove and use results about perpendicular lines. (3.2)
- Write flow proofs and paragraph proofs. (3.2)
- Prove and use results about parallel lines and transversals. (3.3)
- Prove that lines are parallel. (3.4)
- Use properties of parallel lines. (3.4, 3.5)
- Use slope to decide whether lines in a coordinate plane are parallel. (3.6)
- Write an equation of a line parallel to a given line in a coordinate plane. (3.6)
- Use slope to decide whether lines in a coordinate plane are perpendicular. (3.7)
- Write an equation of a line perpendicular to a given line. (3.7)

**WHY did you learn it?**

- Describe lines and planes in real-life objects, such as escalators. (p. 133)
- Lay the foundation for work with angles and proof.
- Solve real-life problems, such as deciding how many angles of a window frame to measure. (p. 141)
- Learn to write and use different types of proof.
- Understand the world around you, such as how rainbows are formed. (p. 148)
- Solve real-life problems, such as predicting paths of sailboats. (p. 152)
- Analyze light passing through glass. (p. 163)
- Use coordinate geometry to show that two segments are parallel. (p. 170)
- Prepare to write coordinate proofs.
- Solve real-life problems, such as deciding whether two stitched lines form a right angle. (p. 176)
- Find the distance from a point to a line. (p. 177)

**How does Chapter 3 fit into the BIGGER PICTURE of geometry?**

In this chapter, you learned about properties of perpendicular and parallel lines. You also learned to write flow proofs and learned some important skills related to coordinate geometry. This work will prepare you to reach conclusions about triangles and other figures and to solve real-life problems in areas such as carpentry, engineering, and physics.

**STUDY STRATEGY**

**How did your study questions help you learn?**

The study questions you wrote, following the study strategy on page 128, may resemble this one.

1. If two lines do not intersect, can you conclude they are parallel?
2. What is the slope of a line perpendicular to $2x - 3y = 6$?
3. If a transversal intersects two parallel lines, which angles are supplementary?
Chapter 3

Chapter Review

VOCABULARY

- parallel lines, p. 129
- skew lines, p. 129
- parallel planes, p. 129
- transversal, p. 131
- corresponding angles, p. 131
- alternate interior angles, p. 131
- alternate exterior angles, p. 131
- consecutive interior angles, p. 131
- same side interior angles, p. 131
- flow proof, p. 136

3.1 LINES AND ANGLES

EXAMPLES

In the figure, \( j \parallel k \), \( h \) is a transversal, and \( h \perp k \).

\( \angle 1 \) and \( \angle 5 \) are corresponding angles.
\( \angle 3 \) and \( \angle 6 \) are alternate interior angles.
\( \angle 1 \) and \( \angle 8 \) are alternate exterior angles.
\( \angle 4 \) and \( \angle 6 \) are consecutive interior angles.

Complete the statement. Use the figure above.

1. \( \angle 2 \) and \( \angle 7 \) are \( \_\_\_\_\_\_\_\_ \) angles.
2. \( \angle 4 \) and \( \angle 5 \) are \( \_\_\_\_\_\_\_\_ \) angles.

Use the figure at the right.

3. Name a line parallel to \( \overline{DH} \).
4. Name a line perpendicular to \( \overrightarrow{AE} \).
5. Name a line skew to \( \overrightarrow{FD} \).

3.2 PROOF AND PERPENDICULAR LINES

EXAMPLE

\( \angle 1 \) and \( \angle 2 \) are complements.

\( \overline{GH} \perp \overline{GJ} \)

\( \angle 1 \) and \( \angle 2 \) are complements.

\( m\angle 1 + m\angle 2 = 90^\circ \)

\( \_\_\_\_\_\_\_\_ \)

\( m\angle HGJ = 90^\circ \)

\( \_\_\_\_\_\_\_\_ \)

\( \angle HGJ \) is a right \( \angle \).

\( \overline{GH} \perp \overline{GJ} \)

6. Copy the flow proof and add a reason for each statement.
3.3 PARALLEL LINES AND TRANSVERSALS

EXAMPLE In the diagram, \( \angle 1 = 75^\circ \).
By the Alternate Exterior Angles Theorem,
\( \angle 8 = \angle 1 = 75^\circ \). Because \( \angle 8 \) and \( \angle 7 \) are a linear pair,
\( \angle 8 + \angle 7 = 180^\circ \).
So, \( \angle 7 = 180^\circ - 75^\circ = 105^\circ \).

7. Find the measures of the other five angles in the diagram above.

Find the value of \( x \). Explain your reasoning.

8. \( \angle 7 = 62^\circ \)
9. \( \angle 8 = 92^\circ \)
10. \( \angle 9 = \frac{44 - 3x}{25^\circ} \)

3.4 PROVING LINES ARE PARALLEL

EXAMPLE GIVEN \( \angle 3 = 125^\circ, \angle 6 = 125^\circ \)
PROVE \( \ell \parallel m \)
Plan for Proof: \( \angle 3 = 125^\circ = \angle 6 \), so \( \angle 3 \equiv \angle 6 \).
So, \( \ell \parallel m \) by the Alternate Exterior Angles Converse.

Use the diagram above to write a proof.

11. GIVEN \( \angle 4 = 60^\circ, \angle 7 = 120^\circ \)
PROVE \( \ell \parallel m \)
12. GIVEN \( \angle 1 \) and \( \angle 7 \) are supplementary.
PROVE \( \ell \parallel m \)

3.5 USING PROPERTIES OF PARALLEL LINES

EXAMPLE In the diagram, \( \ell \perp t, m \perp t, \) and \( m \parallel n \).
Because \( \ell \) and \( m \) are coplanar and perpendicular to the same line, \( \ell \parallel m \). Then, because \( \ell \parallel m \) and \( m \parallel n, \ell \parallel n \).

Which lines must be parallel? Explain.

13. \( \angle 1 \) and \( \angle 2 \) are right angles.
14. \( \angle 3 \equiv \angle 6 \)
15. \( \angle 3 \) and \( \angle 4 \) are supplements.
16. \( \angle 1 \equiv \angle 2, \angle 3 \equiv \angle 5 \)
3.6 PARALLEL LINES IN THE COORDINATE PLANE

**EXAMPLE**

The slope of line \( \ell_1 \) is \( \frac{2 - 0}{1 - 0} = 2 \).

The slope of line \( \ell_2 \) is \( \frac{3 - (-1)}{5 - 3} = \frac{4}{2} = 2 \).

The slopes are the same, so \( \ell_1 \parallel \ell_2 \).

To write an equation for \( \ell_2 \), substitute \((x, y) = (5, 3)\) and \( m = 2 \) into the slope-intercept form.

\[
y = mx + b \\
3 = (2)(5) + b \\
-7 = b
\]

So, an equation for \( \ell_2 \) is \( y = 2x - 7 \).

Find the slope of each line. Are the lines parallel?

17. 18. 19.

20. Find an equation of the line that is parallel to the line with equation \( y = -2x + 5 \) and passes through the point \((-1, -4)\).

3.7 PERPENDICULAR LINES IN THE COORDINATE PLANE

**EXAMPLE**

The slope of line \( j \) is 3. The slope of line \( k \) is \( -\frac{1}{3} \).

\[
3\left(-\frac{1}{3}\right) = -1, \text{ so } j \perp k.
\]

In Exercises 21–23, decide whether lines \( p_1 \) and \( p_2 \) are perpendicular.

21. Lines \( p_1 \) and \( p_2 \) in the diagram.

22. \( p_1: y = \frac{3}{5}x + 2; \quad p_2: y = \frac{5}{3}x - 1 \)

23. \( p_1: 2y - x = 2; \quad p_2: y + 2x = 4 \)

24. Line \( \ell_1 \) has equation \( y = -3x + 5 \). Write an equation of line \( \ell_2 \) which is perpendicular to \( \ell_1 \) and passes through \((-3, 6)\).
In Exercises 1–6, identify the relationship between the angles in the diagram at the right.

1. \( \angle 1 \) and \( \angle 2 \)
2. \( \angle 1 \) and \( \angle 4 \)
3. \( \angle 2 \) and \( \angle 3 \)
4. \( \angle 1 \) and \( \angle 5 \)
5. \( \angle 4 \) and \( \angle 2 \)
6. \( \angle 5 \) and \( \angle 6 \)

7. Write a flow proof.
   **GIVEN** \( m\angle 1 = m\angle 3 = 37^\circ, \overline{BA} \perp \overline{BC} \)
   **PROVE** \( m\angle 2 = 16^\circ \)

8. If \( l \parallel m \), which angles are supplementary to \( \angle 1 \)?

Use the given information and the diagram at the right to determine which lines must be parallel.

9. \( \angle 1 \equiv \angle 2 \)
10. \( \angle 3 \) and \( \angle 4 \) are right angles.
11. \( \angle 1 \equiv \angle 5; \angle 5 \) and \( \angle 7 \) are supplementary.

In Exercises 12 and 13, write an equation of the line described.

12. The line parallel to \( y = -\frac{1}{3}x + 5 \) and with a \( y \)-intercept of 1
13. The line perpendicular to \( y = -2x + 4 \) and that passes through the point \((-1, 2)\)

14. **Writing** Describe a real-life object that has edges that are straight lines. Are any of the lines skew? If so, describe a pair.

15. A carpenter wants to cut two boards to fit snugly together. The carpenter’s squares are aligned along \( EF \), as shown. Are \( AB \) and \( CD \) parallel? State the theorem that justifies your answer.

16. Use the diagram to write a proof.
   **GIVEN** \( \angle 1 \equiv \angle 2, \angle 3 \equiv \angle 4 \)
   **PROVE** \( n \parallel p \)
1. Describe a pattern in the sequence 10, 12, 15, 19, 24, . . . Predict the next number. (1.1)

In the diagram at the right, \( \overrightarrow{AB} \), \( \overrightarrow{AC} \), and \( \overrightarrow{BC} \) are in plane \( M \).

2. Name a point that is collinear with points \( A \) and \( D \). (1.2)
3. Name a line skew to \( \overrightarrow{BC} \). (3.1)
4. Name the ray that is opposite to \( \overrightarrow{GC} \). (1.2)
5. How many planes contain \( A \), \( B \), and \( C \)? Explain. (2.1)

\( \overrightarrow{MN} \) has endpoints \( M(7, -5) \) and \( N(-3, -1) \).

6. Find the length of \( \overrightarrow{MN} \). (1.3)
7. If \( N \) is the midpoint of \( \overrightarrow{MP} \), find the coordinates of point \( P \). (1.5)

In a coordinate plane, plot the points and sketch \( \angle ABC \). Classify the angle as acute, right, or obtuse. (1.4)

8. \( A(-6, 6), B(-2, 2), C(4, 2) \)
9. \( A(2, 1), B(4, 7), C(10, 5) \)
10. \( A(2, 5), B(2, -2), C(5, 4) \)

Find the values of \( x \) and \( y \). (1.5, 1.6, 3.2)

11. \( A \quad 7y - 2 \quad B \quad x + 3 \quad C \quad 9 \quad D \quad 30 \)
12. \( 10(x + y)^\circ \quad 15x^\circ \quad 3x^\circ \)
13. \( (3x + 6)^\circ \quad y^\circ \quad (2x + 4)^\circ \)

In Exercises 14 and 15, find the area of each figure. (1.7)

14. Square with a perimeter of 40 cm
15. Triangle defined by \( R(0, 0), S(6, 8), \) and \( T(10, 0) \)
16. Construct two perpendicular lines. Bisect one of the angles formed. (1.5, 3.1)
17. Rewrite the following statement in if-then form: The measure of a straight angle is 180°. Then write the inverse, converse, and contrapositive of the conditional statement. (2.1)

In Exercises 18–21, find a counterexample that shows the statement is false.

18. If a line intersects two other lines, then all three lines are coplanar. (2.1)
19. Two lines are perpendicular if they intersect. (2.2)
20. If \( AB + BC = AC \), then \( B \) is the midpoint of \( AC \). (2.5)
21. If \( \angle 1 \) and \( \angle 2 \) are supplementary, then \( \angle 1 \) and \( \angle 2 \) form a linear pair. (2.6)
22. Solve the equation \( 3(s - 2) = 15 \) and write a reason for each step. (2.4)
23. Draw a diagram of intersecting lines \( j \) and \( k \). Label each angle with a number. Use the Linear Pair Postulate and the Vertical Angles Theorem to write true statements about the angles formed by the intersecting lines. (2.6)
Let $p$ represent “$x = 0$” and let $q$ represent “$x + x = x$.”

24. Write the biconditional $p \iff q$ in words. Decide whether the biconditional is true. (2.2, 2.3)

25. If the statement $p \to q$ is true and $p$ is true, does it follow that $q$ is true? Explain. (2.3)

Use the diagram at the right.

26. Name four pairs of corresponding angles. (3.1)

27. If $\overrightarrow{AC} \parallel \overrightarrow{DE}$ and $m \angle 2 = 55^\circ$, find $m \angle 6$. (3.3)

28. If $\overrightarrow{BD} \parallel \overrightarrow{CF}$ and $m \angle 3 = 140^\circ$, find $m \angle 4$. (3.3)

29. Which lines must be parallel if $m \angle 3 + m \angle 6 = 180^\circ$? Explain. (3.4)

30. **Proof** Write a proof.

   **Given** $\angle 6 \equiv \angle 9$

   **Prove** $\angle 3$ and $\angle 4$ are supplements. (3.3, 3.4)

In Exercises 31–33, use $A(2, 10)$, $B(22, -5)$, $C(-1, 6)$, and $D(25, -1)$.

31. Show that $\overrightarrow{AD}$ is parallel to $\overrightarrow{BC}$. (3.6)

32. Use slopes to show that $\angle BAC$ is a right angle. (3.7)

33. Write an equation for a line through the point $C$ and parallel to $\overrightarrow{AB}$. (3.6)

34. **Running Track** The inside of the running track in the diagram is formed by a rectangle and two half circles. Find the distance, to the nearest yard, around the inside of the track. Then find the area enclosed by the track. (Use $\pi \approx 3.14$.) (1.7)

35. **Photo Enlargement** A photographer took a 4-inch-by-6-inch photo and enlarged each side to 150% of the original size.

   a. Find the dimensions of the enlarged photo.

   b. Describe the relationship between $\angle 1$ and $\angle 3$. (1.6, 3.2)

   c. Make an accurate diagram of the original photo and the enlargement, as shown. Draw $\overrightarrow{DB}$ and $\overrightarrow{BQ}$. Make a conjecture about the relationship between $\angle 1$ and $\angle 2$. Measure the angles in your diagram to test your conjecture. (1.1, 1.6)

36. **Construction** Two posts support a raised deck. The posts have two parallel braces, as shown.

   a. If $m \angle 1 = 35^\circ$, find $m \angle 2$. (3.3)

   b. If $m \angle 3 = 40^\circ$, what other angle has a measure of 40°? (3.3)

   c. Each post is perpendicular to the deck. Explain how this can be used to show that the posts are parallel to each other. (3.5)